

# Package ‘EfficientMaxEigenpair’

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**Type** Package

**Title** Efficient Initials for Computing the Maximal Eigenpair

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**Description** An implementation for using efficient initials to compute the maximal eigenpair in R. It provides two algorithms to find the efficient initials under two cases: the tridiagonal matrix case and the general matrix case. Besides, it also provides algorithms for the next to the maximal eigenpair under these two cases.

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**URL** <http://github.com/mxjki/EfficientMaxEigenpair>

**BugReports** <http://github.com/mxjki/EfficientMaxEigenpair/issues>

**Depends** R (>= 3.3.2), stats

**Encoding** UTF-8

**RoxygenNote** 5.0.1

**Suggests** knitr, rmarkdown

**VignetteBuilder** knitr

**NeedsCompilation** no

**Repository** CRAN

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eff.ini.maxeig.general

*General matrix maximal eigenpair*

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## Description

Calculate the maximal eigenpair for the general matrix.

## Usage

```
eff.ini.maxeig.general(A, v0_tilde = NULL, z0 = NULL, z0numeric,
    improved = F, xi = 1, digit.thresh = 6)
```

## Arguments

A	The input general matrix.
v0_tilde	The unnormalized initial vector $\tilde{v}_0$ .
z0	The type of initial $z_0$ used to calculate the approximation of $\rho(Q)$ . There are three types: 'fixed', 'Auto' and 'numeric' corresponding to three choices of $z_0$ in paper.
z0numeric	The numerical value assigned to initial $z_0$ as an approximation of $\rho(Q)$ when $z_0='numeric'$ .
improved	With improved=T, the improved algorithm to calculate initial approximating eigenpairs is used.
xi	The coefficient used to form the convex combination of $\delta_1^{-1}$ and $(v_0, -Q*v_0)_\mu$ , it should between 0 and 1.
digit.thresh	The precise level of output results.

## Value

A list of eigenpair object are returned, with components  $z$  and  $v$ .

z	The approximating sequence of the maximal eigenvalue.
v	The approximating sequence of the corresponding eigenvector.

## See Also

[eff.ini.maxeig.tri](#) for the tridiagonal matrix maximal eigenpair.

**Examples**

```

A = matrix(c(1, 1, 3, 2, 2, 2, 3, 1, 1), 3, 3)
eff.ini.maxeig.general(A, v0_tilde = rep(1, dim(A)[1]), z0 = 'fixed')

A = matrix(c(1, 1, 3, 2, 2, 2, 3, 1, 1), 3, 3)
eff.ini.maxeig.general(A, v0_tilde = rep(1, dim(A)[1]), z0 = 'Auto')

##Symmetrizing A converge to second largest eigenvalue
A = matrix(c(1, 3, 9, 5, 2, 14, 10, 6, 0, 11, 7, 0, 0, 1, 8), 4, 4)
S = (t(A) + A)/2
N = dim(S)[1]
a = diag(S[-1, -N])
b = diag(S[-N, -1])
c = rep(NA, N)
c[1] = -diag(S)[1] - b[1]
c[2:(N - 1)] = -diag(S)[2:(N - 1)] - b[2:(N - 1)] - a[1:(N - 2)]
c[N] = -diag(S)[N] - a[N - 1]

z0ini = eff.ini.maxeig.tri(a, b, c, xi = 7/8, improved = TRUE)$z[1]
eff.ini.maxeig.general(A, v0_tilde = rep(1, dim(A)[1]), z0 = 'numeric',
z0numeric = 28 - z0ini)

```

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eff.ini.maxeig.tri      *Tridiagonal matrix maximal eigenpair*

---

**Description**

Calculate the maximal eigenpair for the tridiagonal matrix.

**Usage**

```
eff.ini.maxeig.tri(a, b, c, xi = 1, improved = F, digit.thresh = 6)
```

**Arguments**

a	The lower diagonal vector.
b	The upper diagonal vector.
c	The shifted main diagonal vector. The corresponding unshift diagonal vector is $-c(b[1] + c[1], a[1:N - 1] + b[2:N] + c[2:N], a[N] + c[N + 1])$ where $N+1$ is the dimension of matrix.
xi	The coefficient used to form the convex combination of $\delta_1^{-1}$ and $(v_0, -Q * v_0)_\mu$ , it should between 0 and 1.
improved	With improved=T, the improved algorithm to calculate initial approximating eigenpairs is used.
digit.thresh	The precise level of output results.

**Value**

A list of eigenpair object are returned, with components  $z$  and  $v$ .

$z$                     The approximating sequence of the maximal eigenvalue.  
 $v$                     The approximating sequence of the corresponding eigenvector.

**See Also**

[eff.ini.maxeig.general](#) for the general matrix maximal eigenpair.

**Examples**

```
a = c(1:7)^2
b = c(1:7)^2
c = rep(0, length(a) + 1)
c[length(a) + 1] = 8^2
eff.ini.maxeig.tri(a, b, c, xi = 1)
```

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eff.ini.seceig.general

*General conservative matrix maximal eigenpair*

---

**Description**

Calculate the next to maximal eigenpair for the general conservative matrix.

**Usage**

```
eff.ini.seceig.general(Q, z0 = NULL, c1 = 1000, digit.thresh = 6)
```

**Arguments**

$Q$                     The input general matrix.  
 $z_0$                   The type of initial  $z_0$  used to calculate the approximation of  $\rho(Q)$ . There are two types: 'fixed' and 'Auto' corresponding to two choices of  $z_0$  in paper.  
 $c_1$                     A large constant.  
digit.thresh        The precise level of output results.

**Value**

A list of eigenpair object are returned, with components  $z$  and  $v$ .

$z$                     The approximating sequence of the maximal eigenvalue.  
 $v$                     The approximating sequence of the corresponding eigenvector.

**Note**

The conservativity of matrix  $Q = (q_{ij})$  means that the sums of each row of matrix  $Q$  are all 0.

**See Also**

[eff.ini.seceig.tri](#) for the tridiagonal matrix next to the maximal eigenpair.

**Examples**

```
Q = matrix(c(-30, 1/5, 11/28, 55/3291, 30, -17, 275/42, 330/1097,
0, 84/5, -20, 588/1097, 0, 0, 1097/84, -2809/3291), 4, 4)
eff.ini.seceig.general(Q, z0 = 'Auto', digit.thresh = 5)
eff.ini.seceig.general(Q, z0 = 'fixed', digit.thresh = 5)
```

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`eff.ini.seceig.tri`      *Tridiagonal matrix next to the maximal eigenpair*

---

**Description**

Calculate the next to maximal eigenpair for the tridiagonal matrix whose sums of each row should be 0.

**Usage**

```
eff.ini.seceig.tri(a, b, xi = 1, digit.thresh = 6)
```

**Arguments**

<code>a</code>	The lower diagonal vector.
<code>b</code>	The upper diagonal vector.
<code>xi</code>	The coefficient used in the improved initials to form the convex combination of $\delta_1^{-1}$ and $(v_0, -Q * v_0)_\mu$ , it should be between 0 and 1.
<code>digit.thresh</code>	The precise level of output results.

**Value**

A list of eigenpair object are returned, with components  $z$  and  $v$ .

<code>z</code>	The approximating sequence of the maximal eigenvalue.
<code>v</code>	The approximating sequence of the corresponding eigenvector.

**Note**

The sums of each row of the input tridiagonal matrix should be 0.

**See Also**

[eff.ini.seceig.general](#) for the general conservative matrix next to the maximal eigenpair.

**Examples**

```

a = c(1:7)^2
b = c(1:7)^2

eff.ini.seceig.tri(a, b, xi = 0)
eff.ini.seceig.tri(a, b, xi = 1)
eff.ini.seceig.tri(a, b, xi = 2/5)

```

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EfficientMaxEigenpair *EfficientMaxEigenpair: A package for computing the maximal eigenpair for a matrix.*

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**Description**

The EfficientMaxEigenpair package provides some auxillary functions and four categories of important functions: `tridiag`, `ray.quot`, `eff.ini.maxeig.tri`, `eff.ini.maxeig.general`, `eff.ini.seceig.tri` and `eff.ini.seceig.general`.

**EfficientMaxEigenpair functions**

`tridiag`: Generate tridiagonal matrix Q based on three input vectors.

`ray.quot`: Rayleigh quotient iteration algorithm to computing the maximal eigenpair of matrix Q.

`eff.ini.maxeig.tri`: Calculate the maximal eigenpair for the tridiagonal matrix.

`eff.ini.maxeig.general`: Calculate the maximal eigenpair for the general matrix.

`eff.ini.seceig.tri`: Calculate the next to maximal eigenpair for the tridiagonal matrix whose sums of each row should be 0.

`eff.ini.seceig.general`: Calculate the next to maximal eigenpair for the general conservative matrix.

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ray.quot	<i>Rayleigh quotient iteration</i>
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**Description**

Rayleigh quotient iteration algorithm to computing the maximal eigenpair of matrix Q.

**Usage**

```
ray.quot(Q, mu, v0_tilde, zstart, digit.thresh = 6)
```

**Arguments**

Q	The input matrix to find the maximal eigenpair.
mu	A vector.
v0_tilde	The unnormalized initial vector $\tilde{v}_0$ .
zstart	The initial $z_0$ as an approximation of $\rho(Q)$ .
digit.thresh	The precise level of output results.

**Value**

A list of eigenpair object are returned, with components 'z' and 'v'.

z	The approximating sequence of the maximal eigenvalue.
v	The approximating sequence of the corresponding eigenvector.

**Examples**

```
Q = matrix(c(1, 1, 3, 2, 2, 2, 3, 1, 1), 3, 3)
ray.quot(Q, mu=rep(1,dim(Q)[1]), v0_tilde=rep(1,dim(Q)[1]), zstart=6,
digit.thresh = 6)
```

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tridiag	<i>Tridiagonal matrix</i>
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**Description**

Generate tridiagonal matrix Q based on three input vectors.

**Usage**

```
tridiag(upper, lower, main)
```

**Arguments**

upper	The upper diagonal vector.
lower	The lower diagonal vector.
main	The main diagonal vector.

**Value**

A tridiagonal matrix is returned.

**Examples**

```
a = c(1:7)^2
b = c(1:7)^2
c = -c(1:8)^2
tridiag(b, a, c)
```

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