

# Performance Attribution from Bacon

Matthieu Lestel

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## Abstract

This vignette gives a brief overview of the functions developed in Bacon(2008) to evaluate the performance and risk of portfolios that are included in **PerformanceAnalytics** and how to use them. There are some tables at the end which give a quick overview of similar functions. The page number next to each function is the location of the function in Bacon(2008)

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# 1 Risk Measure

## 1.1 Mean absolute deviation (p.62)

To calculate Mean absolute deviation we take the sum of the absolute value of the difference between the returns and the mean of the returns and we divide it by the number of returns.

$$MeanAbsoluteDeviation = \frac{\sum_{i=1}^n |r_i - \bar{r}|}{n}$$

where  $n$ s the number of observations of the entire series,  $r_i$ s the return in month  $i$  and  $\bar{r}$ s the mean return

```
> data(portfolio_bacon)
> print(MeanAbsoluteDeviation(portfolio_bacon[,1])) #expected 0.0310

[1] 0.03108333
```

## 1.2 Frequency (p.64)

Gives the period of the return distribution (ie 12 if monthly return, 4 if quarterly return)

```
> data(portfolio_bacon)
> print(Frequency(portfolio_bacon[,1])) #expected 12

[1] 12
```

### 1.3 Sharpe Ratio (p.64)

The Sharpe ratio is simply the return per unit of risk (represented by variability). In the classic case, the unit of risk is the standard deviation of the returns.

$$\frac{\overline{(R_a - R_f)}}{\sqrt{\sigma_{(R_a - R_f)}}}$$

```
> data(managers)
> SharpeRatio(managers[,1,drop=FALSE], Rf=.035/12, FUN="StdDev")
```

HAM1

```
StdDev Sharpe (Rf=0.3%, p=95%): 0.3201889
```

### 1.4 Risk-adjusted return: MSquared (p.67)

$M^2$  is a risk adjusted return useful to judge the size of relative performance between different portfolios. With it you can compare portfolios with different levels of risk.

$$M^2 = r_P + SR * (\sigma_M - \sigma_P) = (r_P - r_F) * \frac{\sigma_M}{\sigma_P} + r_F$$

where  $r_P$  is the portfolio return annualized,  $\sigma_M$  is the market risk and  $\sigma_P$  is the portfolio risk

```
> data(portfolio_bacon)
> print(MSquared(portfolio_bacon[,1], portfolio_bacon[,2])) #expected 0.1068
```

```
portfolio.monthly.return....
portfolio.monthly.return.... 0.1068296
```

### 1.5 MSquared Excess (p.68)

$M^2_{excess}$  is the quantity above the standard  $M^2$ . There is a geometric excess return which is better for Bacon and an arithmetic excess return

$$M^2_{excess}(geometric) = \frac{1 + M^2}{1 + b} - 1$$

$$M^2_{excess}(arithmetic) = M^2 - b$$

where  $M^2$  is MSquared and  $b$  is the benchmark annualised return.

```
> data(portfolio_bacon)
> print(MSquaredExcess(portfolio_bacon[,1], portfolio_bacon[,2])) #expected -0.009
```

```

                                portfolio.monthly.return....
portfolio.monthly.return....      -0.009976721

> print(MSquaredExcess(portfolio_bacon[,1], portfolio_bacon[,2], Method="arithmetic")

                                portfolio.monthly.return....
portfolio.monthly.return....      -0.01115381

```

## 2 Regression analysis

### 2.1 Regression equation (p.71)

$$r_P = \alpha + \beta * b + \epsilon$$

### 2.2 Regression alpha (p.71)

"Alpha" purports to be a measure of a manager's skill by measuring the portion of the managers returns that are not attributable to "Beta", or the portion of performance attributable to a benchmark.

```

> data(managers)
> print(CAPM.alpha(managers[,1,drop=FALSE], managers[,8,drop=FALSE], Rf=.035/12))

[1] 0.005960609

```

### 2.3 Regression beta (p.71)

CAPM Beta is the beta of an asset to the variance and covariance of an initial portfolio. Used to determine diversification potential.

```

> data(managers)
> CAPM.beta(managers[, "HAM2", drop=FALSE], managers[, "SP500 TR", drop=FALSE], Rf=.035/12)

[1] 0.3383942

```

## 2.4 Regression epsilon (p.71)

The regression epsilon is an error term measuring the vertical distance between the return predicted by the equation and the real result.

$$\epsilon_r = r_p - \alpha_r - \beta_r * b$$

where  $\alpha_r$ s the regression alpha,  $\beta_r$ s the regression beta,  $r_p$ s the portfolio return and b is the benchmark return

```
> data(managers)
> print(CAPM.epsilon(portfolio_bacon[,1], portfolio_bacon[,2])) #expected -0.013
[1] -0.01313932
```

## 2.5 Jensen's alpha (p.72)

The Jensen's alpha is the intercept of the regression equation in the Capital Asset Pricing Model and is in effect the excess return adjusted for systematic risk.

$$\alpha = r_p - r_f - \beta_p * (b - r_f)$$

where  $r_f$  is the risk free rate,  $\beta_r$  is the regression beta,  $r_p$  is the portfolio return and b is the benchmark return

```
> data(portfolio_bacon)
> print(CAPM.jensenAlpha(portfolio_bacon[,1], portfolio_bacon[,2])) #expected -0.014
[1] -0.01416944
```

## 2.6 Systematic Risk (p.75)

Systematic risk as defined by Bacon(2008) is the product of beta by market risk. Be careful ! It's not the same definition as the one given by Michael Jensen. Market risk is the standard deviation of the benchmark. The systematic risk is annualized

$$\sigma_s = \beta * \sigma_m$$

where  $\sigma_s$  is the systematic risk,  $\beta$  is the regression beta, and  $\sigma_m$ s the market risk

```
> data(portfolio_bacon)
> print(SystematicRisk(portfolio_bacon[,1], portfolio_bacon[,2])) #expected 0.013
[1] 0.132806
```

## 2.7 Specific Risk (p.75)

Specific risk is the standard deviation of the error term in the regression equation.

```
> data(portfolio_bacon)
> print(SpecificRisk(portfolio_bacon[,1], portfolio_bacon[,2])) #expected 0.0329
[1] 0.03293109
```

## 2.8 Total Risk (p.75)

The square of total risk is the sum of the square of systematic risk and the square of specific risk. Specific risk is the standard deviation of the error term in the regression equation. Both terms are annualized to calculate total risk.

$$TotalRisk = \sqrt{SystematicRisk^2 + SpecificRisk^2}$$

```
> data(portfolio_bacon)
> print(TotalRisk(portfolio_bacon[,1], portfolio_bacon[,2])) #expected 0.0134
[1] 0.136828
```

## 2.9 Treynor ratio (p.75)

The Treynor ratio is similar to the Sharpe Ratio, except it uses beta as the volatility measure (to divide the investment's excess return over the beta).

$$TreynorRatio = \frac{\overline{(R_a - R_f)}}{\beta_{a,b}}$$

```
> data(managers)
> print(round(TreynorRatio(managers[,1,drop=FALSE], managers[,8,drop=FALSE], Rf=.05)))
[1] 0.2528
```

## 2.10 Modified Treynor ratio (p.77)

To calculate modified Treynor ratio, we divide the numerator by the systematic risk instead of the beta.

```
> data(portfolio_bacon)
> print(TreynorRatio(portfolio_bacon[,1], portfolio_bacon[,2], modified = TRUE)) #
[1] 0.7806747
```

## 2.11 Appraisal ratio (or Treynor-Black ratio) (p.77)

Appraisal ratio is the Jensen's alpha adjusted for specific risk. The numerator is divided by specific risk instead of total risk.

$$\text{Appraisalratio} = \frac{\alpha}{\sigma_{\epsilon}}$$

where  $\alpha$  is the Jensen's alpha,  $\sigma_{\epsilon}$  is the specific risk

```
> data(portfolio_bacon)
> print(AppraisalRatio(portfolio_bacon[,1], portfolio_bacon[,2], method="appraisal")
[1] -0.4302756
```

## 2.12 Modified Jensen (p.77)

Modified Jensen's alpha is Jensen's alpha divided by beta.

$$\text{ModifiedJensen'salpha} = \frac{\alpha}{\beta}$$

where  $\alpha$  is the Jensen's alpha

```
> data(portfolio_bacon)
> print(AppraisalRatio(portfolio_bacon[,1], portfolio_bacon[,2], method="modified")
[1] -0.01418576
```

## 2.13 Fama decomposition (p.77)

Fama beta is a beta used to calculate the loss of diversification. It is made so that the systematic risk is equivalent to the total portfolio risk.

$$\beta_F = \frac{\sigma_P}{\sigma_M}$$

where  $\sigma_P$  is the portfolio standard deviation and  $\sigma_M$  is the market risk

```
> data(portfolio_bacon)
> print(FamaBeta(portfolio_bacon[,1], portfolio_bacon[,2])) #expected 1.03
portfolio.monthly.return....
portfolio.monthly.return.... 1.030395
```



## 2.14 Selectivity (p.78)

Selectivity is the same as Jensen's alpha

$$Selectivity = r_p - r_f - \beta_p * (b - r_f)$$

where  $r_f$  is the risk free rate,  $\beta_r$  is the regression beta,  $r_p$  is the portfolio return and  $b$  is the benchmark return

```
> data(portfolio_bacon)
> print(Selectivity(portfolio_bacon[,1], portfolio_bacon[,2])) #expected -0.0141
[1] -0.01416944
```

## 2.15 Net selectivity (p.78)

Net selectivity is the remaining selectivity after deducting the amount of return require to justify not being fully diversified

If net selectivity is negative the portfolio manager has not justified the loss of diversification

$$Netselectivity = \alpha - d$$

where  $\alpha$  is the selectivity and  $d$  is the diversification

```
> data(portfolio_bacon)
> print(NetSelectivity(portfolio_bacon[,1], portfolio_bacon[,2])) #expected -0.017
portfolio.monthly.return....
portfolio.monthly.return.... -0.0178912
```

# 3 Relative Risk

## 3.1 Tracking error (p.78)

A measure of the unexplained portion of performance relative to a benchmark.

Tracking error is calculated by taking the square root of the average of the squared deviations between the investment's returns and the benchmark's returns, then multiplying the result by the square root of the scale of the returns.

$$TrackingError = \sqrt{\sum \frac{(R_a - R_b)^2}{len(R_a)\sqrt{scale}}}$$

```
> data(managers)
> TrackingError(managers[,1,drop=FALSE], managers[,8,drop=FALSE])
[1] 0.1131667
```

## 3.2 Information ratio (p.80)

The Active Premium divided by the Tracking Error.

InformationRatio = ActivePremium/TrackingError

This relates the degree to which an investment has beaten the benchmark to the consistency with which the investment has beaten the benchmark.

```
> data(managers)
> InformationRatio(managers[, "HAM1", drop=FALSE], managers[, "SP500 TR", drop=FALSE])
[1] 0.3604125
```

## 4 Return Distribution

### 4.1 Skewness (p.83)

measures the deformation from a normal deformation

$$Skewness = \frac{1}{n} * \sum_{i=1}^n \left( \frac{r_i - \bar{r}}{\sigma_P} \right)^3$$

where  $n$  is the number of return,  $\bar{r}$  is the mean of the return distribution,  $\sigma_P$  is its standard deviation and  $\sigma_{S_P}$  its sample standard deviation

```
> data(managers)
> skewness(managers)
           HAM1    HAM2    HAM3    HAM4    HAM5    HAM6
Skewness -0.6588445  1.45804  0.7908285 -0.4310631  0.07380869 -0.2799993
           EDHEC LS EQ    SP500 TR    US 10Y TR    US 3m TR
Skewness  0.01773013 -0.5531032 -0.4048722 -0.328171
```

### 4.2 Sample skewness (p.84)

$$SampleSkewness = \frac{n}{(n-1) * (n-2)} * \sum_{i=1}^n \left( \frac{r_i - \bar{r}}{\sigma_{S_P}} \right)^3$$

where  $n$  is the number of return,  $\bar{r}$  is the mean of the return distribution,  $\sigma_P$  is its standard deviation and  $\sigma_{S_P}$  is its sample standard deviation

```
> data(portfolio_bacon)
> print(skewness(portfolio_bacon[,1], method="sample")) #expected -0.09
[1] -0.09398414
```

### 4.3 Kurtosis (p.84)

Kurtosis measures the weight or returns in the tails of the distribution relative to standard deviation.

$$Kurtosis(moment) = \frac{1}{n} * \sum_{i=1}^n \left( \frac{r_i - \bar{r}}{\sigma_P} \right)^4$$

where  $n$  is the number of return,  $\bar{r}$  is the mean of the return distribution,  $\sigma_P$  is its standard deviation and  $\sigma_{S_P}$  is its sample standard deviation

```
> data(portfolio_bacon)
> print(kurtosis(portfolio_bacon[,1], method="moment")) #expected 2.43

[1] 2.432454
```

### 4.4 Excess kurtosis (p.85)

$$ExcessKurtosis = \frac{1}{n} * \sum_{i=1}^n \left( \frac{r_i - \bar{r}}{\sigma_P} \right)^4 - 3$$

where  $n$  is the number of return,  $\bar{r}$  is the mean of the return distribution,  $\sigma_P$  is its standard deviation and  $\sigma_{S_P}$  is its sample standard deviation

```
> data(portfolio_bacon)
> print(kurtosis(portfolio_bacon[,1], method="excess")) #expected -0.57

[1] -0.5675462
```

### 4.5 Sample kurtosis (p.85)

$$Samplekurtosis = \frac{n * (n + 1)}{(n - 1) * (n - 2) * (n - 3)} * \sum_{i=1}^n \left( \frac{r_i - \bar{r}}{\sigma_{S_P}} \right)^4$$

where  $n$  is the number of return,  $\bar{r}$  is the mean of the return distribution,  $\sigma_P$  is its standard deviation and  $\sigma_{S_P}$  is its sample standard deviation

```
> data(portfolio_bacon)
> print(kurtosis(portfolio_bacon[,1], method="sample")) #expected 3.03

[1] 3.027405
```

## 4.6 Sample excess kurtosis (p.85)

$$\text{Sample excess kurtosis} = \frac{n * (n + 1)}{(n - 1) * (n - 2) * (n - 3)} * \sum_{i=1}^n \left( \frac{r_i - \bar{r}}{\sigma_{S_P}} \right)^4 - \frac{3 * (n - 1)^2}{(n - 2) * (n - 3)}$$

where  $n$  is the number of return,  $\bar{r}$  is the mean of the return distribution,  $\sigma_P$  is its standard deviation and  $\sigma_{S_P}$  is its sample standard deviation

```
> data(portfolio_bacon)
> print(kurtosis(portfolio_bacon[,1], method="sample_excess")) #expected -0.41
[1] -0.4076603
```

## 5 Drawdown

### 5.1 Pain index (p.89)

The pain index is the mean value of the drawdowns over the entire analysis period. The measure is similar to the Ulcer index except that the drawdowns are not squared. Also, it's different than the average drawdown, in that the numerator is the total number of observations rather than the number of drawdowns. Visually, the pain index is the area of the region that is enclosed by the horizontal line at zero percent and the drawdown line in the Drawdown chart.

$$\text{Pain index} = \sum_{i=1}^n \frac{|D'_i|}{n}$$

where  $n$  is the number of observations of the entire series,  $D'_i$  is the drawdown since previous peak in period  $i$

```
> data(portfolio_bacon)
> print(PainIndex(portfolio_bacon[,1])) #expected 0.04
      portfolio.monthly.return....
Pain Index                        0.0390113
```

### 5.2 Calmar ratio (p.89)

Calmar ratio is another method of creating a risk-adjusted measure for ranking investments similar to the Sharpe ratio.

```
> data(managers)
> CalmarRatio(managers[,1,drop=FALSE])
      HAM1
Calmar Ratio 0.9061697
```

### 5.3 Sterling ratio (p.89)

Sterling ratio is another method of creating a risk-adjusted measure for ranking investments similar to the Sharpe ratio.

```
> data(managers)
> SterlingRatio(managers[,1,drop=FALSE])
```

HAM1

```
Sterling Ratio (Excess = 10%) 0.5462542
```

### 5.4 Burke ratio (p.90)

To calculate Burke ratio we take the difference between the portfolio return and the risk free rate and we divide it by the square root of the sum of the square of the drawdowns.

$$BurkeRatio = \frac{r_P - r_F}{\sqrt{\sum_{t=1}^d D_t^2}}$$

where  $d$  is number of drawdowns,  $r_P$  the portfolio return,  $r_F$  is the risk free rate and  $D_t$  the  $t^{th}$  drawdown.

```
> data(portfolio_bacon)
> print(BurkeRatio(portfolio_bacon[,1])) #expected 0.74
```

```
[1] 0.7447309
```

### 5.5 Modified Burke ratio (p.91)

To calculate the modified Burke ratio we just multiply the Burke ratio by the square root of the number of datas.

$$ModifiedBurkeRatio = \frac{r_P - r_F}{\sqrt{\sum_{t=1}^d \frac{D_t^2}{n}}}$$

where  $n$  is the number of observations of the entire series,  $d$  number of drawdowns,  $r_P$  is the portfolio return,  $r_F$  is the risk free rate and  $D_t$  the  $t^{th}$  drawdown.

```
> data(portfolio_bacon)
> print(BurkeRatio(portfolio_bacon[,1], modified = TRUE)) #expected 3.65
```

```
[1] 3.648421
```

## 5.6 Martin ratio (p.91)

To calculate Martin ratio we divide the difference of the portfolio return and the risk free rate by the Ulcer index

$$\text{Martinratio} = \frac{r_P - r_F}{\sqrt{\sum_{i=1}^n \frac{D_i'^2}{n}}}$$

where  $r_P$  is the annualized portfolio return,  $r_F$  is the risk free rate,  $n$  is the number of observations of the entire series,  $D_i'$  is the drawdown since previous peak in period  $i$

```
> data(portfolio_bacon)
> print(MartinRatio(portfolio_bacon[,1])) #expected 1.70
```

```
portfolio.monthly.return....
Ulcer Index                    1.70772
```

## 5.7 Pain ratio (p.91)

To calculate Pain ratio we divide the difference of the portfolio return and the risk free rate by the Pain index

$$\text{Painratio} = \frac{r_P - r_F}{\sum_{i=1}^n \frac{|D_i'|}{n}}$$

where  $r_P$  is the annualized portfolio return,  $r_F$  is the risk free rate,  $n$  is the number of observations of the entire series,  $D_i'$  is the drawdown since previous peak in period  $i$

```
> data(portfolio_bacon)
> print(PainRatio(portfolio_bacon[,1])) #expected 2.66
```

```
portfolio.monthly.return....
Pain Index                    2.657647
```

# 6 Downside risk

## 6.1 Downside risk (p.92)

Downside deviation, similar to semi deviation, eliminates positive returns when calculating risk. Instead of using the mean return or zero, it uses the Minimum Acceptable Return as proposed by Sharpe (which may be the mean historical return or zero). It measures the

variability of underperformance below a minimum target rate. The downside variance is the square of the downside potential.

$$\text{DownsideDeviation}(R, MAR) = \delta_{MAR} = \sqrt{\sum_{t=1}^n \frac{\min[(R_t - MAR), 0]^2}{n}}$$

$$\text{DownsideVariance}(R, MAR) = \sum_{t=1}^n \frac{\min[(R_t - MAR), 0]^2}{n}$$

$$\text{DownsidePotential}(R, MAR) = \sum_{t=1}^n \frac{\min[(R_t - MAR), 0]}{n}$$

where  $n$  is either the number of observations of the entire series or the number of observations in the subset of the series falling below the MAR.

```
> data(portfolio_bacon)
> MAR = 0.5
> DownsideDeviation(portfolio_bacon[,1], MAR) #expected 0.493
[1] 0.492524
> DownsidePotential(portfolio_bacon[,1], MAR) #expected 0.491
[1] 0.491
```

## 6.2 UpsideRisk (p.92)

Upside Risk is the similar of semideviation taking the return above the Minimum Acceptable Return instead of using the mean return or zero.

$$\text{UpsideRisk}(R, MAR) = \sqrt{\sum_{t=1}^n \frac{\max[(R_t - MAR), 0]^2}{n}}$$

$$\text{UpsideVariance}(R, MAR) = \sum_{t=1}^n \frac{\max[(R_t - MAR), 0]^2}{n}$$

$$\text{UpsidePotential}(R, MAR) = \sum_{t=1}^n \frac{\max[(R_t - MAR), 0]}{n}$$

where  $n$  is either the number of observations of the entire series or the number of observations in the subset of the series falling below the MAR.

```

> data(portfolio_bacon)
> MAR = 0.005
> print(UpsideRisk(portfolio_bacon[,1], MAR, stat="risk")) #expected 0.02937
[1] 0.02937332
> print(UpsideRisk(portfolio_bacon[,1], MAR, stat="variance")) #expected 0.08628
[1] 0.0008627917
> print(UpsideRisk(portfolio_bacon[,1], MAR, stat="potential")) #expected 0.01771
[1] 0.01770833

```

### 6.3 Downside frequency (p.94)

To calculate Downside Frequency, we take the subset of returns that are less than the target (or Minimum Acceptable Returns (MAR)) returns and divide the length of this subset by the total number of returns.

$$DownsideFrequency(R, MAR) = \sum_{t=1}^n \frac{\min[(R_t - MAR), 0]}{R_t * n}$$

where  $n$  is the number of observations of the entire series

```

> data(portfolio_bacon)
> MAR = 0.005
> print(DownsideFrequency(portfolio_bacon[,1], MAR)) #expected 0.458
[1] 0.4583333

```

### 6.4 Bernardo and Ledoit ratio (p.95)

To calculate Bernardo and Ledoit ratio we take the sum of the subset of returns that are above 0 and we divide it by the opposite of the sum of the subset of returns that are below 0

$$BernardoLedoitRatio(R) = \frac{\frac{1}{n} \sum_{t=1}^n \max(R_t, 0)}{\frac{1}{n} \sum_{t=1}^n \max(-R_t, 0)}$$

where  $n$  is the number of observations of the entire series

```

> data(portfolio_bacon)
> print(BernardoLedoitRatio(portfolio_bacon[,1])) #expected 1.78
[1] 1.779783

```



## 6.5 d ratio (p.95)

The d ratio is similar to the Bernado Ledoit ratio but inverted and taking into account the frequency of positive and negative returns.

It has values between zero and infinity. It can be used to rank the performance of portfolios. The lower the d ratio the better the performance, a value of zero indicating there are no returns less than zero and a value of infinity indicating there are no returns greater than zero.

$$DRatio(R) = \frac{n_d * \sum_{t=1}^n \max(-R_t, 0)}{n_u * \sum_{t=1}^n \max(R_t, 0)}$$

where  $n$  is the number of observations of the entire series,  $n_d$  is the number of observations less than zero,  $n_u$  is the number of observations greater than zero

```
> data(portfolio_bacon)
> print(DRatio(portfolio_bacon[,1])) #expected 0.401
```

```
[1] 0.4013329
```

## 6.6 Omega-Sharpe ratio (p.95)

The Omega-Sharpe ratio is a conversion of the omega ratio to a ranking statistic in familiar form to the Sharpe ratio.

To calculate the Omega-Sharpe ration we subtract the target (or Minimum Acceptable Returns (MAR)) return from the portfolio return and we divide it by the opposite of the Downside Deviation.

$$OmegaSharpeRatio(R, MAR) = \frac{r_p - r_t}{\sum_{t=1}^n \frac{\max(r_t - r_i, 0)}{n}}$$

where  $n$  is the number of observations of the entire series

```
> data(portfolio_bacon)
> MAR = 0.005
> print(OmegaSharpeRatio(portfolio_bacon[,1], MAR)) #expected 0.29
```

```
[1] 0.2917933
```

## 6.7 Sortino ratio (p.96)

Sortino proposed an improvement on the Sharpe Ratio to better account for skill and excess performance by using only downside semivariance as the measure of risk.

$$\text{SortinoRatio} = \frac{(\overline{R_a} - MAR)}{\delta_{MAR}}$$

where  $\delta_{MAR}$  is the DownsideDeviation.

```
> data(managers)
> round(SortinoRatio(managers[, 1]), 4)
```

```

                HAM1
Sortino Ratio (MAR = 0%) 0.7649
```

## 6.8 Kappa (p.96)

Introduced by Kaplan and Knowles (2004), Kappa is a generalized downside risk-adjusted performance measure.

To calculate it, we take the difference of the mean of the distribution to the target and we divide it by the l-root of the lth lower partial moment. To calculate the lth lower partial moment we take the subset of returns below the target and we sum the differences of the target to these returns. We then return return this sum divided by the length of the whole distribution.

$$\text{Kappa}(R, MAR, l) = \frac{r_p - MAR}{\sqrt[l]{\frac{1}{n} * \sum_{t=1}^n \max(MAR - R_t, 0)^l}}$$

```
> data(portfolio_bacon)
> MAR = 0.005
> l = 2
> print(Kappa(portfolio_bacon[,1], MAR, l)) #expected 0.157
```

```
[1] 0.1566371
```

## 6.9 Upside potential ratio (p.97)

Sortino proposed an improvement on the Sharpe Ratio to better account for skill and excess performance by using only downside semivariance as the measure of risk. That measure is the Sortino ratio. This function, Upside Potential Ratio, was a further improvement, extending the measurement of only upside on the numerator, and only downside of the denominator of the ratio equation.

$$UPR = \frac{\sum_{t=1}^n (R_t - MAR)}{\delta_{MAR}}$$

where  $\delta_{MAR}$  is the DownsideDeviation.

```
> data(edhec)
> UpsidePotentialRatio(edhec[, 6], MAR=.05/12) #5 percent/yr MAR
```

```
Event Driven
Upside Potential (MAR = 0.4%)    0.5376613
```

## 6.10 Volatility skewness (p.97)

Volatility skewness is a similar measure to omega but using the second partial moment. It's the ratio of the upside variance compared to the downside variance.

$$VolatilitySkewness(R, MAR) = \frac{\sigma_U^2}{\sigma_D^2}$$

where  $\sigma_U$  is the Upside risk and  $\sigma_D$  is the Downside Risk

```
> data(portfolio_bacon)
> MAR = 0.005
> print(VolatilitySkewness(portfolio_bacon[,1], MAR, stat="volatility")) #expected
```

```
[1] 1.323046
```

## 6.11 Variability skewness (p.98)

Variability skewness is the ratio of the upside risk compared to the downside risk.

$$VariabilitySkewness(R, MAR) = \frac{\sigma_U}{\sigma_D}$$

where  $\sigma_U$  is the Upside risk and  $\sigma_D$  is the Downside Risk

```
> data(portfolio_bacon)
> MAR = 0.005
> print(VolatilitySkewness(portfolio_bacon[,1], MAR, stat="variability")) #expected
```

```
[1] 1.150238
```

## 6.12 Adjusted Sharpe ratio (p.99)

Adjusted Sharpe ratio was introduced by Pezier and White (2006) to adjust for skewness and kurtosis by incorporating a penalty factor for negative skewness and excess kurtosis.

$$AdjustedSharpeRatio = SR * [1 + (\frac{S}{6}) * SR - (\frac{K - 3}{24}) * SR^2]$$

where  $SR$  is the sharpe ratio with data annualized,  $S$  is the skewness and  $K$ s the kurtosis

```
> data(portfolio_bacon)
> print(AdjustedSharpeRatio(portfolio_bacon[,1])) #expected 0.81
[1] 0.8084219
```

## 6.13 Skewness-kurtosis ratio (p.99)

Skewness-Kurtosis ratio is the division of Skewness by Kurtosis.' It is used in conjunction with the Sharpe ratio to rank portfolios. The higher the rate the better.

$$SkewnessKurtosisRatio(R, MAR) = \frac{S}{K}$$

where  $S$  is the skewness and  $K$  is the Kurtosis

```
> data(portfolio_bacon)
> print(SkewnessKurtosisRatio(portfolio_bacon[,1])) #expected -0.034
[1] -0.03394204
```

## 6.14 Prospect ratio (p.100)

Prospect ratio is a ratio used to penalise loss since most people feel loss greater than gain

$$ProspectRatio(R) = \frac{\frac{1}{n} * \sum_{i=1}^n (Max(r_i, 0) + 2.25 * Min(r_i, 0) - MAR)}{\sigma_D}$$

where  $n$  is the number of observations of the entire series, MAR is the minimum acceptable return and  $\sigma_D$ s the downside risk

```
> data(portfolio_bacon)
> MAR = 0.05
> print(ProspectRatio(portfolio_bacon[,1], MAR)) #expected -0.134
[1] -0.1347065
```

## 7 Return adjusted for downside risk

### 7.1 M Squared for Sortino (p.102)

M squared for Sortino is a  $M^2$  calculated for Downside risk instead of Total Risk

$$M_S^2 = r_P + \text{Sortinoratio} * (\sigma_{DM} - \sigma_D)$$

where  $M_S^2$  is MSquared for Sortino,  $r_P$  is the annualised portfolio return,  $\sigma_{DM}$  is the benchmark annualised downside risk and  $D$  is the portfolio annualised downside risk

```
> data(portfolio_bacon)
> MAR = 0.005
> print(M2Sortino(portfolio_bacon[,1], portfolio_bacon[,2], MAR)) #expected 0.1035

                    portfolio.monthly.return....
Sortino Ratio (MAR = 0.5%)                      0.1034799
```

### 7.2 Omega excess return (p.103)

Omega excess return is another form of downside risk-adjusted return. It is calculated by multiplying the downside variance of the style benchmark by 3 times the style beta.

$$\omega = r_P - 3 * \beta_S * \sigma_{MD}^2$$

where  $\omega$  is omega excess return,  $\beta_S$  is style beta,  $\sigma_D$  is the portfolio annualised downside risk and  $\sigma_{MD}$  is the benchmark annualised downside risk.

```
> data(portfolio_bacon)
> MAR = 0.005
> print(OmegaExcessReturn(portfolio_bacon[,1], portfolio_bacon[,2], MAR)) #expected

[1] 0.08053795
```

## 8 Tables

### 8.1 Variability risk

Table of Mean absolute difference, Monthly standard deviation and annualised standard deviation

```
> data(managers)
> table.Variability(managers[,1:8])
```

	HAM1	HAM2	HAM3	HAM4	HAM5	HAM6	EDHEC	LS	EQ
Mean Absolute deviation	0.0182	0.0268	0.0268	0.0410	0.0329	0.0187			0.0159
Monthly Std Dev	0.0256	0.0367	0.0365	0.0532	0.0457	0.0238			0.0205
Annualized Std Dev	0.0888	0.1272	0.1265	0.1843	0.1584	0.0825			0.0708
	SP500		TR						
Mean Absolute deviation	0.0333								
Monthly Std Dev	0.0433								
Annualized Std Dev	0.1500								

## 8.2 Specific risk

Table of specific risk, systematic risk and total risk

```
> data(managers)
> table.SpecificRisk(managers[,1:8], managers[,8])
```

	HAM1	HAM2	HAM3	HAM4	HAM5	HAM6	EDHEC	LS	EQ	SP500	TR
Specific Risk	0.0664	NA	0.0946	0.1521	NA	NA			NA		0.00
Systematic Risk	0.0586	0.0515	0.0836	0.1032	0.0477	0.0486		0.0503			0.15
Total Risk	0.0886	NA	0.1262	0.1838	NA	NA			NA		0.15

## 8.3 Information risk

Table of Tracking error, Annualised tracking error and Information ratio

```
> data(managers)
> table.InformationRatio(managers[,1:8], managers[,8])
```

	HAM1	HAM2	HAM3	HAM4	HAM5	HAM6	EDHEC	LS	EQ
Tracking Error	0.0327	0.0443	0.0334	0.0461	0.0520	0.0326			0.0326
Annualised Tracking Error	0.1132	0.1534	0.1159	0.1597	0.1800	0.1128			0.1130
Information Ratio	0.3604	0.5060	0.4701	0.1549	0.1212	0.6723			0.2985
	SP500		TR						
Tracking Error	0								
Annualised Tracking Error	0								
Information Ratio	NaN								

## 8.4 Distributions

Table of Monthly standard deviation, Skewness, Sample standard deviation, Kurtosis, Excess kurtosis, Sample Skewness and Sample excess kurtosis

```
> data(managers)
> table.Distributions(managers[,1:8])
```

	HAM1	HAM2	HAM3	HAM4	HAM5	HAM6	EDHEC	LS	EQ
Monthly Std Dev	0.0256	0.0367	0.0365	0.0532	0.0457	0.0238			0.0205
Skewness	-0.6588	1.4580	0.7908	-0.4311	0.0738	-0.2800			0.0177
Kurtosis	5.3616	5.3794	5.6829	3.8632	5.3143	2.6511			3.9105
Excess kurtosis	2.3616	2.3794	2.6829	0.8632	2.3143	-0.3489			0.9105
Sample skewness	-0.6741	1.4937	0.8091	-0.4410	0.0768	-0.2936			0.0182
Sample excess kurtosis	2.5004	2.5270	2.8343	0.9437	2.5541	-0.2778			1.0013
	SP500 TR								
Monthly Std Dev	0.0433								
Skewness	-0.5531								
Kurtosis	3.5598								
Excess kurtosis	0.5598								
Sample skewness	-0.5659								
Sample excess kurtosis	0.6285								

## 8.5 Drawdowns

Table of Calmar ratio, Sterling ratio, Burke ratio, Pain index, Ulcer index, Pain ratio and Martin ratio

```
> data(managers)
> table.DrawdownsRatio(managers[,1:8])
```

	HAM1	HAM2	HAM3	HAM4	HAM5	HAM6	EDHEC	LS	EQ	SP500	TR
Sterling ratio	0.5463	0.5139	0.3884	0.3136	0.0847	0.7678			0.5688		0.1768
Calmar ratio	0.9062	0.7281	0.5226	0.4227	0.1096	1.7425			1.0982		0.2163
Burke ratio	0.6593	0.8970	0.6079	0.1998	0.1008	1.0788			0.8452		0.2191
Pain index	0.0157	0.0642	0.0674	0.0739	0.1452	0.0183			0.0178		0.1226
Ulcer index	0.0362	0.1000	0.1114	0.1125	0.1828	0.0299			0.0325		0.1893
Pain ratio	8.7789	2.7187	2.2438	1.6443	0.2570	7.4837			6.6466		0.7891
Martin ratio	3.7992	1.7473	1.3572	1.0798	0.2042	4.5928			3.6345		0.5112

## 8.6 Downside risk

Table of Monthly downside risk, Annualised downside risk, Downside potential, Omega, Sortino ratio, Upside potential, Upside potential ratio and Omega-Sharpe ratio

```
> data(managers)
```

```
> table.DownsideRiskRatio(managers[,1:8])
```

	HAM1	HAM2	HAM3	HAM4	HAM5	HAM6	EDHEC	LS	EQ
Monthly downside risk	0.0145	0.0116	0.0174	0.0341	0.0304	0.0121		0.0098	
Annualised downside risk	0.0504	0.0401	0.0601	0.1180	0.1054	0.0421		0.0341	
Downside potential	0.0051	0.0061	0.0079	0.0159	0.0145	0.0054		0.0041	
Omega	3.1907	3.3041	2.5803	1.6920	1.2816	3.0436		3.3186	
Sortino ratio	0.7649	1.2220	0.7172	0.3234	0.1343	0.9102		0.9691	
Upside potential	0.0162	0.0203	0.0203	0.0269	0.0186	0.0165		0.0137	
Upside potential ratio	0.7503	2.2078	1.0852	0.8009	0.7557	1.0003		1.1136	
Omega-sharpe ratio	2.1907	2.3041	1.5803	0.6920	0.2816	2.0436		2.3186	
	SP500	TR							
Monthly downside risk	0.0283								
Annualised downside risk	0.0980								
Downside potential	0.0132								
Omega	1.6581								
Sortino ratio	0.3064								
Upside potential	0.0218								
Upside potential ratio	0.7153								
Omega-sharpe ratio	0.6581								

## 8.7 Sharpe ratio

Table of Annualized Return, Annualized Std Dev, and Annualized Sharpe

```
> data(managers)
```

```
> table.AnnualizedReturns(managers[,1:8])
```

	HAM1	HAM2	HAM3	HAM4	HAM5	HAM6	EDHEC	LS	EQ
Annualized Return	0.1375	0.1747	0.1512	0.1215	0.0373	0.1373		0.1180	
Annualized Std Dev	0.0888	0.1272	0.1265	0.1843	0.1584	0.0825		0.0708	
Annualized Sharpe (Rf=0%)	1.5491	1.3732	1.1955	0.6592	0.2356	1.6642		1.6657	
	SP500	TR							
Annualized Return	0.0967								
Annualized Std Dev	0.1500								
Annualized Sharpe (Rf=0%)	0.6449								