

# Package ‘ThreeWay’

July 2, 2014

**Type** Package

**Title** Three-way component analysis

**Version** 1.1.2

**Date** 2014-04-09

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**Description** Component analysis for three-way data arrays by means of Candecomp/Parafac, Tucker3, Tucker2 and Tucker1 models

**Depends** R(>= 2.8.1), base, graphics, stats

**License** GPL (>= 2)

**LazyLoad** yes

**Repository** CRAN

**NeedsCompilation** no

**Date/Publication** 2014-04-09 14:16:09

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bootstrapCP

*Bootstrap percentile intervals for CANDECOMP/PARAFAC***Description**

Produces percentile intervals for all output parameters. The percentile intervals indicate the instability of the sample solutions.

**Usage**

```
bootstrapCP(X, A, B, C, n, m, p, r, ort1, ort2, ort3, conv, centopt, normopt,
            scaleopt, maxit, laba, labb, labc)
```

**Arguments**

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matrixed array (frontal slices)
A	Component matrix for the A-mode
B	Component matrix for the B-mode
C	Component matrix for the C-mode
n	Number of A-mode entities of X
m	Number of B-mode entities of X
p	Number of C-mode entities of X
r	Number of extracted components
ort1	Type of constraints on A (see <a href="#">CP</a> )
ort2	Type of constraints on B (see <a href="#">CP</a> )
ort3	Type of constraints on C (see <a href="#">CP</a> )
conv	Convergence criterion
centopt	Centering option (see <a href="#">cent3</a> )
normopt	Normalization option (see <a href="#">norm3</a> )
scaleopt	Scaling option (see <a href="#">renormsolCP</a> )
maxit	Maximal number of iterations
laba	Optional vector of length n containing the labels of the A-mode entities
labb	Optional vector of length m containing the labels of the B-mode entities
labc	Optional vector of length p containing the labels of the C-mode entities

**Value**

A list including the following components:

Bint	Bootstrap percentile interval of every element of B
Cint	Bootstrap percentile interval of every element of C
fpint	Bootstrap percentile interval for the goodness of fit index expressed as a percentage

**Note**

The preprocessing must be done in same way as for sample analysis.

The resampling mode must be the A-mode.

The starting points for every bootstrap solution are two: rational (using SVD) and solution from the observed sample.

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**References**

H.A.L. Kiers (2004). Bootstrap confidence intervals for three-way methods. *Journal of Chemometrics* 18:22–36.

**See Also**

[bootstrapT3](#), [CP](#), [percentile95](#)

**Examples**

```
data(TV)
TVdata=TV[[1]]
labSCALE=TV[[2]]
labPROGRAM=TV[[3]]
labSTUDENT=TV[[4]]
# permutation of the modes so that the A-mode refers to students
TVdata <- permnew(TVdata, 16, 15, 30)
TVdata <- permnew(TVdata, 15, 30, 16)
# CP solution
TVcp <- CPfuncprep(TVdata, 30, 16, 15, 2, 1, 1, 1, 0, 1e-6, 10000)
## Not run:
# Bootstrap analysis on CP solution
boot <- bootstrapCP(TVdata, TVcp$A, TVcp$B, TVcp$C, 30, 16, 15, 2, 1, 1, 1,
  1e-6, 0, 0, 0, 10000, labSTUDENT, labSCALE, labPROGRAM)
# Bootstrap analysis on CP solution (when labels are not available)
boot <- bootstrapCP(TVdata, TVcp$A, TVcp$B, TVcp$C, 30, 16, 15, 2, 1, 1, 1,
  1e-6, 0, 0, 0, 10000)

## End(Not run)
```

bootstrapT3

*Bootstrap percentile intervals for Tucker3***Description**

Produces percentile intervals for all output parameters. The percentile intervals indicate the instability of the sample solutions.

**Usage**

```
bootstrapT3(X, A, B, C, G, n, m, p, r1, r2, r3, conv, centopt, normopt,
  optimalmatch, laba, labb, labc)
```

**Arguments**

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matricized array (frontal slices)
A	Component matrix for the A-mode
B	Component matrix for the B-mode
C	Component matrix for the C-mode
G	Matricized core array (frontal slices)
n	Number of A-mode entities of X
m	Number of B-mode entities of X
p	Number of C-mode entities of X
r1	Number of extracted components for the A-mode
r2	Number of extracted components for the B-mode
r3	Number of extracted components for the C-mode
conv	Convergence criterion
centopt	Centering option (see <a href="#">cent3</a> )
normopt	Normalization option (see <a href="#">norm3</a> )
optimalmatch	Binary indicator (0 if the procedure uses matching via orthogonal rotation towards full solutions, 1 if the procedure uses matching via optimal transformation towards full solutions)
laba	Optional vector of length n containing the labels of the A-mode entities
labb	Optional vector of length m containing the labels of the B-mode entities
labc	Optional vector of length p containing the labels of the C-mode entities

**Value**

A list including the following components:

Bint	Bootstrap percentile interval of every element of B
Cint	Bootstrap percentile interval of every element of C
Gint	Bootstrap percentile interval of matricized core array (frontal slices) G
fpint	Bootstrap percentile interval for the goodness of fit index expressed as a percentage

**Note**

The preprocessing must be done in same way as for sample analysis.

The resampling mode must be the A-mode.

The starting points for every bootstrap solution are two: rational (using SVD) and solution from the observed sample.

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**References**

H.A.L. Kiers (2004). Bootstrap confidence intervals for three-way methods. *Journal of Chemometrics* 18:22–36.

**See Also**

[bootstrapCP](#), [percentile95](#), [T3](#)

**Examples**

```
data(Bus)
# labels for Bus data
laba <- rownames(Bus)
labb <- substr(colnames(Bus)[1:5],1,1)
labc <- substr(colnames(Bus)[seq(1,ncol(Bus),5)],3,8)
# T3 solution
BusT3 <- T3funcprep(Bus, 7, 5, 37, 2, 2, 2, 0, 1e-6)
## Not run:
# Bootstrap analysis on T3 solution using matching via optimal transformation
boot <- bootstrapT3(Bus, BusT3$A, BusT3$B, BusT3$C, BusT3$H, 7, 5, 37, 2, 2, 2,
  1e-6, 0, 0, 1, laba, labb, labc)
# Bootstrap analysis on T3 solution using matching via orthogonal rotation
# (when labels are not available)
boot <- bootstrapT3(Bus, BusT3$A, BusT3$B, BusT3$C, BusT3$H, 7, 5, 37, 2, 2, 2,
  1e-6, 0, 0, 0)

## End(Not run)
```

---

Bus

*Bus data*

---

### Description

Three-way data about the process of learning to read of seven first-grade children tested weekly (from week 3 to 47, but weeks 10, 19, 20, 29, 35, 36, 39, 43 were holidays and, thus, data on 37 weeks) with five different tests.

### Usage

```
data(Bus)
```

### Format

A matrix with 7 rows and 185 (5x37) columns.

The rows refer to the pupils.

The columns refer to the combinations of tests and weeks with the tests nested within the weeks.

The matrix contains the frontal slices next to each other of the original array.

The meanings and the ranges of the tests are as follows:

L: letter knowledge test (scores in 0-47);

P: regular orthographic short words (scores in 0-10);

Q: regular orthographic long words (scores in 0-10);

S: regular orthographic long and short words within context (scores in 0-15);

R: irregular orthographic long and short words (scores in 0-15).

### Details

In the literature the Bus data have been analyzed by Tucker3 (see Kroonenberg, 1983; Timmerman, 2001). There is consensus on normalizing the data so to eliminate artificial differences among ranges of tests. Different centering options and numbers of extracted components have been chosen. Specifically, Kroonenberg (1983) suggests averaging over pupils and tests for each time occasions and extracting two components for every mode. Timmerman (2001) suggests to apply Tucker3 to the normalized data with two components for pupils and time occasions and one component for tests.

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### References

P.M. Kroonenberg (1983). *Three-mode Principal Component Analysis. Theory and Applications*. DSWO Press, Leiden.

M.E. Timmerman (2001). *Component Analysis of Multisubject Multivariate Longitudinal Data*. Ph.D. Thesis, University of Groningen.

Cc *Columnwise centering of a matrix*

---

**Description**

Computation of a columnwise centered version of a matrix.

**Usage**

```
Cc(A)
```

**Arguments**

A Matrix of any order

**Value**

Ac Matrix columnwise centered

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**See Also**

[nrm2](#)

**Examples**

```
X <- matrix(rnorm(6*3),ncol=3)  
Y <- Cc(X)  
apply(Y,2,mean)
```

---

ccmat *Columns concatenation*

---

**Description**

Concatenates the columns of two matrices next to each other.

**Usage**

```
ccmat(A, B)
```



**Arguments**

A Matrix of the same order of B  
 B Matrix of the same order of A

**Value**

mat Matrix in which the columns of A and B are concatenated next to each other

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**Examples**

```
X <- matrix(rnorm(6*3),ncol=3)
Y <- matrix(rnorm(6*3),ncol=3)
Z <- ccmat(X,Y)
```

cent3

*Centering of a matricized array***Description**

Centering of a matricized array across one mode (modes indicated by 1,2, or 3).

**Usage**

```
cent3(X, n, m, p, mode)
```

**Arguments**

X Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matricized array (frontal slices)  
 n Number of A-mode entities  
 m Number of B-mode entities  
 p Number of C-mode entities  
 mode Centering option (1 if X is centered across A-mode, 2 if X is centered across B-mode, 3 if X is centered across C-mode)

**Value**

Y Matrix of order (n x mp) containing the centered matricized array (frontal slices)

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**References**

H.A.L. Kiers (2000). Towards a standardized notation and terminology in multiway analysis. *Journal of Chemometrics* 14:105–122.

**See Also**

[Cc](#), [norm3](#)

**Examples**

```
X <- array(c(rnorm(120)),c(6,5,4))
# matricized array
Y <- supermat(X)
# data centered across A-mode
Z <- cent3(Y$Xa, 6, 5, 4, 1)
apply(Z,2,mean)
# data centered also across B-modes (double centering)
Z <- cent3(Z, 6, 5, 4, 2)
apply(Z,1,mean)
apply(Z,2,mean)
```

---

CP

*Interactive Candecomp/Parafac analysis*

---

**Description**

Detects the underlying structure of a three-way array according to the Candecomp/Parafac (CP) model.

**Usage**

```
CP(data, laba, labb, labc)
```

**Arguments**

data	Array of order n by m by p or matrix or data.frame of order (n x mp) containing the matricized array (frontal slices)
laba	Optional vector of length n containing the labels of the A-mode entities
labb	Optional vector of length m containing the labels of the B-mode entities
labc	Optional vector of length p containing the labels of the C-mode entities

**Value**

A list including the following components:

A	Component matrix for the A-mode
B	Component matrix for the B-mode
C	Component matrix for the C-mode
fit	Fit value expressed as a percentage
tripcos	Matrix of the triple cosines among pairs of components (to inspect degeneracy)
fitValues	Fit values expressed as a percentage upon convergence for all the runs of the CP algorithm (see <a href="#">CPfunc</a> )
funcValues	Function values upon convergence for all the runs of the CP algorithm (see <a href="#">CPfunc</a> )
cputime	Computation times for all the runs of the CP algorithm (see <a href="#">CPfunc</a> )
iter	Numbers of iterations upon convergence for all the runs of the CP algorithm (see <a href="#">CPfunc</a> )
fitA	Fit contributions for the A-mode entities (see <a href="#">CPfitpartitioning</a> )
fitB	Fit contributions for the B-mode entities (see <a href="#">CPfitpartitioning</a> )
fitC	Fit contributions for the C-mode entities (see <a href="#">CPfitpartitioning</a> )
Bint	Bootstrap percentile interval of every element of B (see <a href="#">bootstrapCP</a> )
Cint	Bootstrap percentile interval of every element of C (see <a href="#">bootstrapCP</a> )
fpint	Bootstrap percentile interval for the goodness of fit index expressed as a percentage (see <a href="#">bootstrapCP</a> )
Afull	Component matrix for the A-mode (full data) from split-half analysis (see <a href="#">splithalfCP</a> )
As1	Component matrix for the A-mode (split n.1) from split-half analysis (see <a href="#">splithalfCP</a> )
As2	Component matrix for the A-mode (split n.2) from split-half analysis (see <a href="#">splithalfCP</a> )
Bfull	Component matrix for the B-mode (full data) from split-half analysis (see <a href="#">splithalfCP</a> )
Bs1	Component matrix for the B-mode (split n.1) from split-half analysis (see <a href="#">splithalfCP</a> )
Bs2	Component matrix for the B-mode (split n.2) from split-half analysis (see <a href="#">splithalfCP</a> )
Cfull	Component matrix for the C-mode (full data) from split-half analysis (see <a href="#">splithalfCP</a> )
Cs1	Component matrix for the C-mode (split n.1) from split-half analysis (see <a href="#">splithalfCP</a> )
Cs2	Component matrix for the C-mode (split n.2) from split-half analysis (see <a href="#">splithalfCP</a> )
A1	Component matrix for the A-mode from Principal Component Analysis of mean values (see <a href="#">pcamean</a> )
B1	Component matrix for the B-mode from Principal Component Analysis of mean values (see <a href="#">pcamean</a> )
C1	Component matrix for the C-mode from Principal Component Analysis of mean values (see <a href="#">pcamean</a> )
A2	Component matrix for the A-mode from Principal Component Analysis of mean values (see <a href="#">pcamean</a> )

B2	Component matrix for the B-mode from Principal Component Analysis of mean values (see <a href="#">pcamean</a> )
C2	Component matrix for the C-mode from Principal Component Analysis of mean values (see <a href="#">pcamean</a> )
laba	Vector of length n containing the labels of the A-mode entities
labb	Vector of length m containing the labels of the B-mode entities
labc	Vector of length P containing the labels of the C-mode entities
Xprep	Matrix of order (n x mp) containing the matricized array (frontal slices) after preprocessing used for the analysis

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### References

J.D. Carroll and J.J. Chang (1970). Analysis of individual differences in multidimensional scaling via an *N*-way generalization of 'Eckart-Young' decomposition. *Psychometrika* 35:283–319.  
 P. Giordani, H.A.L. Kiers, M.A. Del Ferraro (2014). Three-way component analysis using the R package ThreeWay. *Journal of Statistical Software* 57(7):1–23. <http://www.jstatsoft.org/v57/i07/>.  
 R.A. Harshman (1970). Foundations of the Parafac procedure: models and conditions for an 'explanatory' multi-mode factor analysis. *UCLA Working Papers in Phonetics* 16:1–84.  
 P.M. Kroonenberg (2008). *Applied Multiway Data Analysis*. Wiley, New Jersey.

### See Also

[T3](#), [T2](#), [T1](#)

### Examples

```
data(TV)
TVdata=TV[[1]]
labSCALE=TV[[2]]
labPROGRAM=TV[[3]]
labSTUDENT=TV[[4]]
# permutation of the modes so that the A-mode refers to students
TVdata <- permnew(TVdata, 16, 15, 30)
TVdata <- permnew(TVdata, 15, 30, 16)
## Not run:
# interactive CP analysis
TVcp <- CP(TVdata, labSTUDENT, labSCALE, labPROGRAM)
# interactive CP analysis (when labels are not available)
TVcp <- CP(TVdata)

## End(Not run)
```

---

CPdimensionalityplot *Plot fit of Candecomp/Parafac*

---

### Description

Plots fits against numbers of dimensions, with S as labels and fits against number of effective parameters.

### Usage

```
CPdimensionalityplot(A, n, m, p)
```

### Arguments

A	A matrix with columns: number of components, goodness of fit (%)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities

### Note

A is usually the first and fourth columns of the output of [DimSelector](#).

The number of effective parameters in a Candecomp/Parafac analysis is discussed in Weesie and Van Houwelingen (1983).

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### References

E. Ceulemans & H.A.L. Kiers (2006). Selecting among three-mode principal component models of different types and complexities: A numerical convex hull based method. *British Journal of Mathematical and Statistical Psychology* 59:133–150.

J. Weesie & H. Van Houwelingen (1983). *GEPCAM users' manual (first draft)*. Utrecht, The Netherlands: Institute of Mathematical Statistics, State University of Utrecht.

### See Also

[CP](#), [DimSelector](#)

**Examples**

```

data(TV)
TVdata=TV[[1]]
# permutation of the modes so that the A-mode refers to students
TVdata <- permnew(TVdata, 16, 15, 30)
TVdata <- permnew(TVdata, 15, 30, 16)
# Fit values of CP with different numbers of components (from 1 to 5)
FitCP <- CPrunsFit(TVdata, 30, 16, 15, 5)
OutCP <- FitCP[,c(1,4)]
CPdimensionalityplot(OutCP, 30, 16, 15)

```

---

CPfitpartitioning      *Fit of each entity per mode*

---

**Description**

Computation of fit contributions.

**Usage**

```
CPfitpartitioning(Xprep, n, m, p, A, B, C, laba, labb, labc)
```

**Arguments**

Xprep	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matrixed array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
A	Component matrix for the A-mode
B	Component matrix for the B-mode
C	Component matrix for the C-mode
laba	Optional vector of length n containing the labels of the A-mode entities
labb	Optional vector of length m containing the labels of the B-mode entities
labc	Optional vector of length p containing the labels of the C-mode entities

**Value**

A list including the following components:

fitA	Fit contribution for the A-mode entities
fitB	Fit contribution for the B-mode entities
fitC	Fit contribution for the C-mode entities

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**See Also**

[CP](#)

**Examples**

```
data(TV)
TVdata=TV[[1]]
labSCALE=TV[[2]]
labPROGRAM=TV[[3]]
labSTUDENT=TV[[4]]
# permutation of the modes so that the A-mode refers to students
TVdata <- permnew(TVdata, 16, 15, 30)
TVdata <- permnew(TVdata, 15, 30, 16)
# CP solution
TVcp <- CPfunc(TVdata, 30, 16, 15, 2, 1, 1, 1, 0, 1e-6, 10000)
# Fitpartitioning of the CP solution
FitCP <- CPfitpartitioning(TVdata, 30, 16, 15, TVcp$A, TVcp$B, TVcp$C,
  labSTUDENT, labSCALE, labPROGRAM)
# Fitpartitioning of the CP solution (when labels are not available)
FitCP <- CPfitpartitioning(TVdata, 30, 16, 15, TVcp$A, TVcp$B, TVcp$C)
```

---

CPfunc

*Algorithm for the Candecomp/Parafac (CP) model*

---

**Description**

Alternating Least Squares algorithm for the minimization of the Candecomp/Parafac loss function.

**Usage**

```
CPfunc(X, n, m, p, r, ort1, ort2, ort3, start, conv, maxit, A, B, C)
```

**Arguments**

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matricized array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
r	Number of extracted components

ort1	Type of constraints on A (1 for no constraints, 2 for orthogonality constraints, 3 for zero correlations constraints)
ort2	Type of constraints on B (1 for no constraints, 2 for orthogonality constraints, 3 for zero correlations constraints)
ort3	Type of constraints on C (1 for no constraints, 2 for orthogonality constraints, 3 for zero correlations constraints)
start	Starting point (0 for starting point of the algorithm from SVD's, 1 for random starting point (orthonormalized component matrices), 2 for user specified components)
conv	Convergence criterion
maxit	Maximal number of iterations
A	Optional (necessary if start=2) starting value for A
B	Optional (necessary if start=2) starting value for B
C	Optional (necessary if start=2) starting value for C

**Value**

A list including the following components:

A	Component matrix for the A-mode
B	Component matrix for the B-mode
C	Component matrix for the C-mode
f	Loss function value
fp	Fit value expressed as a percentage
iter	Number of iterations
tripcos	Minimal triple cosine between two components across three component matrices (to inspect degeneracy)
mintripcos	Minimal triple cosine during the iterative algorithm observed at every 10 iterations (to inspect degeneracy)
ftiter	Matrix containing in each row the function value and the minimal triple cosine at every 10 iterations
cputime	Computation time

**Note**

The loss function to be minimized is  $\sum(k) \|X(k) - AD(k)B'\|^2$ , where  $D(k)$  is a diagonal matrix holding the k-th row of C.

CPfunc is the same as CPfuncrep except that all printings are available.

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## References

R.A. Harshman (1970). Foundations of the Parafac procedure: models and conditions for an ‘explanatory’ multi-mode factor analysis. *UCLA Working Papers in Phonetics* 16:1–84.

## See Also

[CP](#), [CPfuncrep](#)

## Examples

```
data(TV)
TVdata=TV[[1]]
# permutation of the modes so that the A-mode refers to students
TVdata <- permnew(TVdata, 16, 15, 30)
TVdata <- permnew(TVdata, 15, 30, 16)
# unconstrained CP solution using two components
# (rational starting point by SVD [start=0])
TVcp <- CPfunc(TVdata, 30, 16, 15, 2, 1, 1, 1, 0, 1e-6, 10000)
# constrained CP solution using two components with orthogonal A-mode
# component matrix (rational starting point by SVD [start=0])
TVcp <- CPfunc(TVdata, 30, 16, 15, 2, 2, 1, 1, 0, 1e-6, 10000)
# constrained CP solution using two components with orthogonal A-mode
# component matrix and zero correlated C-mode component matrix
# (rational starting point by SVD [start=0])
TVcp <- CPfunc(TVdata, 30, 16, 15, 2, 2, 1, 3, 0, 1e-6, 10000)
# unconstrained CP solution using two components
# (random orthonormalized starting point [start=1])
TVcp <- CPfunc(TVdata, 30, 16, 15, 2, 1, 1, 1, 1, 1e-6, 10000)
# unconstrained CP solution using two components (user starting point [start=2])
TVcp <- CPfunc(TVdata, 30, 16, 15, 2, 1, 1, 1, 2, 1e-6, 10000,
  matrix(rnorm(30*2),nrow=30), matrix(rnorm(16*2),nrow=16),
  matrix(rnorm(15*2),nrow=15))
```

---

CPfuncrep

*Algorithm for the Candecomp/Parafac (CP) model*

---

## Description

Alternating Least Squares algorithm for the minimization of the Candecomp/Parafac loss function.

## Usage

CPfuncrep(X, n, m, p, r, ort1, ort2, ort3, start, conv, maxit, A, B, C)

**Arguments**

<code>X</code>	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matricized array (frontal slices)
<code>n</code>	Number of A-mode entities
<code>m</code>	Number of B-mode entities
<code>p</code>	Number of C-mode entities
<code>r</code>	Number of extracted components
<code>ort1</code>	Type of constraints on A (1 for no constraints, 2 for orthogonality constraints, 3 for zero correlations constraints)
<code>ort2</code>	Type of constraints on B (1 for no constraints, 2 for orthogonality constraints, 3 for zero correlations constraints)
<code>ort3</code>	Type of constraints on C (1 for no constraints, 2 for orthogonality constraints, 3 for zero correlations constraints)
<code>start</code>	Starting point (0 for starting point of the algorithm from SVD's, 1 for random starting point (orthonormalized component matrices), 2 for user specified components)
<code>conv</code>	Convergence criterion
<code>maxit</code>	Maximal number of iterations
<code>A</code>	Optional (necessary if start=2) starting value for A
<code>B</code>	Optional (necessary if start=2) starting value for B
<code>C</code>	Optional (necessary if start=2) starting value for C

**Value**

A list including the following components:

<code>A</code>	Component matrix for the A-mode
<code>B</code>	Component matrix for the B-mode
<code>C</code>	Component matrix for the C-mode
<code>f</code>	Loss function value
<code>fp</code>	Fit value expressed as a percentage
<code>iter</code>	Number of iterations
<code>tripcos</code>	Minimal triple cosine between two components across three component matrices (to inspect degeneracy)
<code>mintripcos</code>	Minimal triple cosine during the iterative algorithm observed at every 10 iterations (to inspect degeneracy)
<code>ftiter</code>	Matrix containing in each row the function value and the minimal triple cosine at every 10 iterations
<code>cputime</code>	Computation time

**Note**

The loss function to be minimized is  $\sum(k) \|X(k) - AD(k)B'\|^2$ , where  $D(k)$  is a diagonal matrix holding the k-th row of C.

CPfuncrep is the same as CPfunc except that all printings are suppressed. Thus, CPfuncrep can be helpful for simulation experiments.

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**References**

R.A. Harshman (1970). Foundations of the Parafac procedure: models and conditions for an ‘explanatory’ multi-mode factor analysis. *UCLA Working Papers in Phonetics 16:1–84*.

**See Also**

[CP](#), [CPfunc](#)

**Examples**

```
data(TV)
TVdata=TV[[1]]
# permutation of the modes so that the A-mode refers to students
TVdata <- permnew(TVdata, 16, 15, 30)
TVdata <- permnew(TVdata, 15, 30, 16)
# unconstrained CP solution using two components
# (rational starting point by SVD [start=0])
TVcp <- CPfuncrep(TVdata, 30, 16, 15, 2, 1, 1, 1, 0, 1e-6, 10000)
# constrained CP solution using two components with orthogonal A-mode
# component matrix (rational starting point by SVD [start=0])
TVcp <- CPfuncrep(TVdata, 30, 16, 15, 2, 2, 1, 1, 0, 1e-6, 10000)
# constrained CP solution using two components with orthogonal A-mode
# component matrix and zero correlated C-mode component matrix
# (rational starting point by SVD [start=0])
TVcp <- CPfuncrep(TVdata, 30, 16, 15, 2, 2, 1, 3, 0, 1e-6, 10000)
# unconstrained CP solution using two components
# (random orthonormalized starting point [start=1])
TVcp <- CPfuncrep(TVdata, 30, 16, 15, 2, 1, 1, 1, 1, 1e-6, 10000)
# unconstrained CP solution using two components (user starting point [start=2])
TVcp <- CPfuncrep(TVdata, 30, 16, 15, 2, 1, 1, 1, 2, 1e-6, 10000,
matrix(rnorm(30*2),nrow=30), matrix(rnorm(16*2),nrow=16),
matrix(rnorm(15*2),nrow=15))
```

---

`CPrunsFit`*Candecomp/Parafac solutions*

---

**Description**

Computes all the Candecomp/Parafac solutions (CP) with  $r$  (from 1 to  $\text{maxC}$ ) components.

**Usage**

```
CPrunsFit(X, n, m, p, maxC)
```

**Arguments**

<code>X</code>	Matrix (or data.frame coerced to a matrix) of order $(n \times mp)$ containing the matrixed array (frontal slices)
<code>n</code>	Number of A-mode entities
<code>m</code>	Number of B-mode entities
<code>p</code>	Number of C-mode entities
<code>maxC</code>	Maximum dimensionality for the A-mode

**Value**

<code>out</code>	Matrix with columns: number of components for the A-mode, number of components for the B-mode, number of components for the C-mode, goodness of fit (%), total number of components
------------------	---

**Note**

The structure of `out` is consistent with Tucker models. In CP, the first and fourth columns are sufficient for choosing the optimal number of components.

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**References**

H.A.L. Kiers (1991). Hierarchical relations among three-way methods. *Psychometrika* 56:449–470.

**See Also**

[DimSelector](#), [LineCon](#), [CP](#)

**Examples**

```

data(TV)
TVdata=TV[[1]]
# permutation of the modes so that the A-mode refers to students
TVdata <- permnew(TVdata, 16, 15, 30)
TVdata <- permnew(TVdata, 15, 30, 16)
# Fit values of CP with different numbers of components (from 1 to 5)
FitCP <- CPrunsFit(TVdata, 30, 16, 15, 5)

```

---

DimSelector

*Convex Hull procedure*


---

**Description**

Selects among three-mode principal component models of different complexities.

**Usage**

```
DimSelector(out, n, m, p, model)
```

**Arguments**

out	Matrix with columns: number of components for the A-mode, number of components for the B-mode, number of components for the C-mode, goodness of fit (%), total number of components
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
model	Kind of model (1 for Candecomp/Parafac, 2 for Tucke3, 3 for Tucker2, 4 for Tucker1)

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**References**

E. Ceulemans and H.A.L. Kiers (2006). Selecting among three-mode principal component models of different types and complexities: A numerical convex hull based method. *British Journal of Mathematical and Statistical Psychology* 59:133–150.  
 J. Weesie and H. Van Houwelingen (1983). *GEPCAM users' manual (first draft)*. Utrecht, The Netherlands: Institute of Mathematical Statistics, State University of Utrecht.

**See Also**

[LineCon](#), [T3runsApproxFit](#) [T2runsApproxFit](#) [T1runsFit](#) [CPrunsFit](#)

**Examples**

```
data(Bus)
# Analysis on T3 with different numbers of components (from 1 to 4 for the A-mode,
# from 1 to 3 for the B-mode, from 1 to 5 for the C-mode)
FitT3 <- T3runsApproxFit(Bus,7,5,37,4,4,4)
T3opt <- DimSelector(FitT3,7,5,37,2)
```

---

jointplotgen

*Jointplots*

---

**Description**

Program for producing jointplots in general.

**Usage**

```
jointplotgen(K, A, B, C, fixmode, fixunit, laba, labb, labc)
```

**Arguments**

K	Matricized core array (frontal slices)
A	Component matrix for the A-mode
B	Component matrix for the B-mode
C	Component matrix for the C-mode
fixmode	Mode for which one unit is to be chosen (1 for A-mode, 2 for B-mode, 3 for C-mode)
fixunit	Number of component for which joint plot is desired
laba	Vector of length n containing the labels of the A-mode entities
labb	Vector of length m containing the labels of the B-mode entities
labc	Vector of length p containing the labels of the C-mode entities

**Value**

fit	Percentage of info for component at hand, explained by two-dimensional plot
-----	---

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## References

P.M. Kroonenberg (2008). *Applied Multiway Data Analysis*. Wiley, New Jersey.

## Examples

```
data(Bus)
# labels for Bus data
laba <- rownames(Bus)
labb <- substr(colnames(Bus)[1:5], 1, 1)
labc <- substr(colnames(Bus)[seq(1,ncol(Bus),5)], 3, 8)
# <- T3 solution
BusT3 <- T3funcrep(Bus, 7, 5, 37, 2, 2, 2, 0, 1e-6)
# Joint plot for mode C and component 2
jointplotgen(BusT3$H, BusT3$A, BusT3$B, BusT3$C, 3, 2, laba, labb, labc)
```

---

Kinship

*Kinship terms data*

---

## Description

Three-way proximity data about 15 kinship terms produced by 6 groups of subjects.

## Usage

```
data(Kinship)
```

## Format

An array of order 15 x 15 x 6.

The A-mode and B-mode entities are the kinship terms (Aunt, Brother, Cousin, Daughter, Father, Granddaughter, Grandfather, Grandmother, Grandson, Mother, Nephew, Niece, Sister, Son, Uncle). The C-mode entities are groups of subjects (First female, Second female, First male, Second male, Single female, Single male).

## Details

The original data have been introduced by Rosenberg & Kim (1975). The data were collected by asking to 6 groups of subjects to produce a partition of 15 kinship terms. Two groups (Single female and Single male) were composed by 85 male and 85 female college students, respectively, and provided a single partition. Two additional groups of, respectively, 80 male and 80 female students produced two partitions each (First female, Second female, First male, Second male). In fact, they were informed in advance that, after making the first partition, they should give a new partition of the kinship terms using a different basis of meaning. The array contains similarities. For every group of subjects, the numbers of times in which the kinship terms were grouped together are given.

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**References**

S. Rosenberg \& M.P. Kim (1975). The method of sorting as a data-gathering procedure in multivariate research. *Multivariate Behavioral Research* 10:489–502.

**Examples**

```
data(Kinship)
## The labels are in the data array
laba <- dimnames(Kinship)[[1]]
labb <- dimnames(Kinship)[[2]]
labc <- dimnames(Kinship)[[3]]
## Candecomp/Parafac analysis
## Not run:
CP(Kinship,laba,labb,labc)

## End(Not run)
```

---

LineCon

*Middle point location*

---

**Description**

Checks whether the middle point is located below or on the line connecting its neighbors.

**Usage**

```
LineCon(f1, f2, f3, fp1, fp2, fp3)
```

**Arguments**

f1	Goodness-of-fit value for the first point
f2	Goodness-of-fit value for the second point
f3	Goodness-of-fit value for the third point
fp1	Number of effective parameters for the first point
fp2	Number of effective parameters for the second point
fp3	Number of effective parameters for the third point



**Value**

ret Value that indicates if the middle point is located below or on the line connecting its neighbors (0 if the middle point is not located below the line connecting its neighbors, 1 if the middle point is not located on the line connecting its neighbors)

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**References**

E. Ceulemans and H.A.L. Kiers (2006). Selecting among three-mode principal component models of different types and complexities: A numerical convex hull based method. *British Journal of Mathematical and Statistical Psychology* 59:133–150.  
 J. Weesie and H. Van Houwelingen (1983). *GEPCAM users' manual (first draft)*. Utrecht, The Netherlands: Institute of Mathematical Statistics, State University of Utrecht.

**See Also**

[DimSelector](#)

**Examples**

```
data(Bus)
# T2-AB with 1 component for the A- and B-mode
FitBusT2AB11 <- T2funcrep(Bus, 7, 5, 37, 1, 1, 37, 0, 1e-6,1)$fp
# T2-AB with 2 components for the A-mode and 1 component for the B-mode
FitBusT2AB21 <- T2funcrep(Bus, 7, 5, 37, 2, 1, 37, 0, 1e-6, 1)$fp
# T2-AB with 1 component for the A-mode and 2 components for the B-mode
# T2-AB with 1 component for the A-mode and 2 components for the B-mode
# FitBusT2AB21>FitBusT2AB12
# T2-AB with 2 components for the A- and B-mode
FitBusT2AB22 <- T2funcrep(Bus, 7, 5, 37, 2, 2, 37, 0, 1e-6,1)$fp
# number of effective parameters  $n \times r_1 + m \times r_2 + r_1 \times r_2 \times p - r_1^2 - r_2^2$ 
nepT2AB11 <- 47
nepT2AB21 <- 88
nepT2AB22 <- 164
ret <- LineCon(FitBusT2AB11, FitBusT2AB21, FitBusT2AB22, nepT2AB11, nepT2AB21, nepT2AB22)
```

---

meaudret

*Meaudret data*

---

**Description**

Three-way data about six sampling sites along a small French stream (the Meaudret) on which ten biological and chemical variables are collected four times.

**Usage**

```
data(meaudret)
```

**Format**

An array of order 6 x 10 x 4.

The A-mode entities are sampling sites (Site1, ..., Site6).

The B-mode entities are biological and chemical variables (Temp, Debi, PH, Cond, Oxyg, Biod, Chem, NH4, NO3, PO4).

The C-mode entities are months (June, August, November, February).

**Details**

The ranges of the variables are very different and, therefore, normalization of the raw data is recommended. The data have been used by Kiers (1991) in order to show the existing relations among three-way methods.

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**References**

H.A.L. Kiers (1991). Hierarchical relations among three-way methods. *Psychometrika* 56:449–470.

**Examples**

```
data(meaudret)
## The labels are in the data array
laba <- dimnames(meaudret)[[1]]
labb <- dimnames(meaudret)[[2]]
labc <- dimnames(meaudret)[[3]]
## Candecomp/Parafac analysis
## Not run:
CP(meaudret, laba, labb, labc)
## Tucker3 analysis
T3(meaudret, laba, labb, labc)
## Tucker2 analysis
T2(meaudret, laba, labb, labc)
## Tucker1 analysis
T1(meaudret, laba, labb, labc)

## End(Not run)
```

---

`norm3`*Normalization of a matricized array*

---

**Description**

Normalization of a matricized array within one mode (modes indicated by 1,2, or 3) to sum of squares equal to product of size of other modes.

**Usage**

```
norm3(X, n, m, p, mode)
```

**Arguments**

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matricized array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
mode	Normalization option (1 if X is normalized within A-mode, 2 if X is normalized within B-mode, 3 if X is normalized within C-mode)

**Value**

Y	Matrix of order (n x mp) containing the normalized matricized array (frontal slices)
---	--

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**References**

H.A.L. Kiers (2000). Towards a standardized notation and terminology in multiway analysis. *Journal of Chemometrics* 14:105–122.

**See Also**

[cent3](#), [nrm2](#)

**Examples**

```
X <- array(c(rnorm(120)), c(6,5,4))
# matricized array
Y <- supermat(X)
# data normalized within A-mode
Z <- norm3(Y$Xa, 6, 5, 4, 1)
apply(Z^2,1,sum)
# data normalized within C-mode
Z <- norm3(Y$Xa, 6, 5, 4, 3)
Z <- permnew(Z, 6, 5, 4)
Z <- permnew(Z, 5, 4, 6)
apply(Z^2, 1, sum)
```

---

normvari

*Normalized varimax rotation*

---

**Description**

Produces normalized varimax rotated version of A and rotation matrix T.

**Usage**

```
normvari(A)
```

**Arguments**

A                    Matrix to be rotated

**Value**

A list including the following components:

B                    Rotated version of A ( $B=AT$ )  
T                    Rotation matrix  
f                    Varimax function value

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**References**

H. Kaiser (1958). The varimax criterion for analytic rotation in factor analysis. *Psychometrika* 23:187–200.

**See Also**[varim](#)**Examples**

```
X <- matrix(rnorm(6*3),ncol=3)
Y <- normvari(X)
# normalized varimax rotated version of X
Y$B
# rotation matrix
Y$T
```

---

**nrm2***Columnwise normalization of a matrix*

---

**Description**

Computation of a columnwise normalized version of a matrix.

**Usage**

```
nrm2(A)
```

**Arguments**

A                    Matrix of any order

**Value**

N                    Matrix columnwise normalized

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**See Also**[Cc](#)**Examples**

```
X <- matrix(rnorm(6*3),ncol=3)
Y <- nrm2(X)
apply(Y^2, 2, sum)
```

---

ord	<i>Order</i>
-----	--------------

---

**Description**

In case of vectors, an ordering of its elements in ascending order is produced; in case of matrices, the ordering in ascending order refers to every column.

**Usage**

```
ord(X)
```

**Arguments**

X	Vector or matrix to be ordered
---	--------------------------------

**Value**

A	Vector or matrix with the elements sorted in ascending order
a	Vector or matrix with the ordering indices

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**Examples**

```
# vector
x <- rnorm(6)
y <- ord(x)
# matrix
X <- matrix(rnorm(6*3), ncol=3)
Y <- ord(X)
```

---

orth	<i>Orthonormalization of a matrix</i>
------	---------------------------------------

---

**Description**

Returns an orthonormal basis for the range of A.

**Usage**

```
orth(A)
```

**Arguments**

A                    Matrix to be orthogonalized

**Value**

Q                    Orthonormal basis for the range of A

**Note**

The columns of Q span the same space as the columns of A with  $t(Q)Q=I$ .  
The number of columns of Q is the rank of A.

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**Examples**

```
X <- matrix(rnorm(6*3),ncol=3)
Y <- orth(X)
```

---

orthmax2

*Orthomax Rotation*

---

**Description**

Produces a simultaneous orthomax rotation of two matrices (using one rotation matrix).

**Usage**

```
orthmax2(A1, A2, gam1, gam2, conv)
```

**Arguments**

A1                    First matrix to be rotated with the same number of columns of A2  
A2                    Second matrix to be rotated with the same number of columns of A1  
gam1                  orthomax parameter for A1  
gam2                  orthomax parameter for A2  
conv                  Optional convergence value (default 1e-6)

**Value**

A list including the following components:

B1	Rotated version of A1
B2	Rotated version of A2
T	Rotation matrix
f	Orthomax function value

**Note**

The function to be maximized is  $f = \text{sum}((A1^2) - 1/m1 * \text{gam1} * \text{sum}((\text{sum}(A1^2))^2))^2 + \text{sum}((A2^2) - 1/m2 * \text{gam2} * \text{sum}((\text{sum}(A2^2))^2))^2$ .

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**References**

R. Jennrich (1970). Orthogonal rotation algorithms. *Psychometrika* 35:229–235.

**See Also**

[varim](#)

**Examples**

```
X <- matrix(rnorm(8*3),ncol=3)
Y <- matrix(rnorm(6*3),ncol=3)
orthXY <- orthmax2(X,Y,1,2)
# rotated version of X
orthXY$B1
# rotated version of Y
orthXY$B2
# rotation matrix
orthXY$T
```

---

pcamean

*PCA of the mean matrix*

---

**Description**

Performs Principal Component Analysis (PCA) of the mean matrix aggregated over mode number indicated by aggregmode.



**Usage**

```
pcamean(X, n, m, p, laba, labb, labc)
```

**Arguments**

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matrixed array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
laba	Optional vector of length n containing the labels of the A-mode entities
labb	Optional vector of length m containing the labels of the B-mode entities
labc	Optional vector of length p containing the labels of the C-mode entities

**Value**

A list including the following components:

Y	An object of class <code>matrix</code> containing the mean matrix
ev	A vector containing the eigenvalues of Y
A1	Component matrix for the A mode based on varimax rotation of loadings
B1	Component matrix for the B mode based on varimax rotation of loadings
C1	Component matrix for the C mode based on varimax rotation of loadings
A2	Component matrix for the A mode based on oblique 'HKIC' (Harris-Kaiser Independent Cluster) orthomax rotation of loadings
B2	Component matrix for the B mode based on oblique 'HKIC' (Harris-Kaiser Independent Cluster) orthomax rotation of loadings
C2	Component matrix for the C mode based on oblique 'HKIC' (Harris-Kaiser Independent Cluster) orthomax rotation of loadings

**Note**

`aggmode` denotes the mode over which means are computed (1 for A-mode, 2 for B-mode, 3 for C-mode).

`aggmode` is provided interactively.

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## References

- H. Kaiser (1958). The varimax criterion for analytic rotation in factor analysis. *Psychometrika* 23:187–200.
- C. Harris & H. Kaiser (1964). Some mathematical notes on three-mode factor analysis. *Psychometrika* 29:347–362.

## Examples

```
data(TV)
TVdata=TV[[1]]
labSCALE=TV[[2]]
labPROGRAM=TV[[3]]
labSTUDENT=TV[[4]]
# permutation of the modes so that the A-mode refers to students
TVdata <- permnew(TVdata, 16, 15, 30)
TVdata <- permnew(TVdata, 15, 30, 16)
## Not run:
# PCA on the mean matrix
TVpcamean <- pcamean(TVdata, 30, 16, 15, labSTUDENT, labSCALE, labPROGRAM)
# PCA on the mean matrix (when labels are not available)
TVpcamean <- pcamean(TVdata, 30, 16, 15)

## End(Not run)
```

---

pcasup1

*PCASup Analysis*


---

## Description

Computes PCASup analysis for the direction concerning the reduced mode.

## Usage

```
pcasup1(X, n, m, p, model)
```

## Arguments

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matricized array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
model	Tucker1 model choice (1 for T1-A, 2 for T1-B, 3 for T2-C)

**Value**

A list including the following components:

A	Matrix of the eigenvectors of the supermatrix containing the frontal slices of the array (A-mode)
B	Matrix of the eigenvectors of the supermatrix containing the horizontal slices of the array (B-mode)
C	Matrix of the eigenvectors of the supermatrix containing the lateral slices of the array (C-mode)
1a	Vector of the eigenvalues of the supermatrix containing the frontal slices of the array (A-mode)
1b	Vector of the eigenvalues of the supermatrix containing the horizontal slices of the array (B-mode)
1c	Vector of the eigenvalues of the supermatrix containing the lateral slices of the array (C-mode)

**Note**

pcasup1 computes the Tucker1 solution.

Cumulative sum of eigenvalues and fits from PCAsup applied to the reduced mode are automatically printed.

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**References**

H.A.L. Kiers (1991). Hierarchical relations among three-way methods. *Psychometrika* 56: 449–470.

H.A.L. Kiers (2000). Towards a standardized notation and terminology in multiway analysis. *Journal of Chemometrics* 14:105–122.

L.R Tucker (1966). Some mathematical notes on three-mode factor analysis. *Psychometrika* 31: 279–311.

**See Also**

[T1](#)

**Examples**

```
data(Bus)
# PCA-sup for T1-B
pcasupBus <- pcasup1(Bus, 7, 5, 37, 2)
```

pcasup2

*PCASup Analysis***Description**

Computes PCASup analysis for the directions concerning the reduced modes.

**Usage**

```
pcasup2(X, n, m, p, model)
```

**Arguments**

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matrixed array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
model	Tucker2 model choice (1 for T2-AB, 2 for T2-AC, 3 for T2-BC)

**Value**

A list including the following components:

A	Matrix of the eigenvectors of the supermatrix containing the frontal slices of the array (A-mode)
B	Matrix of the eigenvectors of the supermatrix containing the horizontal slices of the array (B-mode)
C	Matrix of the eigenvectors of the supermatrix containing the lateral slices of the array (C-mode)
1a	Vector of the eigenvalues of the supermatrix containing the frontal slices of the array (A-mode)
1b	Vector of the eigenvalues of the supermatrix containing the horizontal slices of the array (B-mode)
1c	Vector of the eigenvalues of the supermatrix containing the lateral slices of the array (C-mode)

**Note**

Cumulative sum of eigenvalues and fits from PCASup applied to the reduced modes are automatically printed.

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**References**

H.A.L. Kiers (1991). Hierarchical relations among three-way methods. *Psychometrika* 56: 449–470.  
H.A.L. Kiers (2000). Towards a standardized notation and terminology in multiway analysis. *Journal of Chemometrics* 14:105–122.  
L.R Tucker (1966). Some mathematical notes on three-mode factor analysis. *Psychometrika* 31: 279–311.

**See Also**

[T2](#)

**Examples**

```
data(Bus)
# PCA-sup for T2-AB
pcasupBus <- pcasup2(Bus, 7, 5, 37, 1)
```

---

pcasup3

*PCASup Analysis*

---

**Description**

Computes PCASup analysis in all the three directions.

**Usage**

```
pcasup3(X, n, m, p)
```

**Arguments**

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matricized array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities

**Value**

A list including the following components:

A	Matrix of the eigenvectors of the supermatrix containing the frontal slices of the array (A-mode)
B	Matrix of the eigenvectors of the supermatrix containing the horizontal slices of the array (B-mode)
C	Matrix of the eigenvectors of the supermatrix containing the lateral slices of the array (C-mode)
1a	Vector of the eigenvalues of the supermatrix containing the frontal slices of the array (A-mode)
1b	Vector of the eigenvalues of the supermatrix containing the horizontal slices of the array (B-mode)
1c	Vector of the eigenvalues of the supermatrix containing the lateral slices of the array (C-mode)

**Note**

pcasup3 computes the Tucker3 solution according to Tucker (1966).

Cumulative sum of eigenvalues and fits from PCAsup applied to the A-, B- and C-modes are automatically printed.

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**References**

H.A.L. Kiers (1991). Hierarchical relations among three-way methods. *Psychometrika* 56: 449–470.

H.A.L. Kiers (2000). Towards a standardized notation and terminology in multiway analysis. *Journal of Chemometrics* 14:105–122.

L.R Tucker (1966). Some mathematical notes on three-mode factor analysis. *Psychometrika* 31: 279–311.

**See Also**

[T3](#)

**Examples**

```
data(Bus)
## Not run:
# PCA-sup
pcasupBus <- pcasup3(Bus, 7, 5, 37)

## End(Not run)
```

---

percentile95	<i>95% percentile intervals</i>
--------------	---------------------------------

---

**Description**

Computes 2.5% and 97.5% percentiles for all columns of  $X$ .

**Usage**

```
percentile95(X)
```

**Arguments**

$X$	Matrix
-----	--------

**Value**

A list including the following components:

lo	Vector of the 2.5% percentiles of the values in the columns of $X$
up	Vector of the 97.5% percentiles of the values in the columns of $X$

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**See Also**

[bootstrapCP](#), [bootstrapT3](#)

**Examples**

```
X <- matrix(rnorm(50*3), ncol=3)  
perc95X <- percentile95(X)
```

---

permnew                      *Permutation of a matricized array*

---

### Description

Permutes the matricized ( $n \times m \times p$ ) array  $X$  to the matricized array  $Y$  of order ( $m \times p \times n$ ).

### Usage

```
permnew(X, n, m, p)
```

### Arguments

$X$	Matrix (or data.frame coerced to a matrix) containing the matricized array
$n$	Number of A-mode entities of the array $X$
$m$	Number of B-mode entities of the array $X$
$p$	Number of C-mode entities of the array $X$

### Value

$Y$	Matrix containing the permuted matricized array
-----	---

### Author(s)

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### References

H.A.L. Kiers (2000). Towards a standardized notation and terminology in multiway analysis. *Journal of Chemometrics* 14:105–122.

### Examples

```
X <- array(c(rnorm(120)),c(6,5,4))
dim(X)
# matricized array
Xa <- supermat(X)$Xa
# matricized X with the A-mode entities in its rows
dim(Xa)
# matricized X with the B-mode entities in its rows
Xb <- permnew(Xa, 6, 5, 4)
dim(Xb)
# matricized X with the C-mode entities in its rows
Xc <- permnew(Xb, 5, 4, 6)
dim(Xc)
```



---

perms

*Permutation*

---

### Description

Gives all the permutations of the first integer numbers.

### Usage

```
perms(n)
```

### Arguments

n                    Integer

### Value

z                    Matrix containing in its rows all the permutation of the first n integer numbers

### Author(s)

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### References

H.A.L. Kiers (2004). Bootstrap confidence intervals for three-way methods. *Journal of Chemometrics* 18:22–36.

### Examples

```
P <- perms(4)
```

---

phi

*Phi coefficient*

---

### Description

Computes the phi coefficients among columns of two matrices.

### Usage

```
phi(a,b)
```

**Arguments**

a                    Vector or matrix of the same order of b  
 b                    Vector or matrix of the same order of a

**Value**

p                    Matrix containing the phi coefficients

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**References**

L.R Tucker (1951). A method for synthesis of factor analysis studies. *Personnel Research Section Report No. 984*. Department of the Army, Washington, DC.

**Examples**

```
X <- matrix(rnorm(6*3),ncol=3)
Y <- matrix(rnorm(6*3),ncol=3)
P <- phi(X,Y)
```

---

 rarray

*Array reconstruction*


---

**Description**

Produces an array starting from its matricization with all the frontal slices of the array next to each other.

**Usage**

```
rarray(Xa, n, m, p)
```

**Arguments**

Xa                   Matrix (or data.frame coerced to a matrix) containing the elements of the frontal slices of an array  
 n                    Number of A-mode entities  
 m                    Number of B-mode entities  
 p                    Number of C-mode entities

**Value**

X                      Array leading to  $\lambda_a$

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**References**

H.A.L. Kiers (2000). Towards a standardized notation and terminology in multiway analysis. *Journal of Chemometrics* 14:105–122.

**Examples**

```
# matricized array (frontal slice)
Xa <- matrix(1:8,nrow=2)
X <- rarray(Xa, 2, 2, 2)
# original array
X
```

---

renormsolCP

*Scaling of the Candecomp/Parafac solution*


---

**Description**

Scales the Candecomp/Parafac solution producing two component matrices normalized to unit sum of squares (and compensating this scaling in the remaining component matrix).

**Usage**

```
renormsolCP(A, B, C, mode)
```

**Arguments**

A	Component matrix for the A-mode
B	Component matrix for the B-mode
C	Component matrix for the C-mode
mode	Scaling option (1 if scaling for B- and C-modes, 2 if scaling for A- and C-modes, 3 if scaling for A- and B-modes)

**Value**

A list including the following components:

A	Component matrix for the A-mode after normalization
B	Component matrix for the B-mode after normalization
C	Component matrix for the C-mode after normalization

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**Examples**

```
data(TV)
TVdata=TV[[1]]
# permutation of the modes so that the A-mode refers to students
TVdata <- permnew(TVdata, 16, 15, 30)
TVdata <- permnew(TVdata, 15, 30, 16)
# CP solution
TVcp <- CPfuncrep(TVdata, 30, 16, 15, 2, 1, 1, 1, 0, 1e-6, 10000)
# sums of squares of A, B and C
sum(TVcp$A^2)
sum(TVcp$B^2)
sum(TVcp$C^2)
# Renormalization by scaling B- and C-modes
TVcpScalBC <- renormsolCP(TVcp$A, TVcp$B, TVcp$C, 1)
# sums of squares of A, B and C after renormalization
sum(TVcpScalBC$A^2)
sum(TVcpScalBC$B^2)
sum(TVcpScalBC$C^2)
```

---

renormsolT3

*Renormalization of the Tucker3 (and Tucker2) solution*


---

**Description**

Renormalizes the Tucker3 solution producing a core normalized to unit sum of squares (and compensating the core normalization in the component matrices).

**Usage**

```
renormsolT3(A, B, C, G, mode)
```

**Arguments**

A	Component matrix for the A-mode
B	Component matrix for the B-mode
C	Component matrix for the C-mode
G	Matricized core array (frontal slices)
mode	Renormalization option (1 if renormalization with respect to A-mode, 2 if renormalization with respect to B-mode, 3 if renormalization with respect to C-mode)

**Value**

A list including the following components:

A	Component matrix for the A-mode after normalization of the core
B	Component matrix for the B-mode after normalization of the core
C	Component matrix for the C-mode after normalization of the core
H	Normalized matricized core array (frontal slices)

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**Examples**

```
data(Bus)
# labels for Bus data
laba <- rownames(Bus)
labb <- substr(colnames(Bus)[1:5], 1, 1)
labc <- substr(colnames(Bus)[seq(1,ncol(Bus),5)], 3, 8)
# T3 solution
BusT3 <- T3funcrep(Bus, 7, 5, 37, 2, 2, 2, 0, 1e-6)
# sums of squares of A and core
sum(BusT3$A^2)
sum(BusT3$H^2)
# Renormalization with respect to the A-mode
BusT3rA <- renormsolt3(BusT3$A, BusT3$B, BusT3$C, BusT3$H,1)
# sums of squares of A and core after renormalization
sum(BusT3rA$A^2)
sum(BusT3rA$H^2)
```

---

splithalfCP

*Split-Half Analysis*

---

**Description**

Performs split-half analysis for Candecomp/Parafac.

**Usage**

```
splithalfCP(X, n, m, p, r, centopt, normopt, scaleopt, addanal, conv,
maxit, ort1, ort2, ort3, laba, labb, labc)
```

**Arguments**

<code>X</code>	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matrixed array (frontal slices)
<code>n</code>	Number of A-mode entities
<code>m</code>	Number of B-mode entities
<code>p</code>	Number of C-mode entities
<code>r</code>	Number of extracted components
<code>centopt</code>	Centering option (see <a href="#">cent3</a> )
<code>normopt</code>	Normalization option (see <a href="#">norm3</a> )
<code>scaleopt</code>	Scaling option (see <a href="#">renormsolCP</a> )
<code>addanal</code>	Number of additional runs
<code>conv</code>	Convergence criterion
<code>maxit</code>	Maximal number of iterations
<code>ort1</code>	Type of constraints on A (see <a href="#">CP</a> )
<code>ort2</code>	Type of constraints on B (see <a href="#">CP</a> )
<code>ort3</code>	Type of constraints on C (see <a href="#">CP</a> )
<code>laba</code>	Optional vector of length n containing the labels of the A-mode entities
<code>labb</code>	Optional vector of length m containing the labels of the B-mode entities
<code>labc</code>	Optional vector of length p containing the labels of the C-mode entities

**Value**

<code>Afull</code>	Component matrix for the A-mode (full data)
<code>As1</code>	Component matrix for the A-mode (split n.1)
<code>As2</code>	Component matrix for the A-mode (split n.2)
<code>Bfull</code>	Component matrix for the B-mode (full data)
<code>Bs1</code>	Component matrix for the B-mode (split n.1)
<code>Bs2</code>	Component matrix for the B-mode (split n.2)
<code>Cfull</code>	Component matrix for the C-mode (full data)
<code>Cs1</code>	Component matrix for the C-mode (split n.1)
<code>Cs2</code>	Component matrix for the C-mode (split n.2)

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**References**

P.M. Kroonenberg (2008). *Applied Multiway Data Analysis*. Wiley, New Jersey.

**See Also**[CP](#)**Examples**

```

data(TV)
TVdata=TV[[1]]
labSCALE=TV[[2]]
labPROGRAM=TV[[3]]
labSTUDENT=TV[[4]]
# permutation of the modes so that the A-mode refers to students
TVdata <- permnew(TVdata, 16, 15, 30)
TVdata <- permnew(TVdata, 15, 30, 16)
## Not run:
# Split-half analysis on CP solution
splitCP <- splithalfCP(TVdata, 30, 16, 15, 2, 0, 0, 0, 5, 1e-6, 10000, 1, 1, 1,
  labSTUDENT, labSCALE, labPROGRAM)
# Split-half analysis on CP solution (when labels are not available)
splitCP <- splithalfCP(TVdata, 30, 16, 15, 2, 0, 0, 0, 5, 1e-6, 10000, 1, 1, 1)

## End(Not run)

```

splithalfT3

*Split-Half Analysis***Description**

Performs split-half analysis for Tucker3.

**Usage**

```
splithalfT3(X, n, m, p, r1, r2, r3, centopt, normopt, renormmode,
  wa_rel, wb_rel, wc_rel, addanal, conv, laba, labb, labc)
```

**Arguments**

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matrixed array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
r1	Number of extracted components for the A-mode
r2	Number of extracted components for the B-mode
r3	Number of extracted components for the C-mode
centopt	Centering option (see <a href="#">cent3</a> )
normopt	Normalization option (see <a href="#">norm3</a> )

renormmode	Renormalization option (see <a href="#">renormsolT3</a> )
wa_rel	Relative weight for simplicity of A-mode
wb_rel	Relative weight for simplicity of B-mode
wc_rel	Relative weight for simplicity of C-mode
addanal	Number of additional runs
conv	Convergence criterion
laba	Optional vector of length n containing the labels of the A-mode entities
labb	Optional vector of length m containing the labels of the B-mode entities
labc	Optional vector of length p containing the labels of the C-mode entities

**Value**

Afull	Component matrix for the A-mode (full data)
As1	Component matrix for the A-mode (split n.1)
As2	Component matrix for the A-mode (split n.2)
Bfull	Component matrix for the B-mode (full data)
Bs1	Component matrix for the B-mode (split n.1)
Bs2	Component matrix for the B-mode (split n.2)
Cfull	Component matrix for the C-mode (full data)
Cs1	Component matrix for the C-mode (split n.1)
Cs2	Component matrix for the C-mode (split n.2)
Kfull	Matricized core array (frontal slices) (full data)
Ks1	Matricized core array (frontal slices) (split n.1)
Ks2	Matricized core array (frontal slices) (split n.2)
Kss1	Matricized core array (frontal slices) (using full data solutions for A,B and C for split n.1)
Kss2	Matricized core array (frontal slices) (using full data solutions for A,B and C for split n.2)

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**References**

P.M. Kroonenberg (2008). *Applied Multiway Data Analysis*. Wiley, New Jersey.

**See Also**

[T3](#)



**Examples**

```

data(Bus)
# labels for Bus data
laba <- rownames(Bus)
labb <- substr(colnames(Bus)[1:5],1,1)
labc <- substr(colnames(Bus)[seq(1,ncol(Bus),5)],3,8)
## Not run:
# Split-half analysis on T3 solution
splitT3 <- splithalfT3(Bus, 7, 5, 37, 2, 2, 2, 0, 0, 0, 3, 3, 0, 5, 1e-6,
  laba, labb, labc)
# Split-half analysis on T3 solution (when labels are not available)
splitT3 <- splithalfT3(Bus, 7, 5, 37, 2, 2, 2, 0, 0, 0, 3, 3, 0, 5, 1e-6)

## End(Not run)

```

SUM

*Summary***Description**

Summary of the elements of a matrix.

**Usage**

```
SUM(A)
```

**Arguments**

A Matrix or data.frame (coerced to a matrix)

**Value**

A list including the following components:

row	Vector containing the sum of squares of every row
col	Vector containing the sum of squares of every column
mr	Vector containing the mean of every row
mc	Vector containing the mean of every column
minc	Vector containing the minimum of every column
maxc	Vector containing the maximum of every for column
valueMinr	Vector containing the columns corresponding to the minimum values of every row
valueMinc	Vector containing the rows corresponding to the minimum values of every column
valueMaxr	Vector containing the columns corresponding to the maximum values of every row

valueMaxc	Vector containing the rows corresponding to the maximum values of every column
ssq	Sum of squares of the matrix
cumsumr	Matrix containing the cumulative sums of every row
cumsumc	Matrix containing the cumulative sums of every column

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**Examples**

```
X <- matrix(rnorm(6*3),ncol=3)
summary <- SUM(X)
```

---

supermat	<i>Matrix unfolding</i>
----------	-------------------------

---

**Description**

Produces matricizations of a three-way array into matrices denoted as super-matrices.

**Usage**

```
supermat(X)
```

**Arguments**

X                    Array to be unfolded

**Value**

A list including the following components:

Xa	Super-matrix with B-mode entities nested within C-mode entities (all the frontal slices of the array next to each other)
Xb	Super-matrix with C-mode entities nested within A-mode entities (all the horizontal slices of the array next to each other)
Xc	Super-matrix with A-mode entities nested within B-mode entities (all the lateral slices of the array next to each other)

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## References

H.A.L. Kiers (2000). Towards a standardized notation and terminology in multiway analysis. *Journal of Chemometrics* 14:105–122.

## Examples

```
# array (2x2x2) with integers from 1 to 8
X <- array(1:8,c(2,2,2))
Y <- supermat(X)
# matricized arrays
Y$Xa
Y$Xb
Y$Xc
```

---

T1

*Interactive Tucker1 analysis*


---

## Description

Detects the underlying structure of a three-way array according to the Tucker1 (T1) model.

## Usage

```
T1(dati, laba, labb, labc)
```

## Arguments

dati	Array of order n by m by p or matrix or data.frame of order (n x mp) containing the matricized array (frontal slices)
laba	Optional vector of length n containing the labels of the A-mode entities
labb	Optional vector of length m containing the labels of the B-mode entities
labc	Optional vector of length p containing the labels of the C-mode entities

## Value

A list including the following components:

A	Component matrix for the A-mode
B	Component matrix for the B-mode
C	Component matrix for the C-mode
core	Matricized core array (frontal slices)
fit	Fit value expressed as a percentage
fitA	Fit contributions for the A-mode entities (see <a href="#">T3fitpartitioning</a> )
fitB	Fit contributions for the B-mode entities (see <a href="#">T3fitpartitioning</a> )

fitC	Fit contributions for the C-mode entities (see <a href="#">T3fitpartitioning</a> )
laba	Vector of length n containing the labels of the A-mode entities
labb	Vector of length m containing the labels of the B-mode entities
labc	Vector of length P containing the labels of the C-mode entities
Xprep	Matrix of order (n x mp) containing the matricized array (frontal slices) after preprocessing used for the analysis

### Author(s)

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### References

P. Giordani, H.A.L. Kiers, M.A. Del Ferraro (2014). Three-way component analysis using the R package ThreeWay. *Journal of Statistical Software* 57(7):1–23. <http://www.jstatsoft.org/v57/i07/>.  
 P.M. Kroonenberg (2008). *Applied Multiway Data Analysis*. Wiley, New Jersey.  
 L.R. Tucker (1966). Some mathematical notes on three-mode factor analysis. *Psychometrika* 31:279–311.

### See Also

[CP](#), [T3](#), [T2](#)

### Examples

```
data(Bus)
# labels for Bus data
laba <- rownames(Bus)
labb <- substr(colnames(Bus)[1:5],1,1)
labc <- substr(colnames(Bus)[seq(1,ncol(Bus),5)],3,8)
## Not run:
# interactive T1 analysis
BusT1 <- T1(Bus, laba, labb, labc)
# interactive T1 analysis (when labels are not available)
BusT1 <- T1(Bus)

## End(Not run)
```

---

T1runsFit                      *Tucker1 solutions*

---

### Description

Computes all the Tucker1 solutions using PCASup results with  $r_1$  (from 1 to  $max_a$ , if A-mode reduced),  $r_2$  (from 1 to  $max_b$ , if B-mode reduced) and  $r_3$  (from 1 to  $max_c$ , if C-mode reduced) components.

### Usage

```
T1runsFit(X, n, m, p, maxa, maxb, maxc, model)
```

### Arguments

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matrixed array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
maxa	Maximum dimensionality for the A-mode
maxb	Maximum dimensionality for the B-mode
maxc	Maximum dimensionality for the C-mode
model	Tucker1 model choice (1 for T1-A, 2 for T1-B, 3 for T2-C)

### Value

out	Matrix with columns: number of components for the A-mode, number of components for the B-mode, number of components for the C-mode, goodness of fit (%), total number of components
-----	---

### Note

Cumulative sum of eigenvalues and fits from PCASup applied to the reduced mode are automatically printed.

### Author(s)

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### References

H.A.L. Kiers (1991). Hierarchical relations among three-way methods. *Psychometrika* 56:449–470.

**See Also**

[DimSelector](#), [LineCon](#), [pcasup1](#), [T1](#)

**Examples**

```
data(Bus)
# Fit values of T1-A with different numbers of components (from 1 to 5)
FitT1 <- T1runsFit(Bus, 7, 5, 37, 5, 5, 37, 1)
```

---

T2

*Interactive Tucker2 analysis*


---

**Description**

Detects the underlying structure of a three-way array according to the Tucker2 (T2) model.

**Usage**

```
T2(dati, laba, labb, labc)
```

**Arguments**

<code>dati</code>	Array of order $n \times m \times p$ or matrix or data.frame of order $(n \times mp)$ containing the matricized array (frontal slices)
<code>laba</code>	Optional vector of length $n$ containing the labels of the A-mode entities
<code>labb</code>	Optional vector of length $m$ containing the labels of the B-mode entities
<code>labc</code>	Optional vector of length $p$ containing the labels of the C-mode entities

**Value**

A list including the following components:

<code>A</code>	Component matrix for the A-mode
<code>B</code>	Component matrix for the B-mode
<code>C</code>	Component matrix for the C-mode
<code>core</code>	Matricized core array (frontal slices)
<code>fit</code>	Fit value expressed as a percentage
<code>fitValues</code>	Fit values expressed as a percentage upon convergence for all the runs of the CP algorithm (see <a href="#">T2func</a> )
<code>funcValues</code>	Function values upon convergence for all the runs of the CP algorithm (see <a href="#">T2func</a> )
<code>cputime</code>	Computation times for all the runs of the CP algorithm (see <a href="#">T2func</a> )
<code>iter</code>	Numbers of iterations upon convergence for all the runs of the CP algorithm (see <a href="#">T2func</a> )

fitA	Fit contributions for the A-mode entities (see <a href="#">T3fitpartitioning</a> )
fitB	Fit contributions for the B-mode entities (see <a href="#">T3fitpartitioning</a> )
fitC	Fit contributions for the C-mode entities (see <a href="#">T3fitpartitioning</a> )
fitAB	Fit contributions for the A-and mode B component combinations (see <a href="#">T3fitpartitioning</a> )
fitAC	Fit contributions for the A-and mode C component combinations (see <a href="#">T3fitpartitioning</a> )
fitBC	Fit contributions for the B-and mode C component combinations (see <a href="#">T3fitpartitioning</a> )
laba	Vector of length n containing the labels of the A-mode entities
labb	Vector of length m containing the labels of the B-mode entities
labc	Vector of length P containing the labels of the C-mode entities
Xprep	Matrix of order (n x mp) containing the matricized array (frontal slices) after preprocessing used for the analysis

### Author(s)

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 Henk A.L. Kiers <h.a.l.kiers@rug.nl>  
 Paolo Giordani <paolo.giordani@uniroma1.it>

### References

P. Giordani, H.A.L. Kiers, M.A. Del Ferraro (2014). Three-way component analysis using the R package ThreeWay. *Journal of Statistical Software* 57(7):1–23. <http://www.jstatsoft.org/v57/i07/>.  
 P.M. Kroonenberg (2008). *Applied Multiway Data Analysis*. Wiley, New Jersey.  
 L.R Tucker (1966). Some mathematical notes on three-mode factor analysis. *Psychometrika* 31:279–311.

### See Also

[CP](#), [T3](#), [T1](#)

### Examples

```
data(Bus)
# labels for Bus data
laba <- rownames(Bus)
labb <- substr(colnames(Bus)[1:5], 1, 1)
labc <- substr(colnames(Bus)[seq(1,ncol(Bus),5)], 3, 8)
## Not run:
# interactive T2 analysis
BusT2 <- T2(Bus, laba, labb, labc)
# interactive T2 analysis (when labels are not available)
BusT2 <- T2(Bus)

## End(Not run)
```

T2func

*Algorithm for the Tucker2 model***Description**

Alternating Least Squares algorithm for the minimization of the Tucker2 loss function.

**Usage**

```
T2func(X, n, m, p, r1, r2, r3, start, conv, model, A, B, C, H)
```

**Arguments**

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matricized array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
r1	Number of extracted components for the A-mode
r2	Number of extracted components for the B-mode
r3	Number of extracted components for the C-mode
start	Starting point: 0 starting point of the algorithm from generalized eigenvalue decomposition, 1 random starting point (orthonormalized component matrices), 2 if users specified component matrices
conv	Convergence criterion
model	Tucker2 model choice (1 for T2-AB, 2 for T2-AC, 3 for T2-BC)
A	Optional (necessary if start=2) starting value for A
B	Optional (necessary if start=2) starting value for B
C	Optional (necessary if start=2) starting value for C
H	Optional (necessary if start=2) starting value for the matricized core array (frontal slices)

**Value**

A list including the following components:

A	Orthonormal component matrix for the A-mode
B	Orthonormal component matrix for the B-mode
C	Orthonormal component matrix for the C-mode
H	Matricized core array (frontal slices)
f	Loss function value



fp	Fit percentage
iter	Number of iterations
cputime	Computation time
La	Matrix which should be diagonal, and if so, contain ‘intrinsic eigenvalues’ for A-mode
Lb	Matrix which should be diagonal, and if so, contain ‘intrinsic eigenvalues’ for B-mode
Lc	Matrix which should be diagonal, and if so, contain ‘intrinsic eigenvalues’ for C-mode

**Note**

The loss function to be minimized is  $\|X_A - AG_A \text{kron}(C', B')\|^2$  where  $X_A$  and  $G_A$  denote the matricized (frontal slices) data array and core array, respectively, and  $\text{kron}$  stands for the Kronecker product.

T2func is the same as T2funcprep except that all printings are available.

**Author(s)**

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Paolo Giordani <paolo.giordani@uniroma1.it>

**References**

H.A.L. Kiers, P.M. Kroonenberg & J.M.F. ten Berge (1992). An efficient algorithm for TUCK-ALS3 on data with large numbers of observation units. *Psychometrika* 57:415–422.

P.M. Kroonenberg and J. de Leeuw (1980). Principal component analysis of three-mode data by means of alternating least squares algorithms. *Psychometrika* 45:69–97.

**See Also**

[T2](#), [T2funcprep](#)

**Examples**

```
data(Bus)
# labels for Bus data
laba <- rownames(Bus)
labb <- substr(colnames(Bus)[1:5], 1, 1)
labc <- substr(colnames(Bus)[seq(1,ncol(Bus),5)], 3, 8)
# T2-AB solution using two components for the A- and B-modes
# (rational starting point by SVD [start=0])
BusT2 <- T2func(Bus, 7, 5, 37, 2, 2, 37, 0, 1e-6, 1)
# T2-AC solution using two components for the A- and C-modes
# (random orthonormalized starting point [start=1])
BusT2 <- T2func(Bus, 7, 5, 37, 2, 5, 2, 1, 1e-6, 2)
# T2-BC solution using two components for the B- and C- modes
# (user starting point [start=2])
```

```
BusT2 <- T2func(Bus, 7, 5, 37, 7, 2, 2, 1, 1e-6, 3, diag(7),
  matrix(rnorm(5*2),nrow=5), matrix(rnorm(37*2),nrow=37),
  matrix(rnorm(7*4),nrow=7))
```

---

T2funcrep

*Algorithm for the Tucker2 model*


---

### Description

Alternating Least Squares algorithm for the minimization of the Tucker2 loss function.

### Usage

```
T2funcrep(X, n, m, p, r1, r2, r3, start, conv, model, A, B, C, H)
```

### Arguments

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matricized array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
r1	Number of extracted components for the A-mode
r2	Number of extracted components for the B-mode
r3	Number of extracted components for the C-mode
start	Starting point: 0 starting point of the algorithm from generalized eigenvalue decomposition, 1 random starting point (orthonormalized component matrices), 2 if users specified component matrices
conv	Convergence criterion
model	Tucker2 model choice (1 for T2-AB, 2 for T2-AC, 3 for T2-BC)
A	Optional (necessary if start=2) starting value for A
B	Optional (necessary if start=2) starting value for B
C	Optional (necessary if start=2) starting value for C
H	Optional (necessary if start=2) starting value for the matricized core array (frontal slices)

**Value**

A list including the following components:

A	Orthonormal component matrix for the A-mode
B	Orthonormal component matrix for the B-mode
C	Orthonormal component matrix for the C-mode
H	Matricized core array (frontal slices)
f	Loss function value
fp	Fit percentage
iter	Number of iterations
cputime	Computation time
La	Matrix which should be diagonal, and if so, contain ‘intrinsic eigenvalues’ for A-mode
Lb	Matrix which should be diagonal, and if so, contain ‘intrinsic eigenvalues’ for B-mode
Lc	Matrix which should be diagonal, and if so, contain ‘intrinsic eigenvalues’ for C-mode

**Note**

The loss function to be minimized is  $\|X_A - AG_A \text{kron}(C', B')\|^2$  where  $X_A$  and  $G_A$  denote the matricized (frontal slices) data array and core array, respectively, and  $\text{kron}$  stands for the Kronecker product.

T2funcprep is the same as T2func except that all printings are suppressed. Thus, T2funcprep can be helpful for simulation experiments.

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**References**

H.A.L. Kiers, P.M. Kroonenberg & J.M.F. ten Berge (1992). An efficient algorithm for TUCK-ALS3 on data with large numbers of observation units. *Psychometrika* 57:415–422.

P.M. Kroonenberg and J. de Leeuw (1980). Principal component analysis of three-mode data by means of alternating least squares algorithms. *Psychometrika* 45:69–97.

**See Also**

[T2](#), [T2func](#)

**Examples**

```

data(Bus)
# labels for Bus data
laba <- rownames(Bus)
labb <- substr(colnames(Bus)[1:5], 1, 1)
labc <- substr(colnames(Bus)[seq(1,ncol(Bus),5)], 3, 8)
# T2-AB solution using two components for the A- and B-modes
# (rational starting point by SVD [start=0])
BusT2 <- T2funcrep(Bus, 7, 5, 37, 2, 2, 37, 0, 1e-6,1)
# T2-AC solution using two components for for the A- and C-modes
# (random orthonormalized starting point [start=1])
BusT2 <- T2funcrep(Bus, 7, 5, 37, 2, 5, 2, 1, 1e-6, 2)
# T2-BC solution using two components for the B- and C- modes
# (user starting point [start=2])
BusT2 <- T2funcrep(Bus, 7, 5, 37, 7, 2, 2, 1, 1e-6, 3, diag(7),
  matrix(rnorm(5*2),nrow=5), matrix(rnorm(37*2),nrow=37),
  matrix(rnorm(7*4),nrow=7))

```

---

T2runsApproxFit

*Approximated Tucker2 solutions*


---

**Description**

Computes all the approximated Tucker2 solutions using PCASup results with  $r_1$  (from 1 to  $\max_a$ , if A-mode reduced),  $r_2$  (from 1 to  $\max_b$ , if B-mode reduced) and  $r_3$  (from 1 to  $\max_c$ , if C-mode reduced) components.

**Usage**

```
T2runsApproxFit(X, n, m, p, maxa, maxb, maxc, model)
```

**Arguments**

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matricized array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
maxa	Maximum dimensionality for the A-mode
maxb	Maximum dimensionality for the B-mode
maxc	Maximum dimensionality for the C-mode
model	Tucker2 model choice (1 for T2-AB, 2 for T2-AC, 3 for T2-BC)

**Value**

out Matrix with columns: number of components for the A-mode, number of components for the B-mode, number of components for the C-mode, goodness of fit (%), total number of components

**Note**

Cumulative sum of eigenvalues and fits from PCAsup applied to the reduced modes are automatically printed.

**Author(s)**

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Paolo Giordani <paolo.giordani@uniroma1.it>

**References**

H.A.L. Kiers (1991). Hierarchical relations among three-way methods. *Psychometrika* 56:449–470.

**See Also**

[DimSelector](#), [LineCon](#), [pcasup2](#), [T2](#)

**Examples**

```
data(Bus)
# Fit values of T2-AB with different numbers of components
# (from 1 to 3 for the B-mode, from 1 to 5 for the C-mode)
FitT2 <- T2runsApproxFit(Bus, 7, 5, 37, 7, 3, 5, 3)
```

---

T3

*Interactive Tucker3 analysis*

---

**Description**

Detects the underlying structure of a three-way array according to the Tucker3 (T3) model.

**Usage**

```
T3(data, laba, labb, labc)
```

**Arguments**

data	Array of order $n \times m \times p$ or matrix or data.frame of order $(n \times mp)$ containing the matricized array (frontal slices)
laba	Optional vector of length $n$ containing the labels of the A-mode entities
labb	Optional vector of length $m$ containing the labels of the B-mode entities
labc	Optional vector of length $p$ containing the labels of the C-mode entities

**Value**

A list including the following components:

A	Component matrix for the A-mode
B	Component matrix for the B-mode
C	Component matrix for the C-mode
core	Matricized core array (frontal slices)
fit	Fit value expressed as a percentage
fitValues	Fit values expressed as a percentage upon convergence for all the runs of the CP algorithm (see <a href="#">T3func</a> )
funcValues	Function values upon convergence for all the runs of the CP algorithm (see <a href="#">T3func</a> )
cputime	Computation times for all the runs of the CP algorithm (see <a href="#">T3func</a> )
iter	Numbers of iterations upon convergence for all the runs of the CP algorithm (see <a href="#">T3func</a> )
fitA	Fit contributions for the A-mode entities (see <a href="#">T3fitpartitioning</a> )
fitB	Fit contributions for the B-mode entities (see <a href="#">T3fitpartitioning</a> )
fitC	Fit contributions for the C-mode entities (see <a href="#">T3fitpartitioning</a> )
fitAB	Fit contributions for the A-and mode B component combinations (see <a href="#">T3fitpartitioning</a> )
fitAC	Fit contributions for the A-and mode C component combinations (see <a href="#">T3fitpartitioning</a> )
fitBC	Fit contributions for the B-and mode C component combinations (see <a href="#">T3fitpartitioning</a> )
Bint	Bootstrap percentile interval of every element of B (see <a href="#">bootstrapT3</a> )
Cint	Bootstrap percentile interval of every element of C (see <a href="#">bootstrapT3</a> )
Kint	Bootstrap percentile interval of every element of core (see <a href="#">bootstrapT3</a> )
fpint	Bootstrap percentile interval for the goodness of fit index expressed as a percentage (see <a href="#">bootstrapT3</a> )
Afull	Component matrix for the A-mode (full data) from split-half analysis (see <a href="#">splithalfT3</a> )
As1	Component matrix for the A-mode (split n.1) from split-half analysis (see <a href="#">splithalfT3</a> )
As2	Component matrix for the A-mode (split n.2) from split-half analysis (see <a href="#">splithalfT3</a> )
Bfull	Component matrix for the B-mode (full data) from split-half analysis (see <a href="#">splithalfT3</a> )
Bs1	Component matrix for the B-mode (split n.1) from split-half analysis (see <a href="#">splithalfT3</a> )
Bs2	Component matrix for the B-mode (split n.2) from split-half analysis (see <a href="#">splithalfT3</a> )

Cfull	Component matrix for the C-mode (full data) from split-half analysis (see <a href="#">splithalfT3</a> )
Cs1	Component matrix for the C-mode (split n.1) from split-half analysis (see <a href="#">splithalfT3</a> )
Cs2	Component matrix for the C-mode (split n.2) from split-half analysis (see <a href="#">splithalfT3</a> )
Kfull	Matricized core array (frontal slices) (full data) from split-half analysis (see <a href="#">splithalfT3</a> )
Ks1	Matricized core array (frontal slices) (split n.1) from split-half analysis (see <a href="#">splithalfT3</a> )
Ks2	Matricized core array (frontal slices) (split n.2) from split-half analysis (see <a href="#">splithalfT3</a> )
Kss1	Matricized core array (frontal slices) (using full data solutions for A,B and C for split n.1) from split-half analysis (see <a href="#">splithalfT3</a> )
Kss2	Matricized core array (frontal slices) (using full data solutions for A,B and C for split n.2) from split-half analysis (see <a href="#">splithalfT3</a> )
Aplot	Coordinates for plots of the A-mode entities
Bplot	Coordinates for plots of the B-mode entities
Cplot	Coordinates for plots of the C-mode entities
CBplot	Coordinates for plots of the C and B-mode entities using the A-mode projected in it as axes (to be added in plot, i.e. coordinates in ( $\$CBplot, \$A$ ))
ACplot	Coordinates for plots of the A and C-mode entities using the B-mode projected in it as axes (to be added in plot, i.e. coordinates in ( $\$ACplot, \$B$ ))
BAplot	Coordinates for plots of the B and A-mode entities using the C-mode projected in it as axes (to be added in plot, i.e. coordinates in ( $\$BAplot, \$C$ ))
A1	Component matrix for the A-mode from Principal Component Analysis of mean values (see <a href="#">pcamean</a> )
B1	Component matrix for the B-mode from Principal Component Analysis of mean values (see <a href="#">pcamean</a> )
C1	Component matrix for the C-mode from Principal Component Analysis of mean values (see <a href="#">pcamean</a> )
A2	Component matrix for the A-mode from Principal Component Analysis of mean values (see <a href="#">pcamean</a> )
B2	Component matrix for the B-mode from Principal Component Analysis of mean values (see <a href="#">pcamean</a> )
C2	Component matrix for the C-mode from Principal Component Analysis of mean values (see <a href="#">pcamean</a> )
laba	Vector of length n containing the labels of the A-mode entities
labb	Vector of length m containing the labels of the B-mode entities
labc	Vector of length P containing the labels of the C-mode entities
Xprep	Matrix of order (n x mp) containing the matricized array (frontal slices) after preprocessing used for the analysis

**Author(s)**

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Paolo Giordani <paolo.giordani@uniroma1.it>

**References**

P. Giordani, H.A.L. Kiers, M.A. Del Ferraro (2014). Three-way component analysis using the R package ThreeWay. *Journal of Statistical Software* 57(7):1–23. <http://www.jstatsoft.org/v57/i07/>.  
P.M. Kroonenberg (2008). *Applied Multiway Data Analysis*. Wiley, New Jersey.  
L.R Tucker (1966). Some mathematical notes on three-mode factor analysis. *Psychometrika* 31:279–311.

**See Also**

[CP,T2,T1](#)

**Examples**

```
data(Bus)
# labels for Bus data
laba <- rownames(Bus)
labb <- substr(colnames(Bus)[1:5],1,1)
labc <- substr(colnames(Bus)[seq(1,ncol(Bus),5)],3,8)
## Not run:
# interactive T3 analysis
BusT3 <- T3(Bus, laba, labb, labc)
# interactive T3 analysis (when labels are not available)
BusT3 <- T3(Bus)

## End(Not run)
```

---

T3dimensionalityplot *Plot fit of Tucker3*

---

**Description**

Plots fits against numbers of dimensions, with PQR as labels and fits against number of effective paramaters.

**Usage**

```
T3dimensionalityplot(A, n, m, p)
```



**Arguments**

A	Matrix with columns: number of components for the A-mode, number of components for the B-mode, number of components for the C-mode, goodness of fit (%), total number of components
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities

**Note**

A is usually the output of [DimSelector](#).

The number of effective parameters in a Candecomp/Parafac analysis is discussed in Weesie and Van Houwelingen (1983).

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**References**

E. Ceulemans & H.A.L. Kiers (2006). Selecting among three-mode principal component models of different types and complexities: A numerical convex hull based method. *British Journal of Mathematical and Statistical Psychology* 59:133–150.

J. Weesie and H. Van Houwelingen (1983). *GEPCAM users' manual (first draft)*. Utrecht, The Netherlands: Institute of Mathematical Statistics, State University of Utrecht.

**See Also**

[T3](#), [DimSelector](#)

**Examples**

```
data(Bus)
# Fit values of T3 with different numbers of components (from 1 to 4 for the A-mode,
# from 1 to 3 for the B-mode, from 1 to 5 for the C-mode)
FitT3 <- T3runsApproxFit(Bus,7,5,37,4,3,5)
T3dimensionalityplot(FitT3,7,5,37)
```

---

T3fitpartitioning      *Fit of each entity per mode*

---

### Description

Computation of fit contributions by combinations of modes in case of ‘renormalization’.

### Usage

T3fitpartitioning(Xprep, n, m, p, AS, BT, CU, K, renormmode, laba, labb, labc)

### Arguments

Xprep	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matricized array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
AS	Component matrix for the A-mode
BT	Component matrix for the B-mode
CU	Component matrix for the C-mode
K	Matricized core array (frontal slices)
renormmode	Renormalization option (0 for no renormalization, 1 for fit contribution to total fit of each B- and C-mode component combination, 2 for fit contribution to total fit of each A- and C-mode component combination, 3 for fit contribution to total fit of each A- and B-mode component combination)
laba	Optional vector of length n containing the labels of the A-mode entities
labb	Optional vector of length m containing the labels of the B-mode entities
labc	Optional vector of length p containing the labels of the C-mode entities

### Value

A list including the following components:

fitA	Fit contribution for the A-mode entities
fitB	Fit contribution for the B-mode entities
fitC	Fit contribution for the C-mode entities
ABcontr	Contribution to the goodness of fit contributions by combinations of A- and B-modes in case of ‘renormalization’
BCcontr	Contribution to the goodness of fit contributions by combinations of B- and C-modes in case of ‘renormalization’
ACcontr	Contribution to the goodness of fit contributions by combinations of A- and C-modes in case of ‘renormalization’

**Note**

The computation of the fit contributions by combinations of modes is done in case of ‘renormalization’.

In Tucker1, renormmode must be equal to 0.

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**See Also**

[T3](#), [T2](#), [T1](#)

**Examples**

```
data(Bus)
# labels for Bus data
laba <- rownames(Bus)
labb <- substr(colnames(Bus)[1:5], 1, 1)
labc <- substr(colnames(Bus)[seq(1,ncol(Bus),5)], 3, 8)
# T3 solution
BusT3 <- T3func(Bus, 7, 5, 37, 2, 2, 2, 0, 1e-6)
# Fitpartitioning of the T3 solution
FitT3 <- T3fitpartitioning(Bus, 7, 5, 37, BusT3$A, BusT3$B, BusT3$C, BusT3$H, 0,
  laba, labb, labc)
# Fitpartitioning of the T3 solution (when labels are not available)
FitT3 <- T3fitpartitioning(Bus, 7, 5, 37, BusT3$A, BusT3$B, BusT3$C, BusT3$H, 0)
```

---

T3func

*Algorithm for the Tucker3 model*

---

**Description**

Alternating Least Squares algorithm for the minimization of the Tucker3 loss function.

**Usage**

```
T3func(X, n, m, p, r1, r2, r3, start, conv, A, B, C, H)
```

**Arguments**

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matrixed array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities

p	Number of C-mode entities
r1	Number of extracted components for the A-mode
r2	Number of extracted components for the B-mode
r3	Number of extracted components for the C-mode
start	Starting point (0 starting point of the algorithm from generalized eigenvalue decomposition, 1 random starting point (orthonormalized component matrices), 2 if users specified component matrices)
conv	Convergence criterion
A	Optional (necessary if start=2) starting value for A
B	Optional (necessary if start=2) starting value for B
C	Optional (necessary if start=2) starting value for C
H	Optional (necessary if start=2) starting value for the matricized core array (frontal slices)

**Value**

A list including the following components:

A	Orthonormal component matrix for the A-mode
B	Orthonormal component matrix for the B-mode
C	Orthonormal component matrix for the C-mode
H	Matricized core array (frontal slices)
f	Loss function value
fp	Fit percentage
iter	Number of iterations
cputime	Computation time
La	Matrix which should be diagonal, and if so, contain 'intrinsic eigenvalues' for A-mode
Lb	Matrix which should be diagonal, and if so, contain 'intrinsic eigenvalues' for B-mode
Lc	Matrix which should be diagonal, and if so, contain 'intrinsic eigenvalues' for C-mode

**Note**

The loss function to be minimized is  $\|X_A - AG_A \text{kron}(C', B')\|^2$  where  $X_A$  and  $G_A$  denote the matricized (frontal slices) data array and core array, respectively, and  $\text{kron}$  stands for the Kronecker product.

T3func is the same as T3funcprep except that all printings are available.

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## References

- H.A.L. Kiers, P.M. Kroonenberg & J.M.F. ten Berge (1992). An efficient algorithm for TUCK-ALS3 on data with large numbers of observation units. *Psychometrika* 57:415–422.
- P.M. Kroonenberg & J. de Leeuw (1980). Principal component analysis of three-mode data by means of alternating least squares algorithms. *Psychometrika* 45:69–97.

## See Also

[T3](#), [T3funcprep](#)

## Examples

```
data(Bus)
# labels for Bus data
laba <- rownames(Bus)
labb <- substr(colnames(Bus)[1:5], 1, 1)
labc <- substr(colnames(Bus)[seq(1,ncol(Bus),5)], 3, 8)
# T3 solution using two components for all the modes
# (rational starting point by SVD [start=0])
BusT3 <- T3func(Bus, 7, 5, 37, 2, 2, 2, 0, 1e-6)
# T3 solution using two components for all the modes
# (random orthonormalized starting point [start=1])
BusT3 <- T3func(Bus, 7, 5, 37, 2, 2, 2, 1, 1e-6)
# T3 solution using two components for all the modes
# (user starting point [start=2])
BusT3 <- T3func(Bus, 7, 5, 37, 2, 2, 2, 1, 1e-6, matrix(rnorm(7*2),nrow=7),
  matrix(rnorm(5*2),nrow=5), matrix(rnorm(37*2),nrow=37),
  matrix(rnorm(2*4),nrow=2))
```

---

T3funcprep

*Algorithm for the Tucker3 model*

---

## Description

Alternating Least Squares algorithm for the minimization of the Tucker3 loss function.

## Usage

```
T3funcprep(X, n, m, p, r1, r2, r3, start, conv, A, B, C, H)
```

## Arguments

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matricized array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities

p	Number of C-mode entities
r1	Number of extracted components for the A-mode
r2	Number of extracted components for the B-mode
r3	Number of extracted components for the C-mode
start	Starting point (0 starting point of the algorithm from generalized eigenvalue decomposition, 1 random starting point (orthonormalized component matrices), 2 if users specified component matrices)
conv	Convergence criterion
A	Optional (necessary if start=2) starting value for A
B	Optional (necessary if start=2) starting value for B
C	Optional (necessary if start=2) starting value for C
H	Optional (necessary if start=2) starting value for the matricized core array (frontal slices)

### Value

A list including the following components:

A	Orthonormal component matrix for the A-mode
B	Orthonormal component matrix for the B-mode
C	Orthonormal component matrix for the C-mode
H	Matricized core array (frontal slices)
f	Loss function value
fp	Fit percentage
iter	Number of iterations
cputime	Computation time
La	Matrix which should be diagonal, and if so, contain 'intrinsic eigenvalues' for A-mode
Lb	Matrix which should be diagonal, and if so, contain 'intrinsic eigenvalues' for B-mode
Lc	Matrix which should be diagonal, and if so, contain 'intrinsic eigenvalues' for C-mode

### Note

The loss function to be minimized is  $\|X_A - AG_A \text{kron}(C', B')\|^2$  where  $X_A$  and  $G_A$  denote the matricized (frontal slices) data array and core array, respectively, and  $\text{kron}$  stands for the Kronecker product.

T3funcprep is the same as T3func except that all printings are suppressed. Thus, T3funcprep can be helpful for simulation experiments.

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**References**

H.A.L. Kiers, P.M. Kroonenberg \& J.M.F. ten Berge (1992). An efficient algorithm for TUCK-ALS3 on data with large numbers of observation units. *Psychometrika* 57:415–422.  
 P.M. Kroonenberg \& J. de Leeuw (1980). Principal component analysis of three-mode data by means of alternating least squares algorithms. *Psychometrika* 45:69–97.

**See Also**

[T3](#), [T3func](#)

**Examples**

```
data(Bus)
# labels for Bus data
laba <- rownames(Bus)
labb <- substr(colnames(Bus)[1:5], 1, 1)
labc <- substr(colnames(Bus)[seq(1,ncol(Bus),5)], 3, 8)
# T3 solution using two components for all the modes
# (rational starting point by SVD [start=0])
BusT3 <- T3funcprep(Bus, 7, 5, 37, 2, 2, 2, 0, 1e-6)
# T3 solution using two components for all the modes
# (random orthonormalized starting point [start=1])
BusT3 <- T3funcprep(Bus, 7, 5, 37, 2, 2, 2, 1, 1e-6)
# T3 solution using two components for all the modes
# (user starting point [start=2])
BusT3 <- T3funcprep(Bus, 7, 5, 37, 2, 2, 2, 1, 1e-6, matrix(rnorm(7*2),nrow=7),
  matrix(rnorm(5*2),nrow=5), matrix(rnorm(37*2),nrow=37),
  matrix(rnorm(2*4),nrow=2))
```

---

T3runsApproxFit

*Approximated Tucker3 solutions*


---

**Description**

Computes all the approximated Tucker3 solutions using PCASup results with r1 (from 1 to maxa), r2 (from 1 to maxb) and r3 (from 1 to maxc) components.

**Usage**

```
T3runsApproxFit(X, n, m, p, maxa, maxb, maxc)
```

**Arguments**

X	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matrixed array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities
maxa	Maximum dimensionality for the A-mode
maxb	Maximum dimensionality for the B-mode
maxc	Maximum dimensionality for the C-mode

**Value**

out	Matrix with columns: number of components for the A-mode, number of components for the B-mode, number of components for the C-mode, goodness of fit (%), total number of components
-----	---

**Note**

Cumulative sum of eigenvalues and fits from PCAsup applied to the A-, B- and C-modes are automatically printed.

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**References**

H.A.L. Kiers (1991). Hierarchical relations among three-way methods. *Psychometrika* 56:449–470.

**See Also**

[DimSelector](#), [LineCon](#), [pcasup3](#), [T3](#)

**Examples**

```
data(Bus)
# Fit values of T3 with different numbers of components (from 1 to 4 for the A-mode,
# from 1 to 3 for the B-mode, from 1 to 5 for the C-mode)
FitT3 <- T3runsApproxFit(Bus, 7, 5, 37, 4, 3, 5)
```



---

threewayanova	<i>Three-way ANOVA</i>
---------------	------------------------

---

**Description**

Computation of three-way Analysis of Variance (ANOVA).

**Usage**

```
threewayanova(Y, n, m, p)
```

**Arguments**

Y	Matrix (or data.frame coerced to a matrix) of order (n x mp) containing the matrixed array (frontal slices)
n	Number of A-mode entities
m	Number of B-mode entities
p	Number of C-mode entities

**Value**

A list including the following components:

SS.a	Main effect for the A-mode
SS.b	Main effect for the B-mode
SS.c	Main effect for the C-mode
SS.ab	Second order interaction (A- and B-mode)
SS.bc	Second order interaction (B- and C-mode)
SS.ac	Second order interaction (A- and C-mode)
SS.abc	Residual sum of squares after subtraction of second order interactions

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**References**

H.A.L. Kiers & I. Van Mechelen (2001). Three-way component analysis: principles and illustrative applications. *Psychological Methods* 6:84–110.

**Examples**

```
data(TV)  
TVdata=TV[[1]]  
anova3 <- threewayanova(TVdata, 16, 15, 30)
```

---

tr	<i>Trace</i>
----	--------------

---

**Description**

Computes the trace of a matrix.

**Usage**

```
tr(a)
```

**Arguments**

a	Matrix
---	--------

**Value**

t	Trace of A
---	------------

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**Examples**

```
X <- matrix(rnorm(6*6),ncol=6)  
trace <- tr(X)
```

---

TV	<i>TV data</i>
----	----------------

---

**Description**

Three-way data about ratings of 15 American television shows on 16 bipolar scales made by 30 students.

**Usage**

```
data(TV)
```

## Format

A list containing one data.frame and three character vectors.

TV[[1]] is a data.frame with 16 rows and 450 (15 x 30) columns.

The rows refer to the American television shows.

The columns refer to the combinations of scales and students with the scales nested within the students.

The data.frame contains the frontal slices next to each other of the original array.

The labels for the bipolar scales are in the character vector TV[[2]].

The labels for the TV programs are in the character vector TV[[3]].

The labels for the students are in the character vector TV[[4]].

## Details

The original data set consists of ratings made by 40 subjects (psychology students at the University of Western Ontario in 1981). To avoid missing data, only 30 students are considered. The ratings are made on 13-point bipolar scales. Lundy et al. (1989) perform Candecomp/Parafac on the preprocessed data. Details on preprocessing are not reported, but should be centered within TV programs and scales. Three real components are extracted. However, the unconstrained Candecomp/Parafac solution with three components suffers from the so-called degeneracy (obtained solution with highly correlated and uninterpretable dimensions). Degeneracy (see, for instance, Harshman & Lundy, 1984; Stegeman, 2006, 2007; De Silva & Lim, 2008; Rocci & Giordani, 2010) can be overcome by imposing orthogonal constraints in one of the component matrices. The so-obtained solution with three components is meaningful and interpretable as described in Lundy et al. (1989).

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## References

- V. De Silva & L.-H. Lim (2008). Tensor rank and the ill-posedness of the best low-rank approximation problem. *SIAM Journal on Matrix Analysis and Applications* 30:1084–1127.
- R.A. Harshman & M.E. Lundy (1984). Data preprocessing and the extended PARAFAC model. In *H.G. Law, C.W. Snyder Jr, J.A. Hattie, & R.P. McDonald (Eds.): Research methods for multimode data analysis*. Praeger, New York (pp. 216–284).
- M.E. Lundy, R.A. Harshman & J.B. Kruskal (1989). A two-stage procedure incorporating good features of both trilinear and quadrilinear models. In *R. Coppi, S. Bolasco (Eds.): Multiway Data Analysis*. Elsevier, North Holland (pp. 123–130).
- R. Rocci & P. Giordani (2010). A weak degeneracy decomposition for the CANDECOMP/PARAFAC model. *Journal of Chemometrics* 24:57–66.
- A. Stegeman (2006). Degeneracy in Candecomp/Parafac explained for  $p \times p \times 2$  arrays of rank  $p+1$  or higher. *Psychometrika* 71:483–501.
- A. Stegeman (2007). Degeneracy in Candecomp/Parafac and Indscal explained for several three-sliced arrays with a two-valued typical rank. *Psychometrika* 72:601–619.

**Examples**

```
# to perform stability check and produce bootstrap confidence intervals
# it is useful to permute the modes so that the A-mode refers to students
data(TV)
TVdata=TV[[1]]
labSCALE=TV[[2]]
labPROGRAM=TV[[3]]
labSTUDENT=TV[[4]]
TVdata <- permnew(TVdata, 16, 15, 30)
TVdata <- permnew(TVdata, 15, 30, 16)
```

---

varim	<i>Varimax rotation</i>
-------	-------------------------

---

**Description**

Produces varimax rotated version of A and rotation matrix T.

**Usage**

```
varim(A)
```

**Arguments**

A                    Matrix to be rotated

**Value**

A list including the following components:

B                    Rotated version of A ( $B=AT$ )  
T                    Rotation matrix  
f                    Varimax function value

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**References**

H. Kaiser (1958). The varimax criterion for analytic rotation in factor analysis. *Psychometrika* 23:187–200.  
K. Nevels (1986). A direct solution for pairwise rotations in Kaiser's varimax method. *Psychometrika* 51:327–329.

**See Also**[normvari](#)**Examples**

```
X <- matrix(rnorm(6*3),ncol=3)
Y <- varim(X)
# varimax rotated version of X
Y$B
# rotation matrix
Y$T
```

varimcoco

*Varimax Rotation for Tucker3 and Tucker2***Description**

Performs varimax rotation of the core and component matrix rotations to simple structure.

**Usage**

```
varimcoco(A, B, C, H, wa_rel, wb_rel, wc_rel, rot1, rot2, rot3, nanal)
```

**Arguments**

A	Columnwise orthomornal component matrix for the A-mode
B	Columnwise orthomornal component matrix for the B-mode
C	Columnwise orthomornal component matrix for the C-mode
H	Matricized core array (frontal slices)
wa_rel	relative weight ( $\geq 0$ ) for the simplicity of A
wb_rel	relative weight ( $\geq 0$ ) for the simplicity of B
wc_rel	relative weight ( $\geq 0$ ) for the simplicity of C
rot1	binary indicator (1 if the A-mode is rotated, 0 otherwise, default 1)
rot2	binary indicator (1 if the B-mode is rotated, 0 otherwise, default 1)
rot3	binary indicator (1 if the C-mode is rotated, 0 otherwise, default 1)
nanal	Number of random starts, default 5

**Value**

A list including the following components:

AS	Rotated component matrix for the A-mode
BT	Rotated component matrix for the B-mode
CU	Rotated component matrix for the C-mode
K	Rotated matricized core array (frontal slices)
S	Rotation matrix for the A-mode
T	Rotation matrix for the B-mode
U	Rotation matrix for the C-mode
f	Best solution for three-way orthomax function value
f1	Varimax value of H
f2a	Varimax value of AS
f2b	Varimax value of BT
f2c	Varimax value of CU
func	Function values upon convergence for all the runs of the orthomax algorithm

**Note**

The simplicity values f1, f2a, f2b, f2c are based on ‘natural’ weights and therefore comparable across matrices. When multiplied by the relative weights, they give the contribution to the overall simplicity value (they are  $I^2/p$ ,  $J^2/q$  or  $K^2/r$ , respectively, times the sum of the variances of squared values).

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**References**

H.A.L. Kiers (1998). Joint orthomax rotation of the core and component matrices resulting from three-mode principal components analysis. *Journal of Classification* 15:245–263.

**See Also**

[orthmax2](#), [varim](#)

**Examples**

```
data(Bus)
# T3 solution
BusT3 <- T3funcprep(Bus, 7, 5, 37, 2, 2, 2, 0, 1e-6)
# Simplicity of A (with weight = 2.5), B (with weight = 2) and C (with weight = 1.5)
T3vmABC <- varimcoco(BusT3$A, BusT3$B, BusT3$C, BusT3$H, 2.5, 2, 1.5)
```

```
# Simplicity of only A (with weight = 2.5) and B (with weight = 2)
# rot3=0; the value of wc_rel (= 0) does not play an active role
T3vmAB <- varimcoco(BusT3$A, BusT3$B, BusT3$C, BusT3$H, 2.5, 2, 0, 1, 1, 0)
# simplicity repeatedly with different relative weights for A, B and C
T3vm <- list()
weight.a <- c(1, 3, 6)
weight.b <- c(0, 2, 5)
weight.c <- c(1, 4)
i <- 1
for (wa_rel in weight.a){
  for (wb_rel in weight.b){
    for (wc_rel in weight.c){
      T3vm[[i]] <- varimcoco(BusT3$A, BusT3$B, BusT3$C,
        BusT3$H, wa_rel, wb_rel, wc_rel)
      i <- i+1
    }
  }
}
```

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