Package 'mpoly'

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Type Package

Title Symbolic Computation and More with Multivariate Polynomials

Version 1.0.3

URL https://github.com/dkahle/mpoly

BugReports https://github.com/dkahle/mpoly/issues

Description Symbolic computing with multivariate polynomials in R.

Depends stringr (>= 1.0.0)

Imports partitions, plyr, stats, ggplot2, polynom, orthopolynom, tidyr

Suggests testthat, magrittr, dplyr

License GPL-2

RoxygenNote 5.0.1

NeedsCompilation no

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Repository CRAN

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R topics documented:

`as.function.mpoly` *Change a multivariate polynomial into a function.*

Description

Transforms an mpoly object into a function which can be evaluated.

Usage

```
## S3 method for class 'mpoly'
as.function(x, varorder = vars(x), vector
  = TRUE, ...)
```

Arguments

x	an object of class mpoly
varorder	the order in which the arguments of the function will be provided
vector	whether the function should take a vector argument (TRUE) or a series of arguments (FALSE)
...	any additional arguments

See Also

plug

Examples

```
p <- mp("x + 3 x y + z^2 x")
f <- as.function(p)
f(1:3) # -> 16
f(c(1,1,1)) # -> 5

f <- as.function(p, vector = FALSE)
f(1, 2, 3) # -> 16
f(1, 1, 1) # -> 5

f <- as.function(p, varorder = c("z","y","x"), vector = FALSE)
f(3, 2, 1) # -> 16
f(1, 1, 1) # -> 5

# for univariate mpolys, as.function() returns a vectorized function
# that can even apply to arrays
p <- mp("x^2")
f <- as.function(p)
f(1:10)
(mat <- matrix(1:4, 2))
f(mat)


p <- mp("1 2 3 4")
f <- as.function(p)
f(10) # -> 24
```

---

as.function.mpolyList *Change a vector of multivariate polynomials into a function.*

Description

Transforms an mpolyList object into a function which can be evaluated.

Usage

```
## S3 method for class 'mpolyList'
as.function(x, varorder = vars(x),
  vector = TRUE, ...)

## S3 method for class 'bezier'
as.function(x, ...)
```

Arguments

x	an object of class mpolyList
varorder	the order in which the arguments of the function will be provided (default vars(mpoly))
vector	whether the function should take a vector argument (TRUE) or a series of arguments (FALSE)
...	any additional arguments

Examples

```
# basic examples
mpolyList <- mp(c("2 x + 1", "x - z^2"))
f <- as.function(mpolyList)
f(c(1,2)) # -> (2*1 + 1, 1-2^2) = 3 -3

f <- as.function(mpolyList, varorder = c("x","y","z"))
f(c(1,0,2)) # -> 3 -3
f(c(1,4,2)) # -> 3 -3

f <- as.function(mpolyList, varorder = c("x","y","z"), vector = FALSE)
f(1, 0, 2) # -> 3 -3
f(1, 4, 2) # -> 3 -3



# making a gradient function (useful for optim)
mpoly <- mp("x + y^2 + y z")
mpolyList <- gradient(mpoly)
f <- as.function(mpolyList, varorder = vars(mpoly))
f(c(0,2,3)) # -> 1 7 2



# a univariate mpolyList creates a vectorized function
ps <- mp(c("x", "x^2", "x^3"))
f <- as.function(ps)
f
s <- seq(-1, 1, length.out = 11)
f(s)

# another example
```

```
ps <- chebyshev(1:3)
f <- as.function(ps)
f(s)

# the binomial pmf as an algebraic (polynomial) map
# from [0,1] to [0,1]^size
# p |-> {choose(size, x) p^x (1-p)^(size-x)}_{x = 0, ..., size}
abinom <- function(size, indet = "p"){
  chars4mp <- vapply(as.list(0:size), function(x){
    sprintf("%d %s^%d (1-%s)^%d", choose(size, x), indet, x, indet, size-x)
  }, character(1))
  mp(chars4mp)
}
(ps <- abinom(2, "p")) # = mp(c("(1-p)^2", "2 p (1-p)", "p^2"))
f <- as.function(ps)

f(.5) # P[X = 0], P[X = 1], and P[X = 2] for X ~ Bin(2, .5)
dbinom(0:2, 2, .5)

f(.75) # P[X = 0], P[X = 1], and P[X = 2] for X ~ Bin(2, .75)
dbinom(0:2, 2, .75)

# as the degree gets larger, you'll need to be careful when evaluating
# the polynomial.  as.function() is not currently optimized for
# stable numerical evaluation of polynomials; it evaluates them in
# the naive way
all.equal(
  as.function(abinom(10))(.5),
  dbinom(0:10, 10, .5)
)

all.equal(
  as.function(abinom(30))(.5),
  dbinom(0:30, 20, .5)
)


# the function produced is vectorized:
number_of_probs <- 11
probs <- seq(0, 1, length.out = number_of_probs)
(mat <- f(probs))
colnames(mat) <- sprintf("P[X = %d]", 0:2)
rownames(mat) <- sprintf("p = %.2f", s)
mat
```

---

as.mpoly *Convert an object to an mpoly*

Description

mpoly is the most basic function used to create objects of class mpoly.

Usage

```
as.mpoly(x, ...)
```

Arguments

x	an object
...	additional arguments to pass to methods

Value

the object formated as a mpoly object.

Author(s)

David Kahle <david.kahle@gmail.com>

See Also

mp

Examples

```
library(ggplot2); theme_set(theme_classic())
library(dplyr)

n <- 101
s <- seq(-5, 5, length.out = n)



# one dimensional case

df <- data.frame(x = seq(-5, 5, length.out = n)) %>%
  mutate(y = -x^2 + 2*x - 3 + rnorm(n, 0, 2))

(mod <- lm(y ~ x + I(x^2), data = df))
(p <- as.mpoly(mod))
qplot(x, y, data = df) +
  stat_function(fun = as.function(p), colour = "red", size = 1)

(mod <- lm(y ~ poly(x, 2, raw = TRUE), data = df))
(p <- as.mpoly(mod))
qplot(x, y, data = df) +
  stat_function(fun = as.function(p), colour = "red", size = 1)

(mod <- lm(y ~ poly(x, 1, raw = TRUE), data = df))
```

```
(p <- as.mpoly(mod))
qplot(x, y, data = df) +
  stat_function(fun = as.function(p), colour = "red", size = 1)



# two dimensional case with ggplot2

df <- expand.grid(x = s, y = s) %>%
  mutate(z = x^2 - y^2 + 3*x*y + rnorm(n^2, 0, 3))
qplot(x, y, data = df, geom = "raster", fill = z)

(mod <- lm(z ~ x + y + I(x^2) + I(y^2) + I(x*y), data = df))
(mod <- lm(z ~ poly(x, y, degree = 2, raw = TRUE), data = df))
(p <- as.mpoly(mod))
df$fit <- apply(df[,c("x","y")], 1, as.function(p))

qplot(x, y, data = df, geom = "raster", fill = fit)

qplot(x, y, data = df, geom = "raster", fill = z - fit) # residuals
```

`bernstein` *Bernstein polynomials*

Description

Bernstein polynomials

Usage

```
bernstein(k, n, indeterminate = "x")
```

Arguments

`k`	Bernstein polynomial k
`n`	Bernstein polynomial degree
`indeterminate`	indeterminate

Value

a mpoly object

Author(s)

David Kahle

Examples

```
bernstein(0, 0)

bernstein(0, 1)
bernstein(1, 1)

bernstein(0, 1, "t")

bernstein(0:2, 2)
bernstein(0:3, 3)
bernstein(0:3, 3, "t")


bernstein(0:4, 4)
bernstein(0:10, 10)
bernstein(0:10, 10, "t")
bernstein(0:20, 20, "t")

## Not run:   # visualize the bernstein polynomials

library(ggplot2); theme_set(theme_classic())
library(tidyr)

s <- seq(0, 1, length.out = 101)
N <- 10 # number of bernstein polynomials to plot
(bernPolys <- bernstein(0:N, N))

df <- data.frame(s, as.function(bernPolys)(s))
names(df) <- c("x", paste0("B_", 0:N))
head(df)

mdf <- gather(df, degree, value, -x)
head(mdf)

qplot(x, value, data = mdf, geom = "line", color = degree)


## End(Not run)
```

---

bernsteinApprox *Bernstein polynomial approximation*

Description

Bernstein polynomial approximation

Usage

```
bernsteinApprox(f, n, lower = 0, upper = 1, indeterminate = "x")
```

Arguments

f	the function to approximate
n	Bernstein polynomial degree
lower	lower bound for approximation
upper	upper bound for approximation
indeterminate	indeterminate

Value

a mpoly object

Author(s)

David Kahle

Examples

```
## Not run:   # visualize the bernstein polynomials

library(ggplot2); theme_set(theme_bw())
library(reshape2)



f <- function(x) sin(2*pi*x)
p <- bernsteinApprox(f, 20)
round(p, 3)

x <- seq(0, 1, length.out = 101)
df <- data.frame(
  x = rep(x, 2),
  y = c(f(x), as.function(p)(x)),
  which = rep(c("actual", "approx"), each = 101)
)
qplot(x, y, data = df, geom = "line", color = which)
```

```
p <- bernsteinApprox(sin, 20, pi/2, 1.5*pi)
round(p, 4)

x <- seq(0, 2*pi, length.out = 101)
df <- data.frame(
  x = rep(x, 2),
  y = c(sin(x), as.function(p)(x)),
  which = rep(c("actual", "approx"), each = 101)
)
qplot(x, y, data = df, geom = "line", color = which)
```

```
p <- bernsteinApprox(dnorm, 15, -1.25, 1.25)
round(p, 4)

x <- seq(-3, 3, length.out = 101)
df <- data.frame(
  x = rep(x, 2),
  y = c(dnorm(x), as.function(p)(x)),
  which = rep(c("actual", "approx"), each = 101)
)
qplot(x, y, data = df, geom = "line", color = which)
```

```
## End(Not run)
```

`bezier` *Bezier polynomials*

Description

Compute the Bezier polynomials of a given collection of points. Note that using `as.function` on the resulting Bezier polynomials is made numerically stable by taking advantage of de Casteljau's

algorithm; it does not use the polynomial that is printed to the screen. See bezierFunction for details.

Usage

```
bezier(..., indeterminate = "t")
```

Arguments

...	either a sequence of points or a matrix/data frame of points, see examples
indeterminate	the indeterminate of the resulting polynomial

Value

a mpoly object

Author(s)

David Kahle

See Also

bezierFunction

Examples

```
p1 <- c(0,  0)
p2 <- c(1,  1)
p3 <- c(2, -1)
p4 <- c(3,  0)
bezier(p1, p2, p3, p4)


points <- data.frame(x = 0:3, y = c(0,1,-1,0))
bezier(points)


points <- data.frame(x = 0:2, y = c(0,1,0))
bezier(points)









# visualize the bernstein polynomials

library(ggplot2); theme_set(theme_bw())

s <- seq(0, 1, length.out = 101)
```

```
## example 1
points <- data.frame(x = 0:3, y = c(0,1,-1,0))
(bezPolys <- bezier(points))

f <- as.function(bezPolys)
df <- as.data.frame(f(s))

ggplot(aes(x = x, y = y), data = df) +
  geom_point(data = points, color = "red", size = 8) +
  geom_path(data = points, color = "red") +
  geom_path()



## example 1 with weights
f <- as.function(bezPolys, weights = c(1,5,5,1))
df <- as.data.frame(f(s))

ggplot(aes(x = x, y = y), data = df) +
  geom_point(data = points, color = "red", size = 8) +
  geom_path(data = points, color = "red") +
  geom_path()



## example 2
points <- data.frame(x = 0:2, y = c(0,1,0))
(bezPolys <- bezier(points))
f <- as.function(bezPolys)
df <- as.data.frame(f(s))

ggplot(aes(x = x, y = y), data = df) +
  geom_point(data = points, color = "red", size = 8) +
  geom_path(data = points, color = "red") +
  geom_path()



## example 3
points <- data.frame(x = c(-1,-2,2,1), y = c(0,1,1,0))
(bezPolys <- bezier(points))
f <- as.function(bezPolys)
df <- as.data.frame(f(s))

ggplot(aes(x = x, y = y), data = df) +
  geom_point(data = points, color = "red", size = 8) +
  geom_path(data = points, color = "red") +
  geom_path()
```

```
## example 4
points <- data.frame(x = c(-1,2,-2,1), y = c(0,1,1,0))
(bezPolys <- bezier(points))
f <- as.function(bezPolys)
df <- as.data.frame(f(s))

ggplot(aes(x = x, y = y), data = df) +
  geom_point(data = points, color = "red", size = 8) +
  geom_path(data = points, color = "red") +
  geom_path()



## example 5
qplot(speed, dist, data = cars)

s <- seq(0, 1, length.out = 201)
p <- bezier(cars)
f <- as.function(p)
df <- as.data.frame(f(s))
qplot(speed, dist, data = cars) +
  geom_path(data = df, color = "red")

# the curve is not invariant to permutations of the points
# but it always goes through the first and last points
permute_rows <- function(df) df[sample(nrow(df)),]
p <- bezier(permute_rows(cars))
f <- as.function(p)
df <- as.data.frame(f(s))
qplot(speed, dist, data = cars) +
  geom_path(data = df, color = "red")
```

`bezierFunction` *Bezier function*

Description

Compute the Bezier function of a collection of polynomials. By Bezier function we mean the Bezier curve function, a parametric map running from t = 0, the first point, to t = 1, the last point, where the coordinate mappings are linear combinations of Bernstein polynomials.

Usage

```
bezierFunction(points, weights = rep(1L, nrow(points)))
```

Arguments

points	a matrix or data frame of numerics. the rows represent points.
weights	the weights in a weighted Bezier curve

Details

The function returned is vectorized and evaluates the Bezier curve in a numerically stable way with de Castlejau's algorithm (implemented in R).

Value

function of a single parameter

Author(s)

David Kahle

References

http://en.wikipedia.org/wiki/Bezier_curve, http://en.wikipedia.org/wiki/De_Casteljau's_algorithm

See Also

bezier

Examples

```
library(ggplot2); theme_set(theme_bw())


t <- seq(0, 1, length.out = 201)
points <- data.frame(x = 0:3, y = c(0,1,-1,0))


f <- bezierFunction(points)
df <- as.data.frame(f(t))

ggplot(aes(x = x, y = y), data = df) +
  geom_point(data = points, color = "red", size = 8) +
  geom_path(data = points, color = "red") +
  geom_path()
```

```
f <- bezierFunction(points, weights = c(1,5,5,1))
df <- as.data.frame(f(t))

ggplot(aes(x = x, y = y), data = df) +
  geom_point(data = points, color = "red", size = 8) +
  geom_path(data = points, color = "red") +
  geom_path()
```



```
f <- bezierFunction(points, weights = c(1,10,10,1))
df <- as.data.frame(f(t))

ggplot(aes(x = x, y = y), data = df) +
  geom_point(data = points, color = "red", size = 8) +
  geom_path(data = points, color = "red") +
  geom_path()
```

`burst` *Enumerate integer r-vectors summing to n*

Description

Determine all r-vectors with nonnegative integer entries summing to n. Note that this is intended to be optimized.

Usage

```
burst(n, r = n)
```

Arguments

`n`	integer to sum to
`r`	number of components

Value

a matrix whose rows are the n-tuples

Examples

```
burst(4)

burst(4, 4)
burst(4, 3)
burst(4, 2)

rowSums(burst(4))
rowSums(burst(4, 3))
rowSums(burst(4, 2))


burst(10, 4) # all possible 2x2 contingency tables with n=10
burst(10, 4) / 10 # all possible empirical relative frequencies
```

---

chebyshev *Chebyshev polynomials*

Description

Chebyshev polynomials as computed by orthopolynom.

Usage

```
chebyshev(degree, kind = "t", indeterminate = "x", normalized = FALSE)
```

Arguments

degree	degree of polynomial
kind	"t" or "u" (Chebyshev polynomials of the first and second kinds), or "c" or "s"
indeterminate	indeterminate
normalized	provide normalized coefficients

Value

a mpoly object or mpolyList object

Author(s)

David Kahle calling code from the orthopolynom package

See Also

chebyshev.t.polynomials, chebyshev.u.polynomials, chebyshev.c.polynomials, chebyshev.s.polynomials, http://en.wikipedia.org/wiki/Chebyshev_polynomials

Examples

```
chebyshev(0)
chebyshev(1)
chebyshev(2)
chebyshev(3)
chebyshev(4)
chebyshev(5)
chebyshev(6)
chebyshev(10)

chebyshev(0:5)
chebyshev(0:5, normalized = TRUE)
chebyshev(0:5, kind = "u")
chebyshev(0:5, kind = "c")
chebyshev(0:5, kind = "s")
chebyshev(0:5, indeterminate = "t")



# visualize the chebyshev polynomials

library(ggplot2); theme_set(theme_classic())
library(tidyr)

s <- seq(-1, 1, length.out = 201)
N <- 5 # number of chebyshev polynomials to plot
(chebPolys <- chebyshev(0:N))

# see ?bernstein for a better understanding of
# how the code below works

df <- data.frame(s, as.function(chebPolys)(s))
names(df) <- c("x", paste0("T_", 0:N))
mdf <- gather(df, degree, value, -x)
qplot(x, value, data = mdf, geom = "line", color = degree)
```

`components` *Polynomial components*

Description

Compute quantities/expressions related to a multivariate polynomial.

Usage

```
## S3 method for class 'mpoly'
x[ndx]

LT(x, varorder = vars(x), order = "lex")

LC(x, varorder = vars(x), order = "lex")

LM(x, varorder = vars(x), order = "lex")

multideg(x, varorder = vars(x), order = "lex")

totaldeg(x)

monomials(x)

exponents(x, reduced = FALSE)
```

Arguments

x	an object of class mpoly
ndx	a subsetting index
varorder	the order of the variables
order	a total order used to order the terms
reduced	if TRUE, don't include zero degrees
...	additional arguments

Value

An object of class mpoly or mpolyList, depending on the context

Examples

```
(p <- mp("x y^2 + x (x+1) (x+2) x z + 3 x^10"))
p[2]
p[-2]
p[2:3]

LT(p)
LC(p)
LM(p)

multideg(p)
totaldeg(p)
monomials(p)

exponents(p)
exponents(p, reduce = TRUE)
```

```
lapply(exponents(p), is.integer)

homogeneous_components(p)
```

---

deriv.mpoly *Compute partial derivatives of a multivariate polynomial.*

Description

This is a deriv method for mpoly objects. It does not call the deriv function (from package stats).

Usage

```
## S3 method for class 'mpoly'
deriv(expr, var, ...)
```

Arguments

expr	an object of class mpoly
var	character - the partial derivative desired
...	any additional arguments

Value

An object of class mpoly or mpolyList.

Examples

```
m <- mp('x y + y z + z^2')
deriv(m, 'x')
deriv(m, 'y')
deriv(m, 'z')
deriv(m, c('x','y','z'))
deriv(m, 'a')
is.mpoly(deriv(m, 'x'))
is.mpolyList( deriv(m, c('x','y','z')) )
```

gradient *Compute gradient of a multivariate polynomial.*

Description

This is a wrapper for deriv.mpoly.

Usage

```
gradient(mpoly)
```

Arguments

mpoly	an object of class mpoly

Value

An object of class mpoly or mpolyList.

See Also

deriv.mpoly

Examples

```
m <- mp('x y + y z + z^2')
gradient(m)


# gradient descent illustration using the symbolically
# computed gradient of the rosenbrock function
rosenbrock <- mp("(1 - x)^2 + 100 (y - x^2)^2")
fn <- as.function(rosenbrock)
(rosenbrock_gradient <- gradient(rosenbrock))
gr <- as.function(rosenbrock_gradient)

# visualize the function
library(ggplot2)
s <- seq(-5, 5, .05)
df <- expand.grid(x = s, y = s)
df$z <- apply(df, 1, fn)
ggplot(df, aes(x = x, y = y)) +
  geom_raster(aes(fill = z)) +
  scale_fill_continuous(trans = "log10")

# run the gradient descent algorithm using line-search
# step sizes computed with optimize()
current <- steps <- c(-3,-4)
change <- 1
tol <- 1e-5
```

```
while(change > tol){
  last  <- current
  delta <- optimize(
    function(delta) fn(current - delta*gr(current)),
    interval = c(1e-10, .1)
  )$minimum
  current <- current - delta*gr(current)
  steps   <- unname(rbind(steps, current))
  change  <- abs(fn(current) - fn(last))
}
steps <- as.data.frame(steps)
names(steps) <- c("x", "y")

# visualize steps, note the optim at c(1,1)
# the routine took 5748 steps
ggplot(df, aes(x = x, y = y)) +
  geom_raster(aes(fill = z)) +
  geom_path(data = steps, color = "red") +
  geom_point(data = steps, color = "red", size = .5) +
  scale_fill_continuous(trans = "log10")

# it gets to the general region of space quickly
# but once it gets on the right arc, it's terrible
# here's what the end game looks like
last_steps <- tail(steps, 100)
rngx <- range(last_steps$x); sx <- seq(rngx[1], rngx[2], length.out = 201)
rngy <- range(last_steps$y); sy <- seq(rngy[1], rngy[2], length.out = 201)
df <- expand.grid(x = sx, y = sy)
df$z <- apply(df, 1, fn)
ggplot(df, aes(x = x, y = y)) +
  geom_raster(aes(fill = z)) +
  geom_path(data = last_steps, color = "red", size = .25) +
  geom_point(data = last_steps, color = "red", size = 1) +
  scale_fill_continuous(trans = "log10")
```

`grobner` *REMOVED – Compute a grobner basis of a list of multivariate polynomials.*

Description

This function has been removed to eliminate mpoly's dependence on packages that only it uses. To compute a Grobner basis of a collection of multivariate polynomials, checkout the new m2r package, which you can download with the code in the first example.

Usage

```
grobner(mpolyList, varorder = vars(mpolyList), order = "lex")
```

Arguments

mpolyList	an object of class mpolyList
varorder	order of variables
order	total order to be considered for monomials (e.g. lexicographic)

Details

grobner computes a Grobner basis for a collection of multivarite polynomials represented as an object of class mpolyList. Note that the polynomials printed after calculation are unlikely to be properly ordered; this is because the order of the monomials displayed is governed by the print method, not the mpoly's themselves.

Value

An object of class mpolyList.

Examples

```
## Not run:


# code to download m2r:
# note that to do this you should have Macaulay2 installed,
# see https://github.com/musicman3320/m2r and
# http://www.math.uiuc.edu/Macaulay2/Downloads/
if(!require(devtools)) install.packages("devtools")
devtools::install_github("musicman3320/m2r")







## End(Not run)
```

---

hermite *Hermite polynomials*

Description

Hermite polynomials as computed by orthopolynom.

Usage

```
hermite(degree, kind = "he", indeterminate = "x", normalized = FALSE)
```

Arguments

degree	degree of polynomial
kind	"he" (default, probabilists', see Wikipedia article) or "h" (physicists)
indeterminate	indeterminate
normalized	provide normalized coefficients

Value

a mpoly object or mpolyList object

Author(s)

David Kahle calling code from the orthopolynom package

See Also

hermite.h.polynomials, hermite.he.polynomials, http://en.wikipedia.org/wiki/Hermite_polynomials

Examples

```
hermite(0)
hermite(1)
hermite(2)
hermite(3)
hermite(4)
hermite(5)
hermite(6)
hermite(10)

hermite(0:5)
hermite(0:5, normalized = TRUE)
hermite(0:5, indeterminate = "t")



# visualize the hermite polynomials

library(ggplot2); theme_set(theme_classic())
library(tidyr)

s <- seq(-3, 3, length.out = 201)
N <- 5 # number of hermite polynomials to plot
(hermPolys <- hermite(0:N))

# see ?bernstein for a better understanding of
# how the code below works

df <- data.frame(s, as.function(hermPolys)(s))
```

```
names(df) <- c("x", paste0("T_", 0:N))
mdf <- gather(df, degree, value, -x)
qplot(x, value, data = mdf, geom = "line", color = degree)
```

---

homogenize *Homogenize a polynomial*

Description

Homogenize a polynomial.

Usage

```
homogenize(x, var = "t")

dehomogenize(x, var = "t")

is.homogeneous(x)

homogeneous_components(x)
```

Arguments

x	an mpoly object
var	name of homogenization

Value

a (de/homogenized) mpoly or an mpolyList

Examples

```
x <- mp("x^4 + y + 2 x y^2 - 3 z")
is.homogeneous(x)
(xh <- homogenize(x))
is.homogeneous(xh)

homogeneous_components(x)

homogenize(x, "o")

xh <- homogenize(x)
dehomogenize(xh) # assumes var = "t"
plug(xh, "t", 1) # same effect, but dehomogenize is faster
```

```
# the functions are vectorized
(ps <- mp(c("x + y^2", "x + y^3")))
(psh <- homogenize(ps))
dehomogenize(psh)


# demonstrating a leading property of homogeneous polynomials
library(magrittr)
p  <- mp("x^2 + 2 x + 3")
(ph <- homogenize(p, "y"))
lambda <- 3
(d <- totaldeg(p))
ph %>%
  plug("x", lambda*mp("x")) %>%
  plug("y", lambda*mp("y"))
lambda^d * ph
```

`insert` *Insert an element into a vector.*

Description

Insert an element into a vector.

Usage

```
insert(elem, slot, v)
```

Arguments

`elem`	element to insert
`slot`	location of insert
`v`	vector to insert into

Value

vector with element inserted

Examples

```
insert(2, 1, 1)
insert(2, 2, 1)
insert('x', 5, letters)
```

is.wholenumber *Test whether an object is a whole number*

Description

Test whether an object is a whole number.

Usage

```
is.wholenumber(x, tol = .Machine$double.eps^0.5)
```

Arguments

x	object to be tested
tol	tolerance within which a number is said to be whole

Value

Vector of logicals.

Examples

```
is.wholenumber(seq(-3,3,.5))
```

jacobi *Jacobi polynomials*

Description

Jacobi polynomials as computed by orthopolynom.

Usage

```
jacobi(degree, alpha = 1, beta = 1, kind = "p", indeterminate = "x",
  normalized = FALSE)
```

Arguments

degree	degree of polynomial
alpha	the first parameter, also called p
beta	the second parameter, also called q
kind	"g" or "p"
indeterminate	indeterminate
normalized	provide normalized coefficients

Value

a mpoly object or mpolyList object

Author(s)

David Kahle calling code from the orthopolynom package

See Also

jacobi.g.polynomials, jacobi.p.polynomials http://en.wikipedia.org/wiki/Jacobi_polynomials

Examples

```
jacobi(0)
jacobi(1)
jacobi(2)
jacobi(3)
jacobi(4)
jacobi(5)
jacobi(6)
jacobi(10, 2, 2, normalized = TRUE)

jacobi(0:5)
jacobi(0:5, normalized = TRUE)
jacobi(0:5, kind = "g")
jacobi(0:5, indeterminate = "t")



# visualize the jacobi polynomials

library(ggplot2); theme_set(theme_classic())
library(tidyr)

s <- seq(-1, 1, length.out = 201)
N <- 5 # number of jacobi polynomials to plot
(jacPolys <- jacobi(0:N, 2, 2))

df <- data.frame(s, as.function(jacPolys)(s))
names(df) <- c("x", paste0("P_", 0:N))
mdf <- gather(df, degree, value, -x)
qplot(x, value, data = mdf, geom = "line", color = degree)

qplot(x, value, data = mdf, geom = "line", color = degree) +
  coord_cartesian(ylim = c(-30, 30))
```

`laguerre` *Generalized Laguerre polynomials*

Description

Generalized Laguerre polynomials as computed by orthopolynom.

Usage

```
laguerre(degree, alpha = 0, indeterminate = "x", normalized = FALSE)
```

Arguments

`degree`	degree of polynomial
`alpha`	generalization constant
`indeterminate`	indeterminate
`normalized`	provide normalized coefficients

Value

a mpoly object or mpolyList object

Author(s)

David Kahle calling code from the orthopolynom package

See Also

`glaguerre.polynomials`, http://en.wikipedia.org/wiki/Laguerre_polynomials

Examples

```
laguerre(0)
laguerre(1)
laguerre(2)
laguerre(3)
laguerre(4)
laguerre(5)
laguerre(6)

laguerre(2)
laguerre(2, normalized = TRUE)

laguerre(0:5)
laguerre(0:5, normalized = TRUE)
laguerre(0:5, indeterminate = "t")
```

```
# visualize the laguerre polynomials

library(ggplot2); theme_set(theme_classic())
library(tidyr)

s <- seq(-5, 20, length.out = 201)
N <- 5 # number of laguerre polynomials to plot
(lagPolys <- laguerre(0:N))

# see ?bernstein for a better understanding of
# how the code below works

df <- data.frame(s, as.function(lagPolys)(s))
names(df) <- c("x", paste0("L_", 0:N))
mdf <- gather(df, degree, value, -x)
qplot(x, value, data = mdf, geom = "line", color = degree)

qplot(x, value, data = mdf, geom = "line", color = degree) +
  coord_cartesian(ylim = c(-25, 25))
```

LCM *Compute the least common multiple of two numbers.*

Description

A simple algorithm to compute the least common multiple of two numbers

Usage

```
LCM(x, y)
```

Arguments

x	an object of class numeric
y	an object of class numeric

Value

The least common multiple of x and y.

Examples

```
LCM(5,7)
LCM(5,8)
LCM(5,9)
LCM(5,10)
Reduce(LCM, 1:10) # -> 2520
```

`legendre` *Legendre polynomials*

Description

Legendre polynomials as computed by orthopolynom.

Usage

```
legendre(degree, indeterminate = "x", normalized = FALSE)
```

Arguments

`degree`	degree of polynomial
`indeterminate`	indeterminate
`normalized`	provide normalized coefficients

Value

a mpoly object or mpolyList object

Author(s)

David Kahle calling code from the orthopolynom package

See Also

`legendre.polynomials`, http://en.wikipedia.org/wiki/Legendre_polynomials

Examples

```
legendre(0)
legendre(1)
legendre(2)
legendre(3)
legendre(4)
legendre(5)
legendre(6)

legendre(2)
legendre(2, normalized = TRUE)

legendre(0:5)
legendre(0:5, normalized = TRUE)
legendre(0:5, indeterminate = "t")
```

```
# visualize the legendre polynomials

library(ggplot2); theme_set(theme_classic())
library(tidyr)

s <- seq(-1, 1, length.out = 201)
N <- 5 # number of legendre polynomials to plot
(legPolys <- legendre(0:N))

# see ?bernstein for a better understanding of
# how the code below works

df <- data.frame(s, as.function(legPolys)(s))
names(df) <- c("x", paste0("P_", 0:N))
mdf <- gather(df, degree, value, -x)
qplot(x, value, data = mdf, geom = "line", color = degree)
```

---

mp *Define a multivariate polynomial.*

Description

mp is a smart function which attempts to create a formal mpoly object from a character string containing the usual representation of a multivariate polynomial.

Usage

```
mp(string, varorder)
```

Arguments

string	a character string containing a polynomial, see examples
varorder	(optional) order of variables in string

Value

An object of class mpoly.

Author(s)

David Kahle <david.kahle@gmail.com>

See Also

mpoly

Examples

```
( m <- mp("x + y + x y") )
is.mpoly( m )
unclass(m)


mp("x + 2 y + x^2 y + x y z")
mp("x + 2 y + x^2 y + x y z", varorder = c("y", "z", "x"))
# mp("x + 2 y + x^2 y", varorder = c("q", "p")) # -> error

( ms <- mp(c("x + y", "2 x")) )
is.mpolyList(ms)


gradient( mp("x + 2 y + x^2 y + x y z") )
gradient( mp("(x + y)^10") )

# mp and the print methods are kinds of inverses of each other
( polys <- mp(c("x + y", "x - y")) )
strings <- print(polys, silent = TRUE)
strings
mp(strings)
```

`mpoly` *Multivariate polynomials in R.*

Description

A package for symbolic computation and more with multivariate polynomials

mpoly is the most basic function used to create objects of class mpoly. However, it is not a general purpose function; for that see mp.

Usage

```
mpoly(list, varorder)
```

Arguments

`list`	a list from which to construct an mpoly object
`varorder`	(optional) a character vector setting the intrinsic variable order of the polynomial

Value

Object of class mpoly.

Author(s)

David Kahle <david.kahle@gmail.com>

See Also

mp

Examples

```
list <- list(
  c(x = 1, coef = 1, y = 0),
  c(x = 0, y = 1, coef = 2),
  c(y = 1, coef = -6),
  c(z = 1, coef = -3, x = 2),
  c(x = 1, coef = 0, x = 3),
  c(t = 1, coef = 4, t = 2, y = 4),
  c(x = 1),
  c(x = 1),
  c(coef = 5),
  c(coef = 5),
  c(coef = -5)
)

mpoly(list) # 3 x  -  4 y  -  3 x^2 z  +  4 y^4 t^3  +  5
mpoly(list, varorder = c("y", "z", "t", "x"))

list <- list(  c(x = 5, x = 2, coef = 5, coef = 6, y = 0) )
mpoly(list)
```

---

mpolyArithmetic *Arithmetic with multivariate polynomials*

Description

Arithmetic with multivariate polynomials

Usage

```
## S3 method for class 'mpoly'
e1 + e2

## S3 method for class 'mpoly'
e1 - e2
```

```
## S3 method for class 'mpoly'
e1 * e2

## S3 method for class 'mpoly'
e1 ^ e2
```

Arguments

e1	an object of class mpoly
e2	an object of class mpoly

Value

object of class mpoly

Examples

```
p <- mp("x + y")
p + p
p - p
p * p
p^2
p^10


mp("(x+1)^10")
p + 1
2*p
```

---

mpolyEqual *Determine whether two multivariate polynomials are equal.*

Description

Determine whether two multivariate polynomials are equal.

Usage

```
## S3 method for class 'mpoly'
e1 == e2
```

Arguments

e1	an object of class mpoly
e2	an object of class mpoly

Value

A logical value.

Examples

```
p1 <- mp("x + y + 2 z")
p1 == p1

p2 <- reorder(p1, order = "lex", varorder = c("z","y","x"))
p1 == p2
p2 <- reorder(p1, order = "lex", varorder = c("z","w","y","x"))
p1 == p2
p1 == ( 2 * p2 )

p1 <- mp("x + 1")
p2 <- mp("x + 1")
identical(p1, p2)
p1 == p2

mp("x + 1") == mp("y + 1")
mp("2") == mp("1")
mp("1") == mp("1")
mp("0") == mp("-0")
```

`mpolyList` *Define a collection of multivariate polynomials.*

Description

Combine a series of mpoly objects into a mpolyList.

Usage

```
mpolyList(...)
```

Arguments

`...`	a series of mpoly objects.

Value

An object of class mpolyList.

Examples

```
( p1 <- mp("t^4 - x") )
( p2 <- mp("t^3 - y") )
( p3 <- mp("t^2 - z") )
( ms <- mpolyList(p1, p2, p3) )
is.mpolyList( ms )

mpolyList(mp("x + 1"))
p <- mp("x + 1")
mpolyList(p)

ps <- mp(c("x + 1", "y + 2"))
is.mpolyList(ps)


f <- function(){
  a <- mp("1")
  mpolyList(a)
}
f()
```

`mpolyListArithmetic` *Element-wise arithmetic with vectors of multivariate polynomials.*

Description

Element-wise arithmetic with vectors of multivariate polynomials.

Usage

```
## S3 method for class 'mpolyList'
e1 + e2

## S3 method for class 'mpolyList'
e1 - e2

## S3 method for class 'mpolyList'
e1 * e2
```

Arguments

`e1`	an object of class mpolyList
`e2`	an object of class mpolyList

Value

An object of class mpolyList.

Examples

```
( ms1 <- mp( c('x + 1', 'x + 2') ) )
( ms2 <- mp( c('x + 1', 'y + 2') ) )
ms1 + ms2
ms1 - ms2
ms1 * ms2
```

`partitions` *Enumerate the partitions of an integer*

Description

Determine all unrestricted partitions of an integer. This function is equivalent to the function parts in the partitions package.

Usage

```
partitions(n)
```

Arguments

`n`	an integer

Value

a matrix whose rows are the n-tuples

Author(s)

Robin K. S. Hankin, from package partitions

Examples

```
partitions(5)
str(partitions(5))
```

permutations *Determine all permutations of a set.*

Description

An implementation of the Steinhaus-Johnson-Trotter permutation algorithm.

Usage

```
permutations(set)
```

Arguments

set	a set

Value

a matrix whose rows are the permutations of set

Examples

```
permutations(1:3)
permutations(c('first','second','third'))
permutations(c(1,1,3))
apply(permutations(letters[1:6]), 1, paste, collapse = '')
```

plug *Switch indeterminates in a polynomial*

Description

Switch indeterminates in a polynomial

Usage

```
plug(p, indeterminate, value)
```

Arguments

p	a polynomial
indeterminate	the indeterminate in the polynomial to switch
value	the value/indeterminate to substitute

Value

an mpoly object

Examples

```
(p <- mp("(x+y)^3"))
plug(p, "x", 5)
plug(p, "x", "t")
plug(p, "x", "y")
plug(p, "x", mp("2y"))

plug(p, "x", mp("x + y"))
mp("((x+y)+y)^3")
```

---

predicates *mpoly predicate functions*

Description

Various functions to deal with mpoly and mpolyList objects.

Usage

```
is.constant(x)

is.mpoly(x)

is.bernstein(x)

is.bezier(x)

is.chebyshev(x)

is.mpolyList(x)

is.linear(x)
```

Arguments

x	object to be tested

Value

Vector of logicals.

Examples

```
p <- mp("5")
is.mpoly(p)
is.constant(p)

is.constant(mp(c("x + 1", "7", "y - 2")))

p <- mp("x + y")
is.mpoly(p)

is.mpolyList(mp(c("x + 1", "y - 1")))



is.linear(mp("0"))
is.linear(mp("x + 1"))
is.linear(mp("x + y"))
is.linear(mp(c("0", "x + y")))

is.linear(mp("x + x y"))
is.linear(mp(c("x + x y", "x")))


(p <- bernstein(2, 5))
is.mpoly(p)
is.bernstein(p)

(p <- chebyshev(5))
is.mpoly(p)
is.chebyshev(p)
str(p)
```

`print.mpoly` *Pretty printing of multivariate polynomials.*

Description

This is the major function used to view multivariate polynomials.

Usage

```
## S3 method for class 'mpoly'
print(x, varorder, order, stars = FALSE,
  silent = FALSE, ...)
```

Arguments

x	an object of class mpoly
varorder	the order of the variables
order	a total order used to order the monomials in the printing
stars	print the multivariate polynomial in the more computer-friendly asterisk notation (default FALSE)
silent	logical; if TRUE, suppresses output
...	additional parameters to go to cat

Value

Invisible string of the printed object.

Examples

```
p <- mp("2 x^5  -  3 y^2  +  x y^3")
p
print(p) # same
print(p, silent = TRUE)
s <- print(p, silent = TRUE)
s

print(p, order = "lex") # -> 2 x^5  +  x y^3  -  3 y^2
print(p, order = "lex", varorder = c("y","x")) # -> y^3 x  -  3 y^2  +  2 x^5
print(p, varorder = c("y","x")) # -> 2 x^5  -  3 y^2  +  y^3 x

print(p, stars = TRUE)
```

---

print.mpolyList *Pretty printing of a list of multivariate polynomials.*

Description

This function iterates print.mpoly on an object of class mpolyList.

Usage

```
## S3 method for class 'mpolyList'
print(x, varorder = vars(x), order,
  silent = FALSE, ...)
```

Arguments

x	an object of class mpolyList
varorder	the order of the variables
order	a total order used to order the monomials in the printing
silent	logical; if TRUE, suppresses output
...	arguments to pass to print.mpoly

Value

Invisible character vector of the printed objects.

Examples

```
mL <- mp(c('x + 1', 'y - 1', 'x y^2 z  +  x^2 z^2  +  z^2  +  x^3'))
mL

ps <- print(mL, silent = TRUE)
ps

print(mL, order = 'lex')
print(mL, order = 'glex')
print(mL, order = 'grlex')
print(mL, order = 'glex', varorder = c('z','y','x'))
print(mL, order = 'grlex', varorder = c('z','y','x'))
print(mL, varorder = c('z','y','x'))
s <- print(mL, varorder = c('z','y','x'))
str(s)
```

---

reorder.mpoly *Reorder a multivariate polynomial.*

Description

This function is used to set the intrinsic order of a multivariate polynomial. It is used for both the in-term variables and the terms.

Usage

```
## S3 method for class 'mpoly'
reorder(x, varorder = vars(x), order, ...)
```

Arguments

x	an object of class mpoly
varorder	the order of the variables
order	a total order used to order the terms
...	additional arguments

Value

An object of class mpoly.

Examples

```
list <- list(
  c(x = 1, y = 2, z = 1, coef = 1),
  c(x = 2, y = 0, z = 2, coef = 1),
  c(x = 0, y = 0, z = 2, coef = 1),
  c(x = 3, y = 0, z = 0, coef = 1)
)
(p <- mpoly(list)) # -> x y^2 z  +  x^2 z^2  +  z^2  +  x^3
reorder(p) # -> x y^2 z  +  x^2 z^2  +  z^2  +  x^3
reorder(p, varorder = c("x","y","z"), order = "lex")
    # -> x^3  +  x^2 z^2  +  x y^2 z  +  z^2
reorder(p, varorder = c("x","y","z"), order = "glex")
    # -> x^2 z^2  +  x y^2 z  +  x^3  +  z^2
reorder(p, varorder = c("x","y","z"), order = "grlex")
    # -> x y^2 z  +  x^2 z^2  +  x^3  +  z^2

reorder(mp("x + 1"), varorder = c("y","x","z"), order = "lex")
reorder(mp("x + y"), varorder = c("y","x","z"), order = "lex")
reorder(mp("x y + y + 2 x y z^2"), varorder = c("y","x","z"))
reorder(mp("x^2 + y x + y"), order = "lex")
```

---

round.mpoly *Round the coefficients of a polynomial*

Description

Round the coefficients of an mpoly object.

Usage

```
## S3 method for class 'mpoly'
round(x, digits = 3)
```

Arguments

x	an mpoly object
digits	number of digits to round to

Value

the rounded mpoly object

Author(s)

David Kahle <david.kahle@gmail.com>

See Also

mp

Examples

```
p <- mp("x + 3.14159265")^4
p
round(p)
round(p, 0)

## Not run:
library(plyr)
library(ggplot2)
library(stringr)

n <- 101
s <- seq(-5, 5, length.out = n)

# one dimensional case
df <- data.frame(x = s)
df <- mutate(df, y = -x^2 + 2*x - 3 + rnorm(n, 0, 2))
qplot(x, y, data = df)
mod <- lm(y ~ x + I(x^2), data = df)
p <- as.mpoly(mod)
qplot(x, y, data = df) +
  stat_function(fun = as.function(p), colour = 'red')

p
round(p, 1)
qplot(x, y, data = df) +
  stat_function(fun = as.function(p), colour = 'red') +
  stat_function(fun = as.function(round(p,1)), colour = 'blue')



## End(Not run)
```

swap *Swap polynomial indeterminates*

Description

Swap polynomial indeterminates

Usage

```
swap(p, variable, replacement)
```

Arguments

p	polynomial
variable	the variable in the polynomial to replace
replacement	the replacement variable

Value

a mpoly object

Author(s)

David Kahle

Examples

```
(p <- mp("(x + y)^2"))
swap(p, "x", "t")

## the meta data is retained
(p <- bernstein(3, 5))
(p2 <- swap(p, "x", "t"))
is.bernstein(p2)

(p <- chebyshev(3))
(p2 <- swap(p, "x", "t"))
is.chebyshev(p2)
```

terms.mpoly *Extract the terms of a multivariate polynomial.*

Description

Compute the terms of an mpoly object as a mpolyList.

Usage

```
## S3 method for class 'mpoly'
terms(x, ...)
```

Arguments

x	an object of class mpoly
...	additional parameters

Value

An object of class mpolyList.

Examples

```
## Not run:  .Deprecated issues a warning

x <- mp("x^2 - y + x y z")
terms(x)
monomials(x)


## End(Not run)
```

tuples *Determine all n-tuples using the elements of a set.*

Description

Determine all n-tuples using the elements of a set. This is really just a simple wrapper for expand.grid, so it is not optimized.

Usage

```
tuples(set, n = length(set), repeats = FALSE, list = FALSE)
```

Arguments

set	a set
n	length of each tuple
repeats	if set contains duplicates, should the result?
list	tuples as list?

Value

a matrix whose rows are the n-tuples

Examples

```
tuples(1:2, 5)
tuples(1:2, 5, list = TRUE)

apply(tuples(c("x","y","z"), 3), 1, paste, collapse = "")

# multinomial coefficients
r <- 2 # number of variables, e.g. x, y
n <- 2 # power, e.g. (x+y)^2
apply(burst(n,r), 1, function(v) factorial(n)/ prod(factorial(v))) # x, y, xy
mp("x + y")^n

r <- 2 # number of variables, e.g. x, y
n <- 3 # power, e.g. (x+y)^3
apply(burst(n,r), 1, function(v) factorial(n)/ prod(factorial(v))) # x, y, xy
mp("x + y")^n

r <- 3 # number of variables, e.g. x, y, z
n <- 2 # power, e.g. (x+y+z)^2
apply(burst(n,r), 1, function(v) factorial(n)/ prod(factorial(v))) # x, y, z, xy, xz, yz
mp("x + y + z")^n
```

---

vars *Determine the variables in a mpoly object.*

Description

Determine the variables in a mpoly object.

Usage

```
vars(mpoly)
```

Arguments

mpoly	an object of class mpoly

Value

A character vector of the variable names.

Examples

```
list <- list(
  c(x = 5, coef = 2),
  c(y = 2, coef = -3),
  c(x = 1, y = 3, coef = 1)
)
p <- mpoly(list)
vars(p)
```

Index