

# Package ‘bpkde’

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**Title** Back-Projected Kernel Density Estimation

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**Suggests** mvtnorm, KernSmooth

**Description** Nonparametric multivariate kernel density \{\}  
estimation using a back-projected kernel.

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bpk *Back Projected Kernel*

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### Description

Evaluate a d dimensional back projected kernel.

### Usage

```
bpk(X, bandwidths, a = 1.0, kernel = dnorm, ...)
```

### Arguments

X	a numeric matrix of dimension N by d who's rows contain the points where the kernel will be evaluated.
bandwidths	a list with elements alphas, the set of projection directions, and lambdas, their associated bandwidth estimates.
a	a single numeric value containing a common scaling parameter.
kernel	a function for evaluating the univariate kernel.
...	additional arguments are ignored.

### Value

a numeric vector where element i contains the value of the kernel evaluated at row i of X.

### Author(s)

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bpkde *Back-Projected Kernel Density Estimation*

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### Description

Compute a d-dimensional kernel density estimate using a back-projected kernel.

### Usage

```
bpkde(data, alphas, kernel = dnorm, bw = bw.SJ, score.fun = M1, r = 7, padding = 4)
```

**Arguments**

data	a matrix or data frame. The data is coerced to a numeric matrix using the <code>data.matrix</code> function.
alphas	a numeric matrix of dimension $d$ by $K$ whose columns contain the directions (as unit vectors) used for the back projection. If missing, 90 equally space directions spanning the interval $[-\pi/2, \pi/2)$ are used when $d == 2$ and 450 randomly chosen directions are used when $d == 3$ . This argument must be provided for $d >= 4$ .
kernel	a function for evaluating the univariate kernel.
bw	the function used to compute the univariate bandwidth estimates.
score.fun	the function used to compute the least squares cross-validation score for the kernel; see <a href="#">M0</a> and <a href="#">M1</a> .
r	the computations are performed using linear binning and the discrete Fourier transform. The number of the grid points used is $2^r$ .
padding	a positive numeric value specifying the amount of zero-padding in units of bandwidth.

**Value**

a list with class `c("bpkde", "mvkde")` containing the following elements.

axes	a numeric matrix whose columns contain the grid points used along each axis to bin the data.
z	a numeric array containing the discrete kernel density estimate.
params	a list containing the optimal common scaling parameter <code>omega.hat</code> , the input set of directions <code>alphas</code> , and the computed univariate bandwidths <code>lambdas</code> .

**References**

Panaretos, Victor M. and Konis, Kjell (2012). Nonparametric Construction of Multivariate Kernels. *Journal of the American Statistical Association* 107(499):1085-1095.

**Author(s)**

Kjell Konis <kjell.konis@me.com>

**Examples**

```
data(Trimodal2)
f.hat <- bpkde(Trimodal2)
```

discretize.kernel      *Discretize Kernel*

---

### Description

Computes a discrete approximation of the kernel.

### Usage

```
discretize.kernel(grid, kern.fun, ..., grid.fun = NULL, scale = TRUE)
```

### Arguments

grid	a list created by the <code>mvlinbin</code> function.
kern.fun	a function for evaluating the kernel. The first argument must be a matrix whose rows contain the points where the density is to be evaluated. See, for example, the <code>dmvnorm</code> function in the <code>mvtnorm</code> package.
...	additional arguments are passed to <code>kern.fun</code> .
grid.fun	a function that returns the grid points where the kernel will be evaluated.
scale	a logical value. If <code>TRUE</code> then the kernel is scaled so that it integrates to 1 on the provided grid.

### Value

a list with class `kernel` containing the following elements.

axes	a numeric matrix whose columns contain the grid points along each axis where the kernel was evaluated.
z	a numeric array containing the discrete representation of the kernel.
dft	a numeric array containing the discrete Fourier transform of the kernel <code>z</code> .
kern.fun	the name of <code>kern.fun</code> .
params	a list containing the parameters passed to <code>kern.fun</code> through <code>...</code>

### Author(s)

Kjell Konis <kjell.konis@me.com>

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`kernels`*Univariate Kernels*

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**Description**

Evaluate univariate kernels.

**Usage**

```
biweight(x)
epanechnikov(x)
rectangular(x)
triangular(x)
```

**Arguments**

`x` a numeric vector.

**Value**

a numeric vector.

**Author(s)**

Kjell Konis <kjell.konis@me.com>

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`M0`*M0 Least-Squares Cross-Validation*

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**Description**

Compute the M0 least-squares cross-validation score.

**Usage**

```
M0(grid, kern.fun, ...)
```

**Arguments**

`grid` an object of class `linbin`.  
`kern.fun` the density function of the kernel. The first argument must be a matrix whose rows contain the points where the density function will be evaluated. See, for example, the [dmvnorm](#) function in the `mvtnorm` package.  
`...` additional arguments are passed to `kern.fun`.

**Value**

a single numeric value: the M0 least-squares cross-validation score.

**Author(s)**

Kjell Konis <kjell.konis@me.com>

**References**

Silverman, B. W. (1986) *Density Estimation for Statistics and Data Analysis*. London: Chapman and Hall.

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M1

*M1 Least-Squares Cross-Validation*

---

**Description**

Compute the M1 least-squares cross-validation score.

**Usage**

```
M1(grid, kern.fun, ...)
```

**Arguments**

<code>grid</code>	an object of class <code>linbin</code> .
<code>kern.fun</code>	the density function of the kernel. The first argument must be a matrix whose rows contain the points where the density function will be evaluated. See, for example, the <a href="#">dmvnorm</a> function in the <code>mvtnorm</code> package.
<code>...</code>	additional arguments are passed to <code>kern.fun</code> .

**Details**

The computation is done using the Fourier transform of the kernel and the data as described in Silverman (1986).

**Value**

a single numeric value: the M1 least-squares cross-validation score.

**Author(s)**

Kjell Konis <kjell.konis@me.com>

**References**

Silverman, B. W. (1986) *Density Estimation for Statistics and Data Analysis*. London: Chapman and Hall.

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mvlinbin	<i>Multivariate Linear Binning</i>
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**Description**

Compute a binned approximation of the data on a regularly spaced grid using the multivariate linear binning rule described in Wand (1994).

**Usage**

```
mvlinbin(X, r = 7, padding)
```

**Arguments**

X	a numeric matrix.
r	a positive integer value. The number of grid points $M$ in each dimension is given by $M = 2^r$ .
padding	a numeric vector of positive values with length equal to the number of columns of $X$ specifying the amount of zero-padding added to each coordinate direction. No padding is added when this argument is missing.

**Value**

a list with class `mvlinbin` containing the following elements.

axes	a numeric matrix whose columns contain the grid points used along each axis to bin the data.
xi	a numeric array containing the binned approximation of the data.
X	a numeric matrix containing the input data.
deltas	a numeric vector containing the grid spacing.
M	an integer value giving the number of grid points used in each coordinate direction.
n	an integer value containing the number of data points binned.
d	an integer value giving the dimensionality of the data.

**Author(s)**

Kjell Konis <kjell.konis@me.com>

**References**

Wand, M. P. (1994). Fast Computation of Multivariate Kernel Estimators. *Journal of Computational and Graphical Statistics*, 3, 433-445.

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WandJones

*Example Data Sets from Wand & Jones*

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### **Description**

The 12 normal mixture densities considered in Wand and Jones (1993).

### **Usage**

```
data(Uncorrelated.Normal)
data(Correlated.Normal)
data(Skewed)
data(Kurtotic)
data(Bimodal1)
data(Bimodal2)
data(Bimodal3)
data(Bimodal4)
data(Trimodal1)
data(Trimodal2)
data(Trimodal3)
data(Quadrimodal)
```

### **Details**

These 12 data sets consist of 500 points drawn from the Gaussian mixture distributions described in Wand and Jones (1993). The code is provided in the `scripts` subdirectory of this package.

### **Source**

Wand, M. P. and Jones, M. C. (1993). Comparison of Smoothing Parameterizations in Bivariate Kernel Density Estimation. *Journal of the American Statistical Association*, Vol. 88, No. 422, pp. 520-528.



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