

# Package ‘Iso’

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**Title** Functions to perform isotonic regression.

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**Depends** R (>= 1.7.0)

**Description** Linear order and unimodal order (univariate) isotonic regression; bivariate isotonic regression with linear order on both variables.

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biviso	<i>Bivariate isotonic regression.</i>
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## Description

Bivariate isotonic regression with respect to simple (increasing) linear ordering on both variables.

**Usage**

```
biviso(y, w = NULL, eps = NULL, eps2 = 1e-9, ncycle = 50000,
      fatal = TRUE, warn = TRUE)
```

**Arguments**

y	The matrix of observations to be isotonized. It must of course have at least two rows and at least two columns.
w	A matrix of weights, greater than or equal to zero, of the same dimension as y. If left NULL then w is created as a matrix all of whose entries are equal to 1.
eps	Convergence criterion. The algorithm is deemed to have converged if each entry of the output matrix, after the completion of the current iteration, does not differ by more than eps from the corresponding entry of the matrix after the completion of the previous iteration. If this argument is not supplied it defaults to <code>sqrt(.Machine\$double.eps)</code> .
eps2	Criterion used to determine whether isotonicity is “violated”, whence whether (further) application of the “pool adjacent violators” procedure is required.
ncycle	The maximum number of cycles of the iteration procedure. Must be at least 2 (otherwise an error is given). If the procedure has not converged after ncycle iterations then an error is given. (See below.)
fatal	Logical scalar. Should the function stop if the subroutine returns an error code other than 0 or 4? If fatal is FALSE then output is returned by the function even if there was a “serious” fault. One can set fatal=FALSE to inspect the values of the objective matrix at various interim stages prior to convergence. See <b>Examples</b> .
warn	Logical scalar. Should a warning be produced if the subroutine returns a value of ifault equal to 4 (or to any other non-zero value when fatal has been set to FALSE)?

**Details**

See the paper by Bril et al., (*References*) and the references cited therein for details.

**Value**

A matrix of the same dimensions as y containing the corresponding isotonic values. It has an attribute `icycle` equal to the number of cycles required to achieve convergence of the algorithm.

**Error Messages**

The subroutine comprising Algorithm AS 206 produces an error code `ifault` with values from 0 to 6 The meaning of these codes is as follows:

- 0: No error.
- 1: Convergence was not attained in `ncycle` cycles.
- 2: At least one entry of w was negative.

- 3: Either `nrow(y)` or `ncol(y)` was less than 2.
- 4: A near-zero weight less than `delta=0.00001` was replaced by `delta`.
- 5: Convergence was not attained *and* a non-zero weight was replaced by `delta`.
- 6: All entries of `w` were less than `delta`.

If `ifault==4` a warning is given. All of the other non-zero values of `ifault` result in an error being given.

## WARNING

This function appears not to achieve exact isotonicity, at least not quite. For instance one can do:

```
set.seed(42)
u <- matrix(runif(400),20,20)
iu <- biviso(u)
any(apply(iu,2,is.unsorted))
```

and get TRUE. It turns out that columns 13, 14, and 16 of `iu` have exceptions to isotonicity. E.g. six of the values of `diff(iu[,13])` are less than zero. However only one of these is less than `sqrt(.Machine$double.eps)`, and then only “marginally” smaller.

So some of these negative values are “numerically different” from zero, but not by much. The largest in magnitude in this example, from column 16, is  $-2.217624e-08$  — which is probably not of “practical importance”.

Note also that this example occurs in a very artificial context in which there is no actual isotonic structure underlying the data.

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## References

Bril, Gordon; Dykstra, Richard; Pillers Carolyn, and Robertson, Tim ; Isotonic regression in two independent variables; Algorithm AS 206; JRSSC (Applied Statistics), vol. 33, no. 3, pp. 352-357, 1984.

## See Also

[pava\(\)](#) [pava.sa\(\)](#) [ufit\(\)](#)

## Examples

```
x <- 1:20
y <- 1:10
xy <- outer(x,y,function(a,b){a+b+0.5*a*b}) + rnorm(200)
ixy <- biviso(xy)

set.seed(42)
u <- matrix(runif(400),20,20)
```

```
v <- biviso(u)
progress <- list()
for(n in 1:9) progress[[n]] <- biviso(u,ncycle=50*n,fatal=FALSE,warn=FALSE)
```

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pava

*Linear order isotonic regression.*


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## Description

The “pool adjacent violators algorithm” (PAVA) is applied to calculate the isotonic regression of a set of data, with respect to the usual increasing (or decreasing) linear ordering on the indices.

## Usage

```
pava(y, w, decreasing=FALSE, long.out=FALSE, stepfun=FALSE)
pava.sa(y, w, decreasing=FALSE, long.out=FALSE, stepfun=FALSE)
```

## Arguments

y	Vector of data whose isotonic regression is to be calculated.
w	Optional vector of weights to be used for calculating a weighted isotonic regression; if w is not given, all weights are taken to equal 1.
decreasing	Logical scalar; should the isotonic regression be calculated with respect to <i>decreasing</i> (rather than increasing) order?
long.out	Logical argument controlling the nature of the value returned.
stepfun	Logical scalar; if TRUE a step function representation of the isotonic regression is returned.

## Details

The function `pava()` uses dynamically loading of a fortran subroutine "pava" to effect the computations. The function `pava.sa()` ("sa" for "stand-alone") does all of the computations in raw R. Thus `pava.sa()` could be considerably slower for large data sets.

The x values for the step function returned by these functions (if `stepfun` is TRUE) are thought of as being 1, 2, ...,  $n = \text{length}(y)$ . The knots of the step function are the x values (indices) *following* changes in the y values (i.e. the starting indices of the level sets, except for the first level set). The y value corresponding to the first level set is the “left hand” value of y or `yleft`. The step function is formed using the default arguments of `stepfun()`. In particular it is *right* continuous.

## Value

If `long.out` is TRUE then the result returned consists of a list whose components are:

y	the fitted values
w	the final weights

- tr a set of indices made up of the smallest index in each level set, which thus "keeps track" of the level sets.
- h a step function which represents the results of the isotonic regression. This component is present *only if* stepfun is TRUE.

If long.out is FALSE and stepfun is TRUE then only the step function is returned.

If long.out and stepfun are both FALSE then only the vector of fitted values is returned.

### Author(s)

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### References

Robertson, T., Wright, F. T. and Dykstra, R. L. (1988). Order Restricted Statistical Inference. Wiley, New York.

### See Also

[ufit\(\)](#) [stepfun\(\)](#) [biviso\(\)](#)

### Examples

```
# Increasing order:
y <- (1:20) + rnorm(20)
ystar <- pava(y)
plot(y)
lines(ystar, type='s')
# Decreasing order:
z <- NULL
for(i in 4:8) {
  z <- c(z, rep(8-i+1, i)+0.05*(0:(i-1)))
}
zstar <- pava(z, decreasing=TRUE)
plot(z)
lines(zstar, type='s')
# Using the stepfunction:
zstar <- pava(z, decreasing=TRUE, stepfun=TRUE)
plot(z)
plot(zstar, add=TRUE, verticals=FALSE, pch=20, col.points="red")
```

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ufit

*Unimodal isotonic regression.*

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### Description

A "divide and conquer" algorithm is applied to calculate the isotonic regression of a set of data, for a unimodal order. If the mode of the unimodal order is not specified, then the optimal (in terms of minimizing the error sum of squares) unimodal fit is calculated.

**Usage**

```
ufit(y, lmode=NULL, x=NULL, w=NULL, lc=TRUE, rc=TRUE,
     type=c("raw", "stepfun", "both"))
```

**Arguments**

y	Vector of data whose isotonic regression is to be calculated.
lmode	Gives the location of the mode if this is specified; if the location is not specified, then all possible modes are tried and that one giving the smallest error sum of squares is used.
x	A largely notional vector of $x$ values corresponding to the data vector $y$ ; the value of the mode must be given, or will be calculated in terms of these $x$ values. Conceptually the model is $y = m(x) + E$ , where $m()$ is a unimodal function with mode at $lmode$ , and where $E$ is random "error". If $x$ is not specified, it defaults to an equi-spaced sequence on $[0,1]$ .
w	Optional vector of weights to be used for calculating a weighted isotonic regression; if $w$ is not given, all weights are taken to equal 1.
lc	Logical argument; should the isotonization be left continuous? If $lc==FALSE$ then the value of the isotonization just before the mode is set to NA, which causes line plots to have a jump discontinuity at (just to the left of) the mode. The default is $lc=TRUE$ .
rc	Logical argument; should the isotonization be right continuous? If $rc==FALSE$ then the value of the isotonization just after the mode is set to NA, which causes line plots to have a jump discontinuity at (just to the right of) the mode. The default is $rc=TRUE$ .
type	String specifying the type of the output; see "Value". May be abbreviated.

**Details**

Dynamically loads fortran subroutines "pava", "ufit" and "unimode" to do the actual work.

**Value**

If  $type=="raw"$  then the value is a list with components:

x	The argument $x$ if this is specified, otherwise the default value.
y	The fitted values.
lmode	The argument $lmode$ if this is specified, otherwise the value of $lmode$ which is found to minimize the error sum of squares.
mse	The mean squared error.

If  $type=="both"$  then a component  $h$  which is the step function representation of the isotonic regression is added to the foregoing list.

If  $type=="stepfun"$  then only the step function representation  $h$  is returned.

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**References**

Mureika, R. A., Turner, T. R. and Wollan, P. C. (1992). An algorithm for unimodal isotonic regression, with application to locating a maximum. University of New Brunswick Department of Mathematics and Statistics Technical Report Number 92 – 4.

Robertson, T., Wright, F. T. and Dykstra, R. L. (1988). Order Restricted Statistical Inference. Wiley, New York.

Shi, Ning-Zhong. (1988) A test of homogeneity for umbrella alternatives and tables of the level probabilities. Commun. Statist. — Theory Meth. vol. 17, pp. 657 – 670.

Turner, T. R., and Wollan, P. C. (1997) Locating a maximum using isotonic regression. Computational Statistics and Data Analysis vol. 25, pp. 305 – 320.

**See Also**

[pava\(\)](#) [biviso\(\)](#)

**Examples**

```
x <- c(0.00,0.34,0.67,1.00,1.34,1.67,2.00,2.50,3.00,3.50,4.00,4.50,  
      5.00,5.50,6.00,8.00,12.00,16.00,24.00)  
y <- c(0.0,61.9,183.3,173.7,250.6,238.1,292.6,293.8,268.0,285.9,258.8,  
      297.4,217.3,226.4,170.1,74.2,59.8,4.1,6.1)  
z <- ufit(y,x=x,type="b")  
plot(x,y)  
lines(z,col="red")  
plot(z$h,do.points=FALSE,col.hor="blue",col.vert="blue",add=TRUE)
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