

# Package ‘schwartz97’

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**LazyData** no

**Depends** R(>= 2.10), FKF(>= 0.1.0), mvtnorm, methods, RUnit

**Description** This package provides detailed functionality for working with the Schwartz 1997 two-factor commodity model. Essentially, it contains pricing formulas for futures and European options and the standard  $d/p/q/r$  functions for the distribution of the state variables and futures prices. In addition, a parameter estimation procedure is contained together with many utilities as filtering and plotting functionality. This package is accompanied by futures data of ten commodities.

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schwartz97-package	<i>Two-factor Commodity Model</i>
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## Description

This package contains an implementation of the Schwartz two-factor commodity model, that is, the joint dynamics of the spot price and the spot convenience yield according to Schwartz (1997). The parameter estimation function constitutes the core of this package. Once the parameters are estimated, futures and European call and put options can be priced, term structures can be calculated and the usual distribution operations  $d/p/q/r$  can be carried out on the state variables as well as on futures prices. The package is accompanied by a variety of utility functions, futures data of ten commodities, and two vignettes describing technical details and usage of the package.

## Details

Package: schwartz97  
 Type: Package  
 Version: 0.0.4  
 Date: 2011-12-18  
 License: GPL (GNU Public License), Version 2 or later

## Initialization:

`schwartz2f` Initialize a Schwartz two-factor object.

## Density, distribution function, quantile function, random number generation, and trajectories of the state variables:

`dstate` Density of the spot and the convenience yield.

<code>pstate</code>	Distribution of the spot and the convenience yield.
<code>qstate</code>	Quantile of the spot and the convenience yield.
<code>rstate</code>	Random number generation of the spot and the convenience yield.
<code>simstate</code>	Trajectory of the spot and the convenience yield.

### Density, distribution function, quantile function, and random number generation of the futures price:

<code>dfutures</code>	Density of the futures price.
<code>pfutures</code>	Distribution of the futures price.
<code>qfutures</code>	Quantile of the futures price.
<code>rfutures</code>	Random number generation of the futures price.

### Parameter estimation:

<code>fit.schwartz2f</code>	Estimate parameters of the two-factor model.
<code>fitted</code>	Extract the model's fitted values.
<code>resid</code>	Extract model residuals.

### Pricing:

<code>pricefutures</code>	Compute arbitrage-free futures prices.
<code>priceoption</code>	Compute arbitrage-free European option prices.

### Utilities:

<code>coef</code>	Extract model coefficients of <code>schwartz2f</code> -objects.
<code>mean</code>	Extract the mean of <code>schwartz2f</code> -objects.
<code>vcov</code>	Extract the covariance matrix of <code>schwartz2f</code> -objects.
<code>filter.schwartz2f</code>	Filter futures prices to get the spot price and convenience yield.
<code>plot</code>	Plot <code>schwartz2f.fit</code> -objects.
<code>plot</code>	Plot trajectories of <code>schwartz2f</code> -objects.
<code>futures</code>	Use <code>data(futures)</code> to get data of 10 commodities.

### Package vignette:

The R package `schwartz97` contains two vignettes:

The vignette *Technical Document* gives the necessary relations and tools to fully understand the internals of the package.

The vignette *User Guide* discusses implementation details and gives numerous examples and intuitive explanations.

### Author(s)

David Luethi, Philipp Erb, Juri Hinz, Simon Otziger

Maintainer: David Luethi <luethid@gmail.com>

### References

*Stochastic Convenience Yield and the Pricing of Oil Contingent Claims* by Rajna Gibson and Eduardo S. Schwartz  
The Journal of Finance 45, 1990, 959-976

*The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging* by Eduardo S. Schwartz  
Journal of Finance 52, 1997, 923-973

*Pricing of Options on Commodity Futures with Stochastic Term Structures of Convenience Yields and Interest Rates* by Kristian R. Miltersen and Eduardo S. Schwartz  
Journal of Financial and Quantitative Analysis 33, 1998, 33-59

*Valuation of Commodity Futures and Options under Stochastic Convenience Yields, Interest Rates, and Jump Diffusions in the Spot* by Jimmy E. Hilliard and Jorge Reis  
Journal of Financial and Quantitative Analysis 33, 1998, 61-86

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coef-method

*Extract parameters of schwartz2f objects*

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### Description

The function `coef` returns parameters of `schwartz2f` and a `schwartz2f.fit` objects as a list. The function `coefficients` is an alias for `coef`.

### Usage

```
## S4 method for signature 'schwartz2f'
coef(object)
## S4 method for signature 'schwartz2f'
coefficients(object)

## S4 method for signature 'schwartz2f.fit'
coef(object)
## S4 method for signature 'schwartz2f.fit'
coefficients(object)
```

**Arguments**

object            An object from class [schwartz2f](#) or [schwartz2f.fit](#).

**Value**

If object is of class [schwartz2f](#):

s0    Commodity spot price.  
delta0    Convenience yield.  
mu    Drift parameter of the spot price process.  
sigmaS    Diffusion parameter of the spot price process.  
kappa    Speed of mean-reversion of the convenience yield process.  
alpha    Mean-level of the convenience yield process.  
sigmaE    Diffusion parameter of the convenience yield process.  
rho    Correlation coefficient between the Brownian motion driving the spot price and the convenience yield process.

If object is of class [schwartz2f.fit](#):

s0    Commodity spot price.  
delta0    Convenience yield.  
mu    Drift parameter of the spot price process.  
sigmaS    Diffusion parameter of the spot price process.  
kappa    Speed of mean-reversion of the convenience yield process.  
alpha    Mean-level of the convenience yield process.  
sigmaE    Diffusion parameter of the convenience yield process.  
rho    Correlation coefficient between the Brownian motion driving the spot price and the convenience yield process.  
r    Instantaneous risk-free interest rate.  
lambda    Market price of convenience yield risk.  
alphaT    Mean-level of the convenience yield process with respect to the equivalent martingale measure.

The model and its parameters are described in the **Details** section of the [schwartz2f](#)-class documentation and in the package vignette *Technical Document*.

**Author(s)**

Philipp Erb, David Luethi

**See Also**

[schwartz2f](#) classes.

**Examples**

```
# ## coef-method for schwartz2f-objects:
# coef(schwartz2f())
#
```

```

## # coef-method for schwartz2f.fit-objects:
## # Estimate parameters for soybean oil (but stop after 3 iterations).
# data(futures)
# fit.obj <- fit.schwartz2f(futures$soybean.oil$price, futures$soybean.oil$ttm / 260,
#                           deltat = 1 / 260, control = list(maxit = 3))
# coef(fit.obj)

```

---

distribution-futures    *Schwartz two-factor Model: Distribution of Futures Prices*

---

## Description

Density, distribution function, quantile function and random number generation of futures prices.

## Usage

```

## S4 method for signature 'ANY,ANY,ANY,numeric'
dfutures(x, time = 0.1, ttm = 1, s0 = 50, delta0 = 0,
          mu = 0.1, sigmaS = 0.3, kappa = 1, alpha = 0,
          sigmaE = 0.5, rho = 0.75, r = 0.05, lambda = 0,
          alphaT = NULL, measure = c("P", "Q"), ...)

## S4 method for signature 'ANY,ANY,ANY,schwartz2f'
dfutures(x, time = 0.1, ttm = 1, s0, r = 0.05,
          lambda = 0, alphaT = NULL, measure = c("P", "Q"), ...)

## S4 method for signature 'ANY,ANY,ANY,schwartz2f.fit'
dfutures(x, time = 0.1, ttm = 1, s0, measure = c("P", "Q"), ...)

## S4 method for signature 'ANY,ANY,ANY,numeric'
pfutures(q, time = 0.1, ttm = 1, s0 = 50, delta0 = 0,
          mu = 0.1, sigmaS = 0.3, kappa = 1, alpha = 0,
          sigmaE = 0.5, rho = 0.75, r = 0.05, lambda = 0,
          alphaT = NULL, measure = c("P", "Q"), ...)

## S4 method for signature 'ANY,ANY,ANY,schwartz2f'
pfutures(q, time = 0.1, ttm = 1, s0, r = 0.05,
          lambda = 0, alphaT = NULL, measure = c("P", "Q"), ...)

## S4 method for signature 'ANY,ANY,ANY,schwartz2f.fit'
pfutures(q, time = 0.1, ttm = 1, s0, measure = c("P", "Q"), ...)

## S4 method for signature 'ANY,ANY,ANY,numeric'
qfutures(p, time = 0.1, ttm = 1, s0 = 50, delta0 = 0,
          mu = 0.1, sigmaS = 0.3, kappa = 1, alpha = 0,

```

```

        sigmaE = 0.5, rho = 0.75, r = 0.05, lambda = 0,
        alphaT = NULL, measure = c("P", "Q"), ...)

## S4 method for signature 'ANY,ANY,ANY,schwartz2f'
qfutures(p, time = 0.1, ttm = 1, s0, r = 0.05,
         lambda = 0, alphaT = NULL, measure = c("P", "Q"), ...)

## S4 method for signature 'ANY,ANY,ANY,schwartz2f.fit'
qfutures(p, time = 0.1, ttm = 1, s0, measure = c("P", "Q"), ...)

## S4 method for signature 'ANY,ANY,ANY,numeric'
rfutures(n, time = 0.1, ttm = 1, s0 = 50, delta0 = 0,
         mu = 0.1, sigmaS = 0.3, kappa = 1, alpha = 0, sigmaE = 0.5,
         rho = 0.75, r = 0.05, lambda = 0, alphaT = NULL, measure = c("P", "Q"))

## S4 method for signature 'ANY,ANY,ANY,schwartz2f'
rfutures(n, time = 0.1, ttm = 1, s0, r = 0.05,
         lambda = 0, alphaT = NULL, measure = c("P", "Q"))

## S4 method for signature 'ANY,ANY,ANY,schwartz2f.fit'
rfutures(n, time = 0.1, ttm = 1, s0, measure = c("P", "Q"))

```

## Arguments

q, x	vector of quantiles.
p	vector of probabilities.
n	number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
time	Time where the futures process is evaluated (relative to now).
ttm	Time to maturity (relative to now).
s0	Either a numeric representing the initial value of the commodity spot price or an object inheriting from class <code>schwartz2f</code> .
delta0	Initial value of the convenience yield.
mu	Drift term of commodity spot price.
sigmaS	Diffusion parameter of the spot price process.
kappa	Speed of mean-reversion of the convenience yield process.
alpha	Mean-level of the convenience yield process.
sigmaE	Diffusion parameter of the convenience yield process.
rho	Correlation coefficient between the Brownian motion driving the spot price and the convenience yield process.
lambda	Market price of convenience yield risk (see <b>Details</b> ).
alphaT	Mean-level of the convenience yield process with respect to the equivalent martingale measure (see <b>Details</b> ).

r	Instantaneous risk-free interest rate.
measure	under which the functions are computed. “P” denotes the objective measure, “Q” the risk-neutral measure (see <b>Details</b> ).
...	Arguments to be passed to the functions <a href="#">d/p/q-norm</a> .

### Details

Futures prices depend on the spot-price and the convenience yield.

To get the real (i.e. the objective) distribution of futures prices at some date in the future the dynamics is considered under the objective measure  $P$ . The  $P$ -dynamics is

$$dS_t = (\mu - \delta_t)S_t dt + \sigma_S \tilde{S}_t \tilde{dW}_t^1,$$

$$d\delta_t = \kappa(\alpha - \delta_t)dt + \sigma_E dW_t^2$$

$$dW_t^1 \tilde{\cdot} dW_t^2 = \rho dt,$$

where  $W^1$ ,  $W^2$  are Brownian motions under the objective measure, the measure  $P$ .

Options on futures are evaluated based on the risk-neutral dynamics of the spot-price and the convenience yield, i.e. under the measure  $Q$ . The  $Q$ -dynamics is

$$dS_t = (r - \delta_t)S_t dt + \sigma_S \tilde{S}_t \tilde{d\tilde{W}}_t^1$$

$$d\delta_t = \kappa(\tilde{\alpha} - \delta_t)dt + \sigma_E d\tilde{W}_t^2,$$

where  $\tilde{W}^1$ ,  $\tilde{W}^2$  are Brownian motions with respect to  $Q$ .

$\tilde{\alpha} = \alpha - \lambda / \kappa$  where  $\lambda$  is the market price of convenience-yield risk. The market price of convenience yield risk can either be specified explicitly by `lambda` or implicitly by `alphaT`. The relation is `alphaT = alpha - lambda / kappa`. See the package vignette.

### Value

Probabilities, densities, quantiles or samples of the log-normally distributed futures prices as numeric.

### Note

Note that futures and forward prices coincide as the interest rate is assumed to be constant in the Schwartz two-factor model.

### Author(s)

Philipp Erb, David Luethi, Juri Hinz



## References

*The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging* by Eduardo S. Schwartz  
Journal of Finance 52, 1997, 923-973

*Valuation of Commodity Futures and Options under Stochastic Convenience Yields, Interest Rates, and Jump Diffusions in the Spot* by Jimmy E. Hilliard and Jorge Reis  
Journal of Financial and Quantitative Analysis 33, 1998, 61-86

## See Also

[pricefutures](#), [d/p/qstate](#), [r/simstate](#)

## Examples

```
# ## Create a "schwartz2f"-object
# model <- schwartz2f()
#
# ## Probability
# pfutures(q = 10 * 3:9, time = 0.5, ttm = 2, model, lambda = 0.01)
#
# ## Density
# dfutures(x = c(20, 40, 100), time = 0.5, ttm = 2, model, lambda = 0.01)
#
# ## Quantile
# qfutures(p = 0.1 * 2:5, time = 0.5, ttm = 10, model, lambda = 0.01)
#
# ## Sample
# sim <- rfutures(n = 1000, time = 0.5, ttm = 5, model, lambda = 0.01)
#
# hist(sim, prob = TRUE)
# lines(seq(30, 300, length = 100),
#       dfutures(seq(30, 300, length = 100),
#                 time = 0.5, ttm = 5, model, lambda = 0.01), col = "red")
#
# ## At time 0 the futures price is a deterministic function of s0 and
# ## delta0. Therefore 3 times the same value is obtained:
# rfutures(3, time = 0, ttm = 1, model, lambda = 0)
```

## Description

Density, distribution function and quantile function of the state variables. The state variables are the commodity spot price  $s$  and the spot convenience yield  $\delta$ . The commodity log spot price and the convenience yield follow a bivariate normal distribution.

**Usage**

```

## S4 method for signature 'ANY,ANY,numeric'
dstate(x, time = 1, s0 = 50, delta0 = 0,
       mu = 0.1, sigmaS = 0.3, kappa = 1, alpha = 0,
       sigmaE = 0.5, rho = 0.75, ...)

## S4 method for signature 'ANY,ANY,schwartz2f'
dstate(x, time = 1, s0, ...)

## S4 method for signature 'ANY,ANY,ANY,numeric'
pstate(lower, upper, time = 1, s0 = 50, delta0 = 0,
       mu = 0.1, sigmaS = 0.3, kappa = 1, alpha = 0,
       sigmaE = 0.5, rho = 0.75, ...)

## S4 method for signature 'ANY,ANY,ANY,schwartz2f'
pstate(lower, upper, time = 1, s0, ...)

## S4 method for signature 'ANY,ANY,numeric'
qstate(p, time = 1, s0 = 50, delta0 = 0,
       mu = 0.1, sigmaS = 0.3, kappa = 1, alpha = 0,
       sigmaE = 0.5, rho = 0.75, tail = "lower.tail", ...)

## S4 method for signature 'ANY,ANY,schwartz2f'
qstate(p, time = 1, s0, tail = "lower.tail", ...)

```

**Arguments**

x	Vector or matrix of quantiles. If x is a matrix, each row is taken to be a quantile.
time	Time at which the quantity is computed (relative to time zero).
p	Probability, a scalar.
lower	The vector of lower limits of length 2. Note that first component stands for lower limit of the commodity spot price rather than the log-price.
upper	The vector of upper limits of length 2. Note that first component stands for the upper limit of the commodity spot price rather than the log-price.
s0	Either a numeric representing the initial value of the commodity spot price or an object inheriting from class <code>schwartz2f</code> .
delta0	Initial value of the convenience yield.
mu	enters the drift of the commodity spot price.
sigmaS	Diffusion parameter of the spot price-process.
kappa	Speed of mean-reversion of the convenience yield process.
alpha	Mean-level of the convenience yield process.
sigmaE	Diffusion parameter of the convenience yield process.

rho	Correlation coefficient between the Brownian motion driving the spot price and the convenience yield process.
tail	See <a href="#">qmvnorm</a> of package mvtnorm.
...	Further arguments to be passed to methods of package mvtnorm.

### Details

The model and its parameters are described in the **Details** section of the [schwartz2f](#)-class documentation and in the package vignette *Technical Document*.

The above methods rely on the functions [pmvnorm](#), [dmvnorm](#), and [qmvnorm](#) of the package mvtnorm.

### Value

dstate and pstate return a numeric, qstate returns the output of [qmvnorm](#) as a list.

### Author(s)

Philipp Erb, David Luethi, Juri Hinz

### See Also

[schwartz2f](#)-class description, [rstate](#) and [simstate](#) for random number generation, constructors [schwartz2f](#) and [fit.schwartz2f](#).

### Examples

```
# ## Create a "schwartz2f"-object
# model <- schwartz2f()
#
# ## Probability
# pstate(lower = c(0, -Inf), upper = c(45, 0.01), time = 1, model)
#
# ## Density
# dstate(x = c(50, 0.03), time = 2, model)
# dstate(x = rbind(c(50, 0.03), c(50, 0.1)), time = 2, model) # x is a matrix
#
# ## Quantile
# qstate(p = 0.5, s0 = model)
#
# ## Generate random numbers
# object <- schwartz2f(alpha = 0.05)
# samples <- rstate(1000, time = 2, object)
# ## ...and plot histograms
# par(mfrow = c(2, 1))
# hist(samples[,1])
# abline(v = mean(object, time = 2)[1], col = "red")
# hist(samples[,2])
# abline(v = mean(object, time = 2)[2], col = "red")
```

filter-futures

*Schwartz two-factor Model: Filter futures data***Description**

Filter a series of futures prices to obtain state variables.

**Usage**

```
## S4 method for signature 'ANY,ANY,numeric'
filter.schwartz2f(data, ttm, s0 = 50, delta0 = 0,
  mu = 0.1, sigmaS = 0.3, kappa = 1, alpha = 0, sigmaE = 0.5,
  rho = 0.75, r = 0.05, lambda = 0, alphaT = NULL,
  deltat = 1/260, meas.sd = rep(1e-3, ncol(data)),
  P0 = 0.5 * diag(c(log(s0), abs(delta0))))

## S4 method for signature 'ANY,ANY,schwartz2f'
filter.schwartz2f(data, ttm, s0,
  r = 0.05, lambda = 0, alphaT = NULL, deltat = 1/260,
  meas.sd = rep(1e-3, ncol(data)),
  P0 = 0.1 * diag(2))

## S4 method for signature 'ANY,ANY,schwartz2f.fit'
filter.schwartz2f(data, ttm, s0)
```

**Arguments**

data	A matrix with futures prices.
ttm	A matrix with the corresponding time to maturity (see <b>Details</b> ).
s0	Either a numeric representing the initial value of the commodity spot price or an object inheriting from class <code>schwartz2f</code> .
delta0	Initial value of the convenience yield.
mu	enters the drift of the commodity spot price.
sigmaS	Diffusion parameter of the spot price process.
kappa	Speed of mean-reversion of the convenience yield process.
alpha	Mean-level of the convenience yield process.
sigmaE	Diffusion parameter of the convenience yield process.
rho	Correlation coefficient between the Brownian motion driving the spot price and the convenience yield process.
r	Instantaneous risk-free interest rate.
lambda	Market price of convenience yield risk.

alphaT	Mean-level of the convenience yield process with respect to the equivalent martingale measure.
deltat	Time increment.
meas.sd	The standard deviation of the measurement equation (see <b>Details</b> section of <a href="#">fit.schwartz2f</a> ).
P0	Variance of the state variables $s_0$ and $P_0$ .

### Details

The elements of `data` and `ttm` have the following interpretation: `data[i, j]` denotes the futures price whose time to maturity was `ttm[i, j]` when it was observed. The unit of `ttm` is defined by `deltat`.

`deltat` is the time between observation `data[i, j]` and `data[i + 1, j]`. It is assumed to be constant, i.e., that `data` is a regular time-series.

### Value

A list with components:

<code>state</code>	A matrix with colnames are “S” and “delta” giving the the expected spot price and the convenience yield.
<code>fkf.obj</code>	The filter output from the package <a href="#">fkf</a> . Note that the log of the commodity spot price is filtered.

### Author(s)

Philipp Erb, David Luethi, Juri Hinz

### References

*The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging* by Eduardo S. Schwartz  
Journal of Finance 52, 1997, 923-973

### See Also

[fit.schwartz2f](#), [pricefutures](#).

### Examples

```
# data(futures)
#
# ## Estimate parameters for coffee data (stop after 20 iterations)
# fit.obj <- fit.schwartz2f(futures$coffee$price, futures$coffee$ttm / 260,
#                          deltat = 1 / 260, control = list(maxit = 20))
#
# ## Filter the futures data to get the spot price and the convenience yield.
# filtered <- filter.schwartz2f(futures$coffee$price, futures$coffee$ttm / 260, fit.obj)
#
# ## ...and plot it.
```

```
# par(mfrow = c(2, 1))
# plot(filtered$state[,1], ylab = "Spot price", type = "l")
# lines(futures$coffee$price[,1], col = "red") # Closest to maturity futures
# plot(filtered$state[,2], ylab = "Convenience yield", type = "l")
# abline(h = 0)
```

---

fitted-futures

*Extract Model Fitted Values*

---

## Description

Function to extract fitted values from [schwartz2f.fit](#)-objects.

## Usage

```
## S4 method for signature 'schwartz2f.fit'
fitted(object, data, ttm)
```

## Arguments

object	A <a href="#">schwartz2f.fit</a> -object returned from <a href="#">schwartz2f.fit</a> .
data	A matrix with futures prices.
ttm	A matrix with the corresponding times to maturity (see <b>Details</b> ).

## Details

The elements of `data` and `ttm` have the following interpretation: `data[i, j]` denotes the futures price whose time to maturity was `ttm[i, j]` when it was observed. The unit of `ttm` was defined by the argument `deltat` of [fit.schwartz2f](#).

## Value

A matrix containing the fitted values of the same dimension as `data`.

## Author(s)

Philipp Erb, David Luethi, Juri Hinz

## See Also

[fit.schwartz2f](#), [schwartz2f.fit](#)-class, [resid](#).

## Examples

```

# data(futures)
#
# ## Estimate parameters for lumber data. Fit only 'mu', 'sigmaS',
# ## 'sigmaE', and 'rho' (and stop after 100 iterations).
# fit.obj <- fit.schwartz2f(futures$lumber$price, futures$lumber$ttm / 260,
#                          opt.pars = c(s0 = FALSE, delta0 = FALSE, mu = TRUE,
#                                       sigmaS = TRUE, kappa = FALSE, alpha = FALSE,
#                                       sigmaE = TRUE, rho = TRUE, lambda = FALSE),
#                          alpha = 0, kappa = 1.2, lambda = 0,
#                          deltat = 1 / 260, control = list(maxit = 100))
# fit.obj
#
# ## Get the fitted values
# fitted.futures <- fitted(fit.obj, futures$lumber$price, futures$lumber$ttm / 260)
#
# par(mfrow = c(1, 3))
# ## Plot futures prices
# plot(as.ts(futures$lumber$price), plot.type = "single", ylab = "Futures prices",
#      col = gray(seq(0.1, 0.9, length = 4)))
# ## Plot fitted values
# plot(as.ts(fitted.futures), plot.type = "single", ylab = "Fitted values",
#      col = gray(seq(0.1, 0.9, length = 4)))
# ## Plot relative difference
# plot(as.ts((fitted.futures - futures$lumber$price) / fitted.futures), plot.type = "single",
#      ylab = "Relative difference", col = gray(seq(0.1, 0.9, length = 4)))

```

---

futures-data

*Daily futures prices*


---

## Description

Futures prices, time to maturity, open interest, volume, underlying tickers, and last trade date of ten different commodities: corn, wheat, soybean, soybean meal, soybean oil, lumber, live cattle, coffee, heating oil, copper.

There are, depending on the liquidity of the commodity, between 4 and 10 ‘clean’ closest to maturity futures price series.

## Usage

```
data(futures)
```

## Format

A list containing ten commodities as lists: “corn”, “wheat”, “soybean”, “soybean.meal”, “soybean.oil”, “lumber”, “live.cattle”, “coffee”, “heating.oil”, “copper”.

Each list contains six dimnamed matrices:

`price` Daily futures prices.

`ttm` The time to maturity of the futures contracts in units of days (see **Details**.)

`oi` Open interest.

`vol` Volume.

`underl.tickers` Underlying tickers / contracts.

`last.trade.dt` Last trade date as character in the ISO 8601 international standard format.

The *i*-th column of each matrix contains data for the *i*-th closest to maturity contract. The *i*-th column name is the ticker of the *i*-th generic futures.

Commodity	# Contracts	Exchange	Start date	End date
Corn	6	CBOT	1997-01-02	2010-04-07
Wheat	5	CBOT	1995-01-03	2010-04-07
Soybean	7	CBOT	1995-01-03	2010-04-07
Soybean meal	6	CBOT	2000-01-03	2010-04-07
Soybean oil	6	CBOT	1995-01-03	2010-04-07
Lumber	4	CME	1995-01-03	2010-04-07
Live cattle	6	CME	2004-07-01	2010-04-07
Coffee	5	ICE	1995-01-03	2010-04-07
Heating oil	10	NYMEX	1995-01-03	2010-03-31
Copper	8	COMEX	1996-01-02	2010-02-24

## Details

The elements of `price` and `ttm` have the following interpretation: `price[i, j]` denotes the futures price whose time to maturity was `ttm[i, j]` days when it was observed.

## Author(s)

Philipp Erb, David Luethi, Juri Hinz

## See Also

[futuresplot](#), [fit.schwartz2f](#).

## Examples

```
# data(futures)
#
# ## Plot forward curves of lumber
# futuresplot(futures$lumber, type = "forward.curve")
#
# ## Plot time to maturity of heating oil data
# futuresplot(futures$heating.oil, type = "ttm")
#
```



```

### Make 'futures' weekly, take Wednesday data
# futures.w <- rapply(futures, function(x)x[format(as.Date(rownames(x)), "%w") == 3,],
#                   classes = "matrix", how = "list")
#
#
### Make 'futures' monthly, take the 28th day of the month
# futures.m <- rapply(futures, function(x)x[format(as.Date(rownames(x)), "%d") == 28,],
#                   classes = "matrix", how = "list")
#
#
### Plot weekly lumber and monthly soybean data
# futuresplot(futures.w$lumber, type = "forward.curve", main = "Lumber")
# futuresplot(futures.m$soybean, type = "forward.curve", main = "Soybean")
#
#
### Convert to zoo-objects:
# require(zoo)
# futures.zoo <- rapply(futures, function(x)zoo(x, as.Date(rownames(x))),
#                      classes = "matrix", how = "list")
#
#
### ...and plot it nicely using plot.zoo:
# plot(futures.zoo$heating.oil$ttm)
# plot(futures.zoo$wheat$vol)
# plot(futures.zoo$copper$oi)
#
#
### Estimate soybean meal parameters (stop after 100 iterations).
### ttm (time-to-maturity) is divided by 260 as it is in unit of days and
### deltat = 1/52 because weekly price observations are used.
# soybean.meal.fit <- fit.schwartz2f(data = futures.w$soybean.meal$price,
#                                   ttm = futures.w$soybean.meal$ttm / 260,
#                                   deltat = 1 / 52, r = 0.04,
#                                   control = list(maxit = 100))
# soybean.meal.fit

```

---

futures-plot

*Visualization of Futures Data*


---

## Description

Visualization of historical commodity futures prices and remaining time to maturity. This function is intended to be fed with the futures data contained in this package (see [futures-data](#)).

## Usage

```
futuresplot(futures, type = c("forward.curve", "ttm"), ...)
```

## Arguments

futures	A list with elements price and ttm. Usually an element of <a href="#">futures</a> .
type	What shall be plotted. "forward.curve" or "ttm" (time to maturity).
...	Optional arguments passed to plot.

**Author(s)**

Philipp Erb, David Luethi, Juri Hinz

**See Also**

[futures-data](#)

**Examples**

```
# data(futures)
#
# ## Plot time to maturity of corn data
# futuresplot(futures$corn, type = "ttm")
#
# ## Plot forward curves of wheat data since Jan 2010
# wheat.2010 <- lapply(futures$wheat,
#                     function(x)x[as.Date(rownames(x)) > "2010-01-01",])
# futuresplot(wheat.2010, type = "forward.curve")
```

---

mean-vcov-methods

*Expected value and variance-covariance*

---

**Description**

The function `mean` returns the expected value of the spot price and the convenience yield for some time in the future. The function `vcov` returns the covariance matrix of the *log* spot price and the convenience yield.

**Usage**

```
## S4 method for signature 'schwartz2f'
mean(x, time = 1)

## S4 method for signature 'schwartz2f'
vcov(object, time = 1)
```

**Arguments**

`x`, `object`      An object inheriting from class `schwartz2f`.

`time`              The point in time for which the mean or covariance matrix is computed. `mean` accepts a vector, `vcov` a scalar only.

**Value**

Either the expected value or variance-covariance matrix.

**Note**

vcov returns the variance-covariance matrix of [*log* spot price, convenience yield](time).

**Author(s)**

David Luethi

**See Also**

[schwartz2f](#) to create Schwartz 2 factor objects, [plot](#) to show trajectories of the state variables including means and standard deviations.

**Examples**

```
# mean(schwartz2f(mu = 0.1), time = 1)
#
# mean(schwartz2f(mu = 0.2), time = 0:3)
#
# vcov(schwartz2f(), time = 10)
#
# ## Plot a schwartz2f-object including means and standard deviations
# plot(schwartz2f(sigmaE = 0.1), n = 50, time = 5, dt = 1 / 52)
```

---

parameter-estimation    *Schwartz 1997 two factor parameter estimation*

---

**Description**

Fit the Schwartz 1997 two factor commodity model to futures data.

**Usage**

```
fit.schwartz2f(data, ttm, deltat = 1 / 260,
               s0 = data[1,1], delta0 = 0,
               mu = 0.1, sigmaS = 0.3, kappa = 1, alpha = 0, sigmaE = 0.3,
               rho = 0.7, lambda = 0,
               meas.sd = rep(0.1, ncol(data)),
               opt.pars = c(s0 = FALSE, delta0 = FALSE, mu = TRUE, sigmaS = TRUE,
                           kappa = TRUE, alpha = TRUE, sigmaE = TRUE,
                           rho = TRUE, lambda = FALSE),
               opt.meas.sd = c("scalar", "all", "none"),
               r = 0.03, silent = FALSE, ...)
```

**Arguments**

<code>data</code>	A matrix with futures prices. NA-values are allowed.
<code>ttm</code>	A matrix with the corresponding time to maturity (see <b>Details</b> ).
<code>deltat</code>	Time increment (see <b>Details</b> ).
<code>s0</code>	Initial value of the commodity spot price.
<code>delta0</code>	Initial value of the convenience yield.
<code>mu</code>	Initial value of the drift parameter of the commodity spot price.
<code>kappa</code>	Initial parameter of the speed of mean-reversion of the convenience yield process.
<code>alpha</code>	Initial parameter of the mean-level of the convenience yield process.
<code>sigmaS</code>	Initial parameter of the diffusion parameter of the spot price process.
<code>sigmaE</code>	Initial parameter of the diffusion parameter of the convenience yield process.
<code>rho</code>	Initial parameter of the correlation coefficient between the Brownian motion driving the spot price and the convenience yield process.
<code>meas.sd</code>	Initial parameter of the standard deviation of the measurement equation (see <b>Details</b> ).
<code>lambda</code>	Initial value of the market price of convenience yield risk.
<code>opt.pars</code>	A logical vector giving the parameters which shall be fitted. The order is as given in the function header, names are discarded.
<code>opt.meas.sd</code>	States how the standard deviation of the measurement equation should be treated (see <b>Details</b> ).
<code>r</code>	Instantaneous risk-free interest rate.
<code>silent</code>	If FALSE the log-likelihood and parameters will be printed in each iteration.
<code>...</code>	Arguments passed to <code>optim</code> .

**Details**

The elements of `data` and `ttm` have the following interpretation: `data[i, j]` denotes the futures price whose time to maturity was `ttm[i, j]` when it was observed (in units of `deltat`).

The time increment between observation `data[i, j]` and `data[i + 1, j]` is denoted with `deltat`. Note that this specification requires a regularly spaced data series.

`opt.meas.sd` specifies how measurement uncertainty is treated in the fit: According to the model there should be a one-to-one correspondence between the spot and the futures price. In reality, the term structure does not fully match for any set of parameters. This is reflected in the measurement uncertainty-vector `meas.sd`. All components of `meas.sd` can be fitted. However, it might be sufficient to fit only a scalar where the measurement uncertainty is parametrized by `scalar * meas.sd`. In this case define the vector `meas.sd` and set `opt.meas.sd` to “scalar”. `meas.sd` can be set to a vector with each component set to, e.g., 2%, giving each point in the term structure equal weight. Another reasonable specification takes open interest or volumes into account: The higher the volume, the higher the weight and therefore the lower the corresponding component of `meas.sd`. If all components of `meas.sd` shall be fitted choose “all”. If the measurement uncertainty is known set `meas.sd` to “none”. Note that the measurement errors are assumed to be independent in this implementation (even though the model and the filter do not require independence).

The model and its parameters are described in the **Details** section of the `schwartz2f`-class documentation and in the package vignette *Technical Document*.

### Value

An object of class `schwartz2f.fit`.

### Note

Parameter estimation is statistically fragile and computationally demanding. Multiple local maxima of the likelihood may exist which can result in absurd parameter estimates as, e.g., a yearly drift of 300% and or a market price of convenience yield risk of -200%. Therefore, a reasonable parameter estimation is most likely an iteration where several initial values are used and different combinations of parameters are held constant during estimation. Also, simulation studies showed that a fairly large sample is required to get adequate estimates (e.g. 20000 daily observations, depending on the number of parameters which shall be estimated). For this reason the default is to hold `s0`, `delta0`, and `lambda` constant.

Several utility functions may help to investigate the fit (see e.g. `fitted`, `resid`, `plot`, `coef`).

The fitting procedure generally requires a large number of iterations to achieve a reasonable tolerance level. Each optimization iteration involves the filtering of the data set by the Kalman filter. Therefore, an efficient implementation of the Kalman filter is key. Hence, the `fkf` function of the package `FKF` can be considered as the backbone of the estimation procedure.

### Author(s)

Philipp Erb, David Luethi, Juri Hinz

### References

*The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging* by Eduardo S. Schwartz  
Journal of Finance 52, 1997, 923-973

### See Also

`schwartz2f.fit` class, `fitted`, `resid`, `plot`, `coef`), `pricefutures`, `futures-data`.

### Examples

```
# data(futures)
#
# ## Estimate parameters for wheat data.
# ## (little precision required with reltol = 1e-3)
# fit.obj <- fit.schwartz2f(futures$wheat$price, futures$wheat$ttm / 260,
#                          deltat = 1 / 260, control = list(reltol = 1e-3))
#
# ## See how parameter values evolved during the fit
# plot(fit.obj, type = "trace.pars")
#
```

```

### Do the same but with lower tolerance level:
high.precision.fit <- fit.schwartz2f(futures$wheat$price, futures$wheat$ttm / 260,
#                                     control = list(maxit = 3000, reltol = 5e-8))
#
# high.precision.fit
#
# plot(high.precision.fit, type = "trace.pars") # ...concerning parameter evolution.
#
### Alpha becomes nonsensically high, kappa (speed of mean-reversion
### of the convenience yield) goes to zero. Solution: Choose different
### initial values and/or hold kappa constant at 1.
#
constrained.fit <- fit.schwartz2f(futures$wheat$price, futures$wheat$ttm / 260,
#                                     opt.pars = c(s0 = FALSE, delta0 = FALSE, mu = TRUE,
#                                     sigmaS = TRUE, kappa = FALSE, alpha = TRUE,
#                                     sigmaE = TRUE, rho = TRUE, lambda = TRUE),
#                                     alpha = 0, kappa = 1,
#                                     control = list(maxit = 3000, reltol = 5e-8))
#
# constrained.fit
#
# plot(constrained.fit, type = "trace.pars")
#
### The above parameters based on a fit - where kappa was held constant at 1 -
### look more reasonable.
#
### These residuals should be iid standard normal
model.resid <- resid(fit.obj, data = futures$wheat$price, ttm = futures$wheat$ttm / 260,
#                                     type = "filter.std")
# acf(model.resid, na.action = na.pass)
# par(mfrow = c(3, 2))
# apply(model.resid, 2, function(x)plot(density(na.omit(x))))
### ...but are anything but iid standard normal.
#
### ...though the fitted values look fine:
fitted.futures <- fitted(fit.obj, futures$wheat$price, futures$wheat$ttm / 260)
# par(mfrow = c(1, 3))
### Plot futures prices
# plot(as.ts(futures$wheat$price), plot.type = "single", ylab = "Futures prices",
#       col = gray(seq(0.1, 0.9, length = 4)))
### Plot fitted values
# plot(as.ts(fitted.futures), plot.type = "single", ylab = "Fitted values",
#       col = gray(seq(0.1, 0.9, length = 4)))
### Plot relative difference
# plot(as.ts((fitted.futures - futures$wheat$price) / fitted.futures), plot.type = "single",
#       ylab = "Relative difference", col = gray(seq(0.1, 0.9, length = 4)))
#
#
### Try with kappa = 1, alpha = 0, and flexible standard deviations of
### the measurement errors. Stop at 200 iterations.
fit.obj.2 <- fit.schwartz2f(futures$wheat$price, futures$wheat$ttm / 260,
#                                     opt.pars = c(s0 = FALSE, delta0 = FALSE, mu = TRUE,
#                                     sigmaS = TRUE, kappa = FALSE, alpha = FALSE,

```

```

#                               sigmaE = TRUE, rho = TRUE, lambda = TRUE),
#                               alpha = 0, kappa = 1, opt.meas.sd = "all",
#                               deltat = 1 / 260, control = list(maxit = 200))
#
#
# model.resid.2 <- resid(fit.obj.2, data = futures$wheat$price, ttm = futures$wheat$ttm / 260,
#                       type = "filter.std")
# ## Residuals seem to be better:
# acf(model.resid.2, na.action = na.pass)
# par(mfrow = c(3, 2))
# apply(model.resid.2, 2, function(x)plot(density(na.omit(x))))
#
#
# ## The schwartz2f.fit-object 'fit.obj' can be used to do further calculations as
# ## pricing a put option on the wheat futures which matures in 1.1
# ## years. The option expires in 1 year. The strike price is the
# ## expected futures price in 1.1 years.
# priceoption("put", time = 1, Time = 1.1, K = pricefutures(1.1, fit.obj),
#             fit.obj)
#
#
#
# ## Parameter estimation for weekly soybean meal data:
# ## Get Wednesday observations:
# futures.w <- rapply(futures, function(x)x[format(as.Date(rownames(x)), "%w") == 3,],
#                   classes = "matrix", how = "list")
#
# ## Estimate soybean meal parameters (stop after 500 iterations).
# ## ttm (time-to-maturity) is divided by 260 as it is in unit of days and
# ## deltat = 1/52 because weekly price observations are used.
# ## Estimate all measurement error standard deviations (opt.meas.sd == "all"),
# ## but hold kappa, alpha, and lambda constant.
# soybean.meal.fit <- fit.schwartz2f(data = futures.w$soybean.meal$price,
#                                   ttm = futures.w$soybean.meal$ttm / 260,
#                                   opt.meas.sd = "all",
#                                   mu = 0, kappa = 1, alpha = 0.03, r = 0.04,
#                                   opt.pars = c(s0 = FALSE, delta0 = FALSE, mu = TRUE,
#                                               sigmaS = TRUE, kappa = FALSE, alpha = FALSE,
#                                               sigmaE = TRUE, rho = TRUE, lambda = FALSE),
#                                   deltat = 1 / 52, control = list(maxit = 500))
#
# soybean.meal.fit
#
# plot(soybean.meal.fit, type = "trace.pars") # plot the parameter evolution
#
# ## Plot real and predicted forward curves:
# par(mfrow = c(1, 2))
# futuresplot(futures.w$soybean.meal, type = "forward.curve")
# plot(soybean.meal.fit, type = "forward.curve", data = futures.w$soybean.meal$price,
#       ttm = futures.w$soybean.meal$ttm / 260)

```

---

plot.fit-method

*Plot Schwartz two-factor fit-objects*


---

### Description

This function plots the parameter evolution during the fit, the filtered state variables (i.e. the spot price and the convenience yield), forward curves, or trajectories of the state variables.

### Usage

```
## S4 method for signature 'schwartz2f.fit,missing'
plot(x, type = c("trace.pars", "state", "forward.curve", "sim"),
     data, ttm, ...)
```

### Arguments

x	A <a href="#">schwartz2f.fit</a> object.
type	What shall be plotted (see <b>Details</b> ).
data	A matrix containing futures prices to which parameters were fitted.
ttm	A matrix with the corresponding time to maturity (see <b>Details</b> ).
...	Arguments passed to <a href="#">plot</a> .

### Details

If type == "trace.pars", the parameter evolution of the estimation is plotted. The horizontal lines denote the final value.

If type == "state", the filtered state variables are plotted and overlaid with the futures prices.

If type == "forward.curve", fitted forward curves are plotted.

If type == "sim", a bunch of simulated trajectories of the state variables are plotted.

The elements of data and ttm have the following interpretation: data[i, j] denotes the futures price whose time to maturity was ttm[i, j] when it was observed. The time unit was defined by the argument deltat of the function [fit.schwartz2f](#) (stored in x@deltat).

### Author(s)

David Luethi

### See Also

[fit.schwartz2f](#) for parameter estimation, [plot](#)-method for [schwartz2f](#)-objects.



**Examples**

```

# data(futures)
#
# ## Estimate parameters for lumber data (stop after 100 iterations)
# fit.obj <- fit.schwartz2f(futures$lumber$price, futures$lumber$ttm / 260,
#                          deltat = 1 / 260,
#                          control = list(maxit = 100))
#
# ## Plot parameter evolution
# plot(fit.obj, type = "trace.pars")
#
# ## Plot the state variables
# plot(fit.obj, type = "state", data = futures$lumber$price,
#      ttm = futures$lumber$ttm / 260)
#
# ## Plot fitted and real forward curves of wheat data since Jan 2010.
# lumber.1995 <- lapply(futures$lumber, function(x)x[as.Date(rownames(x)) < "2000-01-01",])
# par(mfrow = c(1, 2))
# plot(fit.obj, type = "forward.curve", data = lumber.1995$price,
#      ttm = lumber.1995$ttm / 260)
# futuresplot(lumber.1995)
#
# ## Plot trajectories from the state variables
# plot(fit.obj, type = "sim")

```

---

plot.state-method

*Plot Schwartz two-factor trajectories*


---

**Description**

This function plots trajectories of the Schwartz two-factor model including the means and confidence intervals at 99%, 95% and 90% levels.

**Usage**

```

## S4 method for signature 'schwartz2f,missing'
plot(x, n = 100, time = 2, dt = 1/52)

```

**Arguments**

x	A <code>schwartz2f</code> -object.
n	Number of trajectories.
time	Time span of the simulation.
dt	Time step.

**Author(s)**

David Luethi

**See Also**[schwartz2f](#) constructor.**Examples**

```
# object <- schwartz2f(s0 = 1, mu = 0.1, sigmaS = 0.2,
#                       delta0 = 0, kappa = 2, alpha = 0.05, sigmaE = 0.1,
#                       rho = 0.5)
#
# plot(object, n = 50, time = 2, dt = 1/52)
```

pricing-futures

*Schwartz two-factor Model: Futures Prices***Description**

Compute arbitrage-free futures prices.

**Usage**

```
## S4 method for signature 'ANY,numeric'
pricefutures(ttm = 1, s0 = 50, delta0 = 0, sigmaS = 0.3,
             kappa = 1, alpha = 0, sigmaE = 0.5, rho = 0.75,
             r = 0.03, lambda = 0, alphaT = NULL)

## S4 method for signature 'ANY,schwartz2f'
pricefutures(ttm = 1, s0, r = 0.03,
             lambda = 0, alphaT = NULL)

## S4 method for signature 'ANY,schwartz2f.fit'
pricefutures(ttm = 1, s0)
```

**Arguments**

ttm	Time to maturity.
s0	Either a numeric representing the initial value of the commodity spot price or an object inheriting from class <a href="#">schwartz2f</a> .
delta0	Initial value of the convenience yield.
sigmaS	Diffusion parameter of the spot price-process.

kappa	Speed of mean-reversion of the convenience-yield process.
alpha	Mean-level of the convenience-yield process.
sigmaE	Diffusion parameter of the convenience-yield process.
rho	Correlation coefficient between the Brownian motion driving the spot-price and the convenience-yield process.
r	Instantaneous risk-free interest rate.
lambda	Market price of convenience yield risk (see <b>Details</b> ).
alphaT	Mean-level of the convenience yield process with respect to the equivalent martingale measure (see <b>Details</b> ).

### Details

The model and its parameters are described in the **Details** section of the [schwartz2f](#)-class documentation and in the package vignette *Technical Document*.

### Value

A numeric containing futures prices.

### Author(s)

Philipp Erb, David Luethi, Juri Hinz

### References

*The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging* by Eduardo S. Schwartz  
Journal of Finance 52, 1997, 923-973

*Valuation of Commodity Futures and Options under Stochastic Convenience Yields, Interest Rates, and Jump Diffusions in the Spot* by Jimmy E. Hilliard and Jorge Reis  
Journal of Financial and Quantitative Analysis 33, 1998, 61-86

### See Also

[priceoption](#) to price options, [d/p/q/rfutures](#) to work with futures, [schwartz2f](#)-constructor, [fit.schwartz2f](#) for parameter estimation, [futures-data](#).

### Examples

```
## function call by atomic arguments
# forward.curve <- pricefutures(ttm = 0.2 * 1:10, s0 = 10, delta0 = 0,
#                               alpha = 0, lambda = 0.02, r = 0)
# plot(forward.curve, type = "b")
#
## function call via schwartz2f-object.
# obj <- schwartz2f(delta0 = 0, sigmaE = 1e-5) # Make convenience yield inactive
```

```
# forward.curve <- pricefutures(ttm = 0.2 * 1:10, s0 = obj, r = 0, alphaT = 0)
# plot(forward.curve, type = "b")
```

pricing-options

*Schwartz two-factor Model: European Option Prices***Description**

Compute arbitrage-free prices of European call and put options on commodity futures contracts.

**Usage**

```
## S4 method for signature 'ANY,ANY,ANY,ANY,numeric'
priceoption(type = c("call", "put"), time = 0.5, Time = 1, K = 40,
            g0 = 50, sigmaS = 0.3, kappa = 1, sigmaE = 0.5,
            rho = 0.75, r = 0.03)

## S4 method for signature 'ANY,ANY,ANY,ANY,schwartz2f'
priceoption(type = c("call", "put"),
            time = 0.5, Time = 1, K = 40,
            g0, r = 0.03, lambda = 0, alphaT = NULL)

## S4 method for signature 'ANY,ANY,ANY,ANY,schwartz2f.fit'
priceoption(type = c("call", "put"),
            time = 0.5, Time = 1, K = 40, g0)
```

**Arguments**

type	Either a European "call" or a "put" option on a futures contract.
time	Exercise time of the option.
Time	Maturity date of the underlying futures (see <b>Details</b> ).
K	Strike price.
g0	The current <i>futures price</i> or an object inheriting from class <a href="#">schwartz2f</a> .
sigmaS	Diffusion parameter of the spot price-process.
kappa	Speed of mean-reversion of the convenience-yield process.
sigmaE	Diffusion parameter of the convenience-yield process.
rho	Correlation coefficient between the Brownian motion driving the spot-price and the convenience-yield process.
r	Instantaneous risk-free interest rate.
lambda	Market price of convenience yield risk (see <b>Details</b> ).
alphaT	Mean-level of the convenience yield process with respect to the equivalent martingale measure (see <b>Details</b> ).

## Details

The price of an option on the spot commodity is obtained by setting `time == Time`. This is because of the convergence of the futures price towards the spot price at maturity. In general the option expires before the futures contract (`time < Time`).

If `g0` is either of class `schwartz2f` or class `schwartz2f.fit` the futures price `g0` is computed first and then plugged into the pricing function with signature `ANY, ANY, ANY, ANY, numeric`.

The model and its parameters are described in the **Details** section of the `schwartz2f`-class documentation and in the package vignette *Technical Document*.

## Value

A numeric containing the option prices.

## Note

Since the two-factor model assumes a constant interest rate, futures and forwards always have the same value and therefore also any derivative of futures or forwards.

## Author(s)

Philipp Erb, David Luethi, Juri Hinz

## References

*The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging* by Eduardo S. Schwartz  
Journal of Finance 52, 1997, 923-973

*Pricing of Options on Commodity Futures with Stochastic Term Structures of Convenience Yields and Interest Rates* by Kristian R. Miltersen and Eduardo S. Schwartz  
Journal of Financial and Quantitative Analysis 33, 1998, 33-59

*Valuation of Commodity Futures and Options under Stochastic Convenience Yields, Interest Rates, and Jump Diffusions in the Spot* by Jimmy E. Hilliard and Jorge Reis  
Journal of Financial and Quantitative Analysis 33, 1998, 61-86

## See Also

[pricefutures](#) to price futures, [d/p/q/rfutures](#) to work with futures, [schwartz2f](#)-constructor, [fit.schwartz2f](#) for parameter estimation, [futures-data](#).

## Examples

```
## The option expires in 0.5 years and the futures contract in 1 year.  
# priceoption(type = "call", time = 0.5, Time = 1, K = 40, g0 = 50)
```

```
#
# ## The price of a European put option on the spot which expires in 2.5
# ## years.
# priceoption(type = "put", time = 2.5, Time = 2.5, K = 900, lambda = 0.02,
#             g0 = schwartz2f(s0 = 1000))
```

---

rand-state

*Schwartz two-factor Model: Sampling from the State Variables*


---

### Description

Random number and trajectory generation of the state variables. The state variables are the commodity spot price  $s_0$  and the spot convenience yield  $\delta_0$ . The commodity log spot price and the convenience yield follow a bivariate normal distribution.

### Usage

```
## S4 method for signature 'ANY,ANY,numeric'
rstate(n, time = 1, s0 = 50, delta0 = 0,
       mu = 0.1, sigmaS = 0.3, kappa = 1, alpha = 0,
       sigmaE = 0.5, rho = 0.75, method = "chol")

## S4 method for signature 'ANY,ANY,swartz2f'
rstate(n, time = 1, s0, method = "chol")

## S4 method for signature 'ANY,ANY,numeric'
simstate(n, time = 1, s0 = 50, delta0 = 0,
        mu = 0.1, sigmaS = 0.3, kappa = 1, alpha = 0,
        sigmaE = 0.5, rho = 0.75, method = "chol")

## S4 method for signature 'ANY,ANY,swartz2f'
simstate(n, time = 1, s0, method = "chol")
```

### Arguments

n	Number of observations.
time	at which random numbers of the state variables are drawn (rstate) or horizon of the trajectory (simstate) relative to now.
s0	Either a numeric representing the initial value of the commodity spot price or an object inheriting from class <code>swartz2f</code> .
delta0	Initial value of the convenience yield.
mu	enters the drift of the commodity spot price.
sigmaS	Diffusion parameter of the spot price-process.

kappa	Speed of mean-reversion of the convenience yield process.
alpha	Mean-level of the convenience yield process.
sigmaE	Diffusion parameter of the convenience yield process.
rho	Correlation coefficient between the Brownian motion driving the spot price and the convenience yield process.
method	See <a href="#">rmvnorm</a> of package mvtnorm.

### Details

The model and its parameters are described in the **Details** section of the [schwartz2f](#)-class documentation and in the package vignette *Technical Document*.

The above methods rely on the functions [pmvnorm](#), [dmvnorm](#), [qmvnorm](#) and [rmvnorm](#) of the package mvtnorm.

### Value

Samples or trajectories of the commodity spot price and instantaneous spot convenience yield as matrix.

### Author(s)

Philipp Erb, David Luethi, Juri Hinz

### See Also

[schwartz2f](#)-class description. [d/p/q/state](#) for the density, distribution, and quantile function of the state variables.

### Examples

```
# ## Create a "schwartz2f"-object
# model <- schwartz2f()
#
# ## and sample from its distribution at time = 2.5.
# sim <- rstate(n = 1000, s0 = model, time = 2.5)
# par(mfrow = c(1, 2))
# hist(sim[,1], main = "Distribution of Spot Price")
# hist(sim[,2], main = "Distribution of Convenience Yield")
#
#
# ## Create a trajectory over a 6 years horizon sampled on a weekly basis.
# trajectory <- simstate(6 * 52, time = 6, s0 = model)
# par(mfrow = c(1, 2))
# plot(trajectory[,1], main = "Spot Price", type = "l")
# plot(trajectory[,2], main = "Convenience Yield", type = "l")
```

---

resid-futures                      *Extract Model Residuals*

---

### Description

Function to extract model residuals from [schwartz2f.fit](#)-objects.

### Usage

```
## S4 method for signature 'schwartz2f.fit'  
resid(object, data, ttm, type = c("filter", "filter.std", "real"))
```

### Arguments

object	A <a href="#">schwartz2f.fit</a> -object returned from <a href="#">schwartz2f.fit</a> .
data	A matrix with futures prices.
ttm	A matrix with the corresponding time to maturity (see <b>Details</b> ).
type	What kind of residuals shall be returned (see <b>Details</b> ).

### Details

If `type == "filter"`, then the residuals from the measurement equation are returned. If `type == "filter.std"`, standardized residuals from the measurement equation are returned (note that these residuals should be standard multivariate normal). If `type == "real"`, the difference between the observed futures prices and the fitted values (see [fitted](#)) are returned.

The model and its parameters are described in the **Details** section of the [schwartz2f](#)-class documentation and in the package vignette *Technical Document*.

### Value

A matrix containing the residuals.

### Author(s)

Philipp Erb, David Luethi, Juri Hinz

### See Also

[fit.schwartz2f](#), [schwartz2f.fit](#)-class, [fitted](#).



**Examples**

```

# data(futures)
#
# ## Estimate parameters for live.cattle data.
# ## (little precision required with reltol = 1e-3)
# fit.obj <- fit.schwartz2f(futures$live.cattle$price, futures$live.cattle$ttm / 260,
#                           deltata = 1 / 260,
#                           control = list(maxit = 100, reltol = 1e-3))
#
# ## Standardized residuals
# resid.std <- resid(fit.obj, data = futures$live.cattle$price, ttm =
#                   futures$live.cattle$ttm / 260, type = "filter.std")
# acf(resid.std, na.action = na.pass) # ...are not independent
#
# ## Real differences
# resid.real <- resid(fit.obj, data = futures$live.cattle$price, ttm =
#                   futures$live.cattle$ttm / 260, type = "real")
#
# plot(as.ts(resid.real / futures$live.cattle$price)) # ...are 'relatively' accurate.

```

---

 schwartz2f-class-hierarchy

*Classes schwartz2f and schwartz2f.fit*

---

**Description**

The `schwartz2f` class stores parameters which determine initial values and the dynamics of the state variables. The class `schwartz2f.fit` inherits from the `schwartz2f` class. The class `schwartz2f.fit` adds slots which contain data regarding the estimation procedure and parameters of the risk-neutral dynamics. In particular, it adds the market price of convenience yield risk  $\lambda$  and the interest rate.

**Objects from the Class**

Objects should only be created by calls to the constructors `schwartz2f` and `fit.schwartz2f`.

**Slots****Slots of class “schwartz2f”:**

`call`: The function-call of class `call`.

`s0`: Initial commodity spot-price of class `numeric`.

`deltat0`: Initial value of the convenience yield of class `numeric`.

`mu`: Enters the drift of the commodity spot price (under the objective measure, see **Details**) of class `numeric`.

`sigmaS`: Diffusion parameter of the spot price process of class `numeric`.

`kappaE`: Speed of mean-reversion of the convenience-yield process of class `numeric`.

`alpha`: Mean level of the convenience-yield process of class `numeric`.

`sigmaE`: Diffusion parameter of the convenience-yield process of class `numeric`.

`rhoSE`: Correlation between the two Brownian motions which drive the spot price and convenience-yield processes of class `numeric`.

**Slots added by class “`schwartz2f.fit`”:**

`n.iter`: The number of iterations of class `numeric`.

`llh`: The log likelihood value of class `numeric`.

`converged`: A logical stating whether the fit converged or not.

`error.code`: An error code of class `numeric`. The value of `optim`’s “convergence”. If an unknown error occurs the value -1 is returned.

`error.message`: An error message of class character, if any.

`fitted.params`: A logical vector stating which parameters were fitted.

`trace.pars`: Contains the parameter value evolution during the estimation procedure of class `matrix`.

`r`: The risk-free interest rate of class `numeric`.

`alphaT`: The mean-value of the convenience yield process under the equivalent martingale measure of class `numeric` (see **Details**).

`lambda`: The market price of convenience yield risk of class `numeric`.

`meas.sd`: The standard deviation of the measurement equation of class `numeric`.

`deltat`: The time-increment of the transition equation of class `numeric`.

**Details**

The joint dynamics of the spot-price and the convenience yield are given by the stochastic differential equations

$$\begin{aligned} dS_t &= (\mu - \delta_t)S_t dt + \sigma_S \tilde{S}_t \tilde{dW}_t^1, \\ d\delta_t &= \kappa(\alpha - \delta_t)dt + \sigma_E dW_t^2 \\ dW_t^1 \tilde{\cdot} dW_t^2 &= \rho dt, \end{aligned}$$

where  $W^1, W^2$  are Brownian motions under the objective measure.

Under an equivalent martingale measure (the pricing measure) the dynamics is

$$\begin{aligned} dS_t &= (r - \delta_t)S_t dt + \sigma_S \tilde{S}_t \tilde{d\tilde{W}}_t^1 \\ d\delta_t &= \kappa(\tilde{\alpha} - \delta_t)dt + \sigma_E d\tilde{W}_t^2, \end{aligned}$$

where  $\tilde{W}^1, \tilde{W}^2$  are Brownian motions with respect to the martingale measure.

$\tilde{\alpha} = \alpha - \lambda/\kappa$  where  $\lambda$  is the market price of convenience-yield risk.

**Extends**

Class “`schwartz2f.fit`” extends class “`schwartz2factor`”, directly.

**Author(s)**

Philipp Erb, David Luethi, Juri Hinz

**References**

*Stochastic Convenience Yield and the Pricing of Oil Contingent Claims* by Rajna Gibson and Eduardo S. Schwartz  
The Journal of Finance 45, 1990, 959-976

*The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging* by Eduardo S. Schwartz  
Journal of Finance 52, 1997, 923-973

*Pricing of Options on Commodity Futures with Stochastic Term Structures of Convenience Yields and Interest Rates* by Kristian R. Miltersen and Eduardo S. Schwartz  
Journal of Financial and Quantitative Analysis 33, 1998, 33-59

*Valuation of Commodity Futures and Options under Stochastic Convenience Yields, Interest Rates, and Jump Diffusions in the Spot* by Jimmy E. Hilliard and Jorge Reis  
Journal of Financial and Quantitative Analysis 33, 1998, 61-86

**See Also**

[schwartz2f](#) to initialize schwartz2f-objects. [fit.schwartz2f](#) to fit the two-factor model to data and get a schwartz2f.fit object, [schwartz97-package](#) for an overview.

**Examples**

```
# obj <- schwartz2f() # create an object of class schwartz2f
# obj           # print it
# coef(obj)    # get coefficients
# unclass(obj) # see the slots
#
# ## create an object of class schwartz2f.fit
# data(futures)
# fit.obj <- fit.schwartz2f(futures$wheat$price, futures$wheat$ttm / 260,
#                           deltat = 1 / 260, control = list(maxit = 3))
# fit.obj           # print it
# coef(fit.obj)    # get coefficients
# unclass(fit.obj) # see the slots
```

---

schwartz2f-constructor

*Create schwartz2f objects*

---

## Description

Create objects of class [schwartz2f](#).

## Usage

```
schwartz2f(s0 = 100, delta0 = 0, mu = 0.1, sigmaS = 0.3,  
          kappa = 1, alpha = 0, sigmaE = 0.3, rho = 0.5)
```

## Arguments

s0	Initial value of the commodity spot price.
delta0	Initial value of the convenience yield.
mu	enters the drift of the commodity spot price.
sigmaS	Diffusion parameter of the spot price-process.
kappa	Speed of mean-reversion of the convenience yield process.
alpha	Mean-level of the convenience yield process.
sigmaE	Diffusion parameter of the convenience yield process.
rho	Correlation coefficient between the Brownian motion driving the spot price and the convenience yield process.

## Details

The dynamics of the Schwartz two-factor model is explained in the [schwartz2f](#) class documentation or in the package vignette in the doc-folder.

## Value

An object of class [schwartz2f](#).

## Author(s)

Philipp Erb, David Luethi, Juri Hinz

## References

*The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging* by Eduardo S. Schwartz  
Journal of Finance 52, 1997, 923-973

**See Also**

[fit.schwartz2f](#) for parameter estimation. [d/p/q/r/simstate](#) for the density, distribution, and quantile function of the state variables and random number generation. [plot-method](#) for [schwartz2f](#)-objects.

**Examples**

```
# ## Initialize a 'schwartz2f' object with high convenience yield volatility:
# obj <- schwartz2f(sigmaE = 0.7)
#
# plot(obj) # plot it
#
# rstate(10, time = 1, s0 = obj) # generate 10 random variates.
#
# ## Get the probability of the event 'the spot price is >= 100 and the
# ## convenience yield is >= 0':
# pstate(c(0, -Inf), c(100, 0), time = 10, s0 = obj)
```

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