

# Package ‘ExtDist’

May 30, 2015

**Version** 0.6-3

**License** GPL (>= 2)

**Title** Extending the Range of Functions for Probability Distributions

**Description** A consistent, unified and extensible framework for estimation of parameters for probability distributions, including parameter estimation procedures that allow for weighted samples; the current set of distributions included are: the standard beta, The four-parameter beta, Burr, gamma, Gumbel, Johnson SB and SU, Laplace, logistic, normal, symmetric truncated normal, truncated normal, symmetric-reflected truncated beta, standard symmetric-reflected truncated beta, triangular, uniform, and Weibull distributions; decision criteria and selections based on these decision criteria.

**Repository** CRAN

**Author** Haizhen Wu <h.wu2@massey.ac.nz>,  
A. Jonathan R. Godfrey <A.J.Godfrey@massey.ac.nz>,  
Kondaswamy Govindaraju <k.govindaraju@massey.ac.nz>,  
Sarah Pirikahu <s.pirikahu@massey.ac.nz>

**Maintainer** A. Jonathan R. Godfrey <a.j.godfrey@massey.ac.nz>

**Imports** numDeriv, optimx

**Suggests** ggplot2, knitr, PerformanceAnalytics, stats, SuppDists,  
truncdist, VGAM, xtable

**VignetteBuilder** knitr

**NeedsCompilation** no

**Date/Publication** 2015-05-30 11:42:04

## R topics documented:

ExtDist-package . . . . .	2
bestDist . . . . .	3
Beta . . . . .	5
Beta_ab . . . . .	7
Burr . . . . .	10
compareDist . . . . .	13
DistSelCriteria . . . . .	13

eDist . . . . .	14
eval.estimation . . . . .	16
Exponential . . . . .	17
Gamma . . . . .	19
Gumbel . . . . .	22
JohnsonSB . . . . .	24
JohnsonSU . . . . .	27
Laplace . . . . .	30
Logistic . . . . .	32
Normal . . . . .	35
Normal_sym_trunc_ab . . . . .	38
Normal_trunc_ab . . . . .	39
SRTB_ab . . . . .	41
SSRTB . . . . .	43
Triangular . . . . .	45
Uniform . . . . .	46
Weibull . . . . .	49
wmle . . . . .	51

<b>Index</b>	<b>53</b>
--------------	-----------

---

ExtDist-package	<i>Extended Probability Distribution Functions</i>
-----------------	--

---

## Description

The package provides a consistent, unified and extensible framework for parameter estimation of probability distributions; it extends parameter estimation procedures to allow for weighted samples; moreover, it extends the gallery of available distributions.

## Details

Package:	ExtDist
Version:	0.6
License:	GPL (>= 2)
Imports:	numDeriv, optimx, VGAM, SuppDists, truncdist
Suggests:	knitr, ggplot2, xtable, PerformanceAnalytics
Roxygen:	list(wrap = FALSE)
VignetteBuilder:	knitr
Built:	R 3.1.1; ; 2015-03-31 03:52:27 UTC; unix

## Index:

Beta	The standard Beta Distribution.
Beta_ab	The four-Parameter beta Distribution.
Burr	The Burr's Distribution.

DistSelCriteriaValues	Distribution Selection Criteria Values.
ExtDist-package	Extended Probability Distribution Functions
Gamma	The Gamma Distribution.
Gumbel	The Gumbel Distribution.
JohnsonSB	The Johnson SB Distribution.
JohnsonSU	The Johnson SU Distribution.
Laplace	The Laplace Distribution.
Logistic	The Logistic Distribution.
Normal	The Normal Distribution.
Normal_sym_trunc_ab	The symmetric truncated normal distribution.
Normal_trunc_ab	The truncated normal distribution.
SRTB_ab	The Symmetric-Reflected Truncated Beta (SRTB) Distribution.
SSRTB	The standard Symmetric-Reflected Truncated Beta (SRTB) Distribution.
Triangular	The Triangular Distribution.
Uniform	The Uniform Distribution.
Weibull	The Weibull Distribution.
bestDist	Best distribution for (weighted) sample.
compareDist	Compare sample and fitted distributions
eDist	S3 methods for manipulating eDist objects.
eval.estimation	Parameter Estimation Evaluation.
wmle	Weighted Maximum Likelihood Estimation.

Further information is available in the following vignettes:

Distributions-Beta	Distributions-Beta (source)
Distributions-Normal	Distributions-Normal (source)
Distributions	Distributions-Index (source)
ParaEst-and-DistSel-by-ExtDist	Parameter-Estimation-and-Distribution-Selection-by-ExtDist (source)

### Author(s)

Haizhen Wu <h.wu2@massey.ac.nz>, A. Jonathan R. Godfrey <A.J.Godfrey@massey.ac.nz>, Kondaswamy Govindaraju <k.govindaraju@massey.ac.nz>, Sarah Pirikahu <s.pirikahu@massey.ac.nz>

Maintainer: Jonathan Godfrey <a.j.godfrey@massey.ac.nz>

---

bestDist

*Finding the best distribution for a (weighted) sample.*

---

### Description

This function chooses the best fitted distribution, based on a specified criterion.

**Usage**

```
bestDist(X, w = rep(1, length(X))/length(X), candDist = c("Beta_ab",  
  "Laplace", "Normal"), criterion = c("AICc", "logLik", "AIC", "BIC", "MDL"))
```

**Arguments**

X	Sample observations.
w	An optional vector of sample weights.
candDist	A vector of candidate distributions.
criterion	The basis on which the best fitted distribution is chosen.

**Details**

When comparing models fitted by maximum likelihood to the same data, the smaller the AIC, BIC or MDL, the better the fit. When comparing models using the log-likelihood criterion, the larger the log-likelihood the better the fit.

**Value**

An object of class character containing the name of the best distribution and its corresponding parameter estimates.

**Note**

The MDL criterion only works for parameter estimation by numerical maximum likelihood.

**Author(s)**

Haizhen Wu and A. Jonathan R. Godfrey.

**Examples**

```
X <- rBeta_ab(30, a = 0, b = 1, shape1 = 2, shape2 = 10)  
  
# Determining the best distribution from the list of candidate distributions for the data X  
Best.Dist <- bestDist(X, candDist = c("Laplace", "Normal", "Beta_ab"), criterion = "logLik")  
  
# Printing the parameter estimates of the best distribution  
attributes(Best.Dist)$best.dist.par
```

**Description**

Density, distribution, quantile, random number generation, and parameter estimation functions for the beta distribution with parameters shape1 and shape2. Parameter estimation can be based on a weighted or unweighted i.i.d. sample and can be carried out analytically or numerically.

**Usage**

```
dBeta(x, shape1 = 2, shape2 = 3, params = list(shape1, shape2), ...)
pBeta(q, shape1 = 2, shape2 = 3, params = list(shape1, shape2), ...)
qBeta(p, shape1 = 2, shape2 = 3, params = list(shape1, shape2), ...)
rBeta(n, shape1 = 2, shape2 = 3, params = list(shape1, shape2), ...)
eBeta(X, w, method = c("MOM", "numerical.MLE"), ...)
lBeta(X, w, shape1 = 2, shape2 = 3, params = list(shape1, shape2),
      logL = TRUE, ...)
sBeta(X, w, shape1 = 2, shape2 = 3, params = list(shape1, shape2), ...)
iBeta(X, w, shape1 = 2, shape2 = 3, params = list(shape1, shape2), ...)
```

**Arguments**

x, q	Vector of quantiles.
shape1, shape2	Shape parameters.
params	A list that includes all named parameters.
...	Additional parameters.
p	Vector of probabilities.
n	Number of observations.
X	Sample observations.
w	Optional vector of sample weights.
method	Parameter estimation method.
logL	logical, if TRUE lBeta gives the log-likelihood, otherwise the likelihood is given.

### Details

The `dBeta()`, `pBeta()`, `qBeta()`, and `rBeta()` functions serve as wrappers of the standard `dbeta`, `pbeta`, `qbeta`, and `rbeta` functions in the **stats** package. They allow for the shape parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The beta distribution with parameters  $\text{shape1}=\alpha$  and  $\text{shape2}=\beta$  is given by

$$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)}$$

where  $0 \leq x \leq 1$ ,  $\alpha > 0$ ,  $\beta > 0$ , and  $B$  is the **beta** function.

Analytical parameter estimation is conducted using the method of moments. The parameter estimates for  $\alpha$  and  $\beta$  are as given in the **Engineering Statistics Handbook**.

The log-likelihood function of the beta distribution is given by

$$l(\alpha, \beta | x) = (\alpha - 1) \sum_i \ln(x_i) + (\beta - 1) \sum_i \ln(1 - x_i) - \ln B(\alpha, \beta).$$

Aryal & Nadarajah (2004) derived the score function and Fisher's information matrix for the 4-parameter beta function, from which the 2-parameter cases can be obtained.

### Value

`dBeta` gives the density, `pBeta` the distribution function, `qBeta` the quantile function, `rBeta` generates random deviates, and `eBeta` estimates the parameters. `lBeta` provides the log-likelihood function, `sBeta` the observed score function, and `iBeta` the observed information matrix.

### Author(s)

Haizhen Wu and A. Jonathan R. Godfrey.  
Updates and bug fixes by Sarah Pirikahu.

### References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, volume 2, chapter 25, Wiley, New York.

**Engineering Statistics Handbook**

Bury, K. (1999) Statistical Distributions in Engineering, Chapter 14, pp.253-255, Cambridge University Press.

Aryal, G. and Nadarajah, S. (2004) Information Matrix for Beta Distributions, *Serdica Math. J.* 30, 513-526.

### See Also

**ExtDist** for other standard distributions.

**Examples**

```

# Parameter estimation for a distribution with known shape parameters
x <- rBeta(n=500, params=list(shape1=2, shape2=2))
est.par <- eBeta(x); est.par
plot(est.par)

# Fitted density curve and histogram
dens <- dBeta(x=seq(0,1,length=100), params=list(shape1=2, shape2=2))
hist(x, breaks=10, probability=TRUE, ylim = c(0,1.2*max(dens)))
lines(seq(0,1,length=100), dens, col="blue")
lines(density(x), lty=2)

# Extracting shape parameters
est.par[attributes(est.par)$par.type=="shape"]

# Parameter estimation for a distribution with unknown shape parameters
# Example from; Bury(1999) pp.253-255, parameter estimates as given by Bury are
# shape1 = 4.222 and shape2 = 6.317
data <- c(0.461, 0.432, 0.237, 0.113, 0.526, 0.278, 0.275, 0.309, 0.67, 0.428, 0.556,
0.402, 0.472, 0.226, 0.632, 0.533, 0.309, 0.417, 0.495, 0.241)
est.par <- eBeta(X=data, method="numerical.MLE"); est.par
plot(est.par)

# Log-likelihood, score function, and observed information matrix
lBeta(data, param=est.par)
sBeta(data, param=est.par)
iBeta(data, param=est.par)

# Evaluating the precision of parameter estimation by the Hessian matrix.
H <- attributes(est.par)$nll.hessian;H
var <- solve(H)
se <- sqrt(diag(var)); se

```

Beta\_ab

*The four-parameter beta distribution.***Description**

Density, distribution, quantile, random number generation, and parameter estimation functions for the 4-parameter beta distribution. Parameter estimation can be based on a weighted or unweighted i.i.d sample and can be performed numerically.

**Usage**

```

dBeta_ab(x, shape1 = 2, shape2 = 3, a = 0, b = 1,
  params = list(shape1, shape2, a, b), ...)

pBeta_ab(q, shape1 = 2, shape2 = 3, a = 0, b = 1, params = list(shape1
  = 2, shape2 = 5, a = 0, b = 1), ...)

```

```
qBeta_ab(p, shape1 = 2, shape2 = 3, a = 0, b = 1, params = list(shape1
= 2, shape2 = 5, a = 0, b = 1), ...)
```

```
rBeta_ab(n, shape1 = 2, shape2 = 3, a = 0, b = 1,
params = list(shape1, shape2, a, b), ...)
```

```
eBeta_ab(X, w, method = "numerical.MLE", ...)
```

```
lBeta_ab(X, w, shape1 = 2, shape2 = 3, a = 0, b = 1,
params = list(shape1, shape2, a, b), logL = TRUE, ...)
```

```
sBeta_ab(X, w, shape1 = 2, shape2 = 3, a = 0, b = 1,
params = list(shape1, shape2, a, b), ...)
```

### Arguments

x, q	A vector of quantiles.
shape1, shape2	Shape parameters.
a, b	Boundary parameters.
params	A list that includes all named parameters.
...	Additional parameters.
p	A vector of probabilities.
n	Number of observations.
X	Sample observations.
w	An optional vector of sample weights.
method	Parameter estimation method.
logL	logical; if TRUE, lBeta_ab gives the log-likelihood, otherwise the likelihood is given.

### Details

The dBeta\_ab(), pBeta\_ab(), qBeta\_ab(), and rBeta\_ab() functions serve as wrappers of the standard dbeta, pbeta, qbeta and rbeta functions in the **stats** package. They allow for the parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The four-parameter beta distribution with parameters shape1=p, shape2=q, a = a and b=b has probability density function

$$f(x) = \frac{1}{B(p, q)} \frac{(x - a)^{(p-1)}(b - x)^{(q-1)}}{((b - a)^{(p+q-1)})}$$

with  $p > 0$ ,  $q > 0$ ,  $a \leq x \leq b$  and where B is the **beta** function, Johnson et.al (p.210).

The log-likelihood function of the four-parameter beta distribution is

$$l(p, q, a, b|x) = -\ln B(p, q) + ((p - 1)\ln(x - a) + (q - 1)\ln(b - x)) - (p + q - 1)\ln(b - a).$$



Johnson et.al (p.226) provides the Fisher's information matrix of the four-parameter beta distribution in the regular case where  $p, q > 2$ .

### Value

dBeta\_ab gives the density, pBeta\_ab the distribution function, qBeta\_ab the quantile function, rBeta\_ab generates random deviates, and eBeta\_ab estimates the parameters. lBeta\_ab provides the log-likelihood function, sBeta\_ab the observed score function and iBeta\_ab the observed information matrix.

### Author(s)

Haizhen Wu and A. Jonathan R. Godfrey  
Updates and bug fixes by Sarah Pirikahu.

### References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, volume 2, chapter 25, Wiley, New York.

Bury, K. (1999) Statistical Distributions in Engineering, Chapter 14, pp.261-262, Cambridge University Press.

### See Also

[ExtDist](#) for other standard distributions.

### Examples

```
# Parameter estimation for a distribution with known shape parameters
X <- rBeta_ab(n=500, shape1=2, shape2=5, a=1, b=2)
est.par <- eBeta_ab(X); est.par
plot(est.par)

# Fitted density curve and histogram
den.x <- seq(min(X),max(X),length=100)
den.y <- dBeta_ab(den.x,params = est.par)
hist(X, breaks=10, probability=TRUE, ylim = c(0,1.1*max(den.y)))
lines(den.x, den.y, col="blue") # Original data
lines(density(X), lty=2) # Fitted density curve

# Extracting boundary and shape parameters
est.par[attributes(est.par)$par.type=="boundary"]
est.par[attributes(est.par)$par.type=="shape"]

# Parameter Estimation for a distribution with unknown shape parameters
# Example from: Bury(1999) pp.261-262, parameter estimates as given by Bury are
# shape1 = 4.088, shape2 = 10.417, a = 1.279 and b = 2.407.
# The log-likelihood for this data and Bury's parameter estimates is 8.598672.
data <- c(1.73, 1.5, 1.56, 1.89, 1.54, 1.68, 1.39, 1.64, 1.49, 1.43, 1.68, 1.61, 1.62)
est.par <- eBeta_ab(X=data, method="numerical.MLE");est.par
```

```

plot(est.par)

# Estimates calculated by eBeta_ab differ from those given by Bury(1999).
# However, eBeta_ab's parameter estimates appear to be an improvement, due to a larger
# log-likelihood of 9.295922 (as given by lBeta_ab below).

# log-likelihood and score functions
lBeta_ab(data,param = est.par)
sBeta_ab(data,param = est.par)

```

---

Burr

*The Burr Distribution.*


---

### Description

Density, distribution, quantile, random number generation, and parameter estimation functions for the Burr distribution with parameters location, scale and inequality. Parameter estimation can be based on a weighted or unweighted i.i.d sample and can be performed numerically.

### Usage

```

dBurr(x, b = 1, g = 2, s = 2, params = list(b = 1, g = 2, s = 2), ...)

pBurr(q, b = 1, g = 2, s = 2, params = list(b = 1, g = 2, s = 2), ...)

qBurr(p, b = 1, g = 2, s = 2, params = list(b = 1, g = 2, s = 2), ...)

rBurr(n, b = 1, g = 2, s = 2, params = list(b = 1, g = 2, s = 2), ...)

eBurr(X, w, method = "numerical.MLE", ...)

lBurr(X, w, b = 1, g = 2, s = 2, params = list(b = 1, g = 2, s = 2),
      logL = TRUE, ...)

```

### Arguments

x,q	A vector of quantiles.
b	Scale parameters.
g,s	Shape parameters.
params	A list that includes all named parameters.
...	Additional parameters.
p	A vector of probabilities.
n	Number of observations.
X	Sample observations.
w	An optional vector of sample weights.

method	Parameter estimation method.
logL	logical; if TRUE, lBurr gives the log-likelihood, otherwise the likelihood is given.

### Details

The Burr distribution is a special case of the Pareto(IV) distribution where the location parameter is equal 0 and inequality parameter is equal to  $1/g$ , Brazauskas (2003).

The dBurr(), pBurr(), qBurr(), and rBurr() functions serve as wrappers of the [dparetoIV](#), [pparetoIV](#), [qparetoIV](#), and [rparetoIV](#) functions in the **VGAM** package. They allow for the parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The Burr distribution is most simply defined in terms of its cumulative distribution function (Johnson et.al p.576)

$$F(x) = [1 + (x/b)^g]^{-s}$$

where  $b, g$  and  $s > 0$ . Parameter estimation can only be implemented numerically.

The log-likelihood and score functions are as given by Watkins (1999) and the information matrix is as given by Brazauskas (2003).

### Value

dBurr gives the density, pBurr the distribution function, qBurr the quantile function, rBurr generates random deviates, and eBurr estimate the distribution parameters. lBurr provides the log-likelihood function.

### Author(s)

Haizhen Wu and A. Jonathan R. Godfrey.  
Updates and bug fixes by Sarah Pirikahu.

### References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, volume 1, chapter 20, Wiley, New York.

Brazauskas, V. (2003) Information matrix for Pareto(IV), Burr, and related distributions. Comm. Statist. Theory and Methods 32, 315-325.

Watkins A.J. (1999) An algorithm for maximum likelihood estimation in the three parameter Burr XII distribution, Computational Statistics & Data Analysis, 32, 19-27.

**Mathworks: [Matlab documentation for Burr Type XII distribution](#)**

### See Also

[ExtDist](#) for other standard distributions.

**Examples**

```

# Parameter estimation for a distribution of known shape parameters
X <- rBurr(n=500, b = 1, g = 2, s = 2)
est.par <- eBurr(X); est.par
plot(est.par)

# Fitted density curve and histogram
den.x <- seq(min(X),max(X),length=100)
den.y <- dBurr(den.x, b=est.par$b, g=est.par$g, s=est.par$s)
hist(X, breaks=10, probability=TRUE, ylim = c(0,1.1*max(den.y)))
lines(den.x, den.y, col="blue")
lines(density(X), lty=2)

# Extracting shape or scale parameters
est.par[attributes(est.par)$par.type=="scale"]
est.par[attributes(est.par)$par.type=="shape"]

# Parameter Estimation for a distribution with unknown shape parameters
# Example from: Matlab Statistical Toolbox package
# (See: http://au.mathworks.com/help/stats/burr-type-xii-distribution.html)
# Parameter estimates given are: b = 80.4515, g = 18.9251 and s = 0.4492.
QRS.duration <- c(91,81,138,100,88,100,77,78,84,89,102,77,78,91,77,75,82,70,91,82,83,90,71,75,82,
109,94,95,90,96,85,71,75,78,82,69,103,85,80,94,80,79,92,84,86,73,75,73,78,80,81,
83,103,92,88,77,79,90,91,83,80,78,76,82,81,80,82,71,73,87,76,101,93,90,87,88,94,
94,90,78,83,92,93,100,83,163,96,114,170,137,84,82,79,72,97,87,102,85,84,78,79,91,
98,86,72,97,82,78,97,94,82,78,79,87,93,75,106,96,88,90,74,85,90,71,75,77,87,95,
74,99,89,83,78,100,80,87,79,102,80,85,81,85,95,82,97,92,102,86,80,85,85,111,89,63,
70,92,75,93,83,84,91,81,113,92,81,74,78,80,82,95,106,95,100,90,88,71,78,77,87,79,
85,91,92,98,68,84,92,110,108,153,73,81,87,87,95,73,95,100,96,97,76,62,86,71,99,68,
90,146,86,80,90,93,91,111,89,79,77,73,92,98,78,87,98,84,82,90,85,71,84,85,77,93,
74,89,89,103,85,88,81,84,96,90,98,78,93,80,85,67,74,69,105,95,87,108,99,79,86,82,
91,93,80,84,90,81,90,78,98,81,90,85,79,61,90,79,83,84,78,86,72,87,91,102,80,82,104,
85,83,81,94,84,91,99,101,132,79,103,97,131,91,90,121,78,84,97,94,96,91,80,97,92,90,
90,123,105,85,77,83,92,85,96,69,88,84,107,91,74,89,109,80,83,92,100,113,105,99,84,
74,76,87,87,96,88,80,85,90,74,95,86,74,95,74,73,104,92,105,97,101,83,84,98,81,93,
84,102,94,91,100,92,94,98,146,84,77,82,84,76,106,70,87,118,86,82,96,89,93,82,97,
86,188,93,72,107,81,76,83,147,82,110,108,82,93,95,80,185,73,78,71,86,85,76,93,
87,96,86,78,87,80,98,75,78,82,94,83,94,140,87,55,133,83,77,123,79,88,80,88,79,
77,87,88,94,88,74,85,88,81,91,81,80,100,108,93,79)
est.par <- eBurr(QRS.duration); est.par
plot(est.par)

# log-likelihood function
lBurr(QRS.duration,param = est.par)

# Evaluation of the precision of the parameter estimates by the Hessian matrix
H <- attributes(est.par)$nll.hessian
var <- solve(H)
se <- sqrt(diag(var)); se

```

---

compareDist	<i>Compare a sample to one or more fitted distributions</i>
-------------	---

---

**Description**

Compare a sample to one or more fitted distributions

**Usage**

```
compareDist(X, Dist1, Dist2 = NULL, Dist3 = NULL)
```

**Arguments**

X	An unweighted sample
Dist1, Dist2, Dist3	The fitted distribution, specified as either the objects of class eDist or names of the distribution to be fitted.

**Value**

compareDist returns an object of class histogram comparing the sample distribution to the specified fitted distribution(s).

**Author(s)**

Haizhen Wu and A. Jonathan R. Godfrey.

**Examples**

```
X <- rBeta(n=100, params=list(shape1=1, shape2=2))
compareDist(X, "Beta", "Normal", eNormal(X))
```

---

DistSelCriteria	<i>Distribution Selection Criteria.</i>
-----------------	---

---

**Description**

A function to calculate the distribution selection criteria for a list of candidate fits.

**Usage**

```
DistSelCriteria(X, w = rep(1, length(X))/length(X), candDist = c("Beta_ab",
  "Laplace", "Normal"), criteria = c("logLik", "AIC", "AICc", "BIC", "MDL"))
```

**Arguments**

<code>x</code>	Sample observations.
<code>w</code>	An optional vector of sample weights.
<code>candDist</code>	A vector of names of candidate distributions.
<code>criteria</code>	A vector of criteria to be calculated.

**Details**

When comparing models fitted by maximum likelihood to the same data, the smaller the AIC, BIC or MDL, the better the fit. When comparing models using the log-likelihood criterion, the larger the log-likelihood the better the fit.

**Value**

An object of class `matrix`, containing the listed distribution selection criteria for the named distributions.

**Note**

The MDL criterion only works for parameter estimation by numerical maximum likelihood.

**Author(s)**

Haizhen Wu and A. Jonathan R. Godfrey.

**Examples**

```
Ozone <- airquality$Ozone
Ozone <- Ozone[!is.na(Ozone)] # Removing the NA's from Ozone data
DistSelCriteria(Ozone, candDist = c("Gamma", "Weibull", "Normal", "Exp"),
criteria = c("logLik", "AIC", "AICc", "BIC"))
```

---

eDist

*S3 methods for manipulating eDist objects.*

---

**Description**

S3 methods for manipulating eDist objects

**Usage**

```

## S3 method for class 'eDist'
logLik(object, ...)

## S3 method for class 'eDist'
AIC(object, ..., k = 2)

AICc(object)

## S3 method for class 'eDist'
AICc(object, ...)

## S3 method for class 'eDist'
vcov(object, ..., corr = FALSE)

BIC(object)

## S3 method for class 'eDist'
BIC(object, ...)

MDL(object)

## S3 method for class 'eDist'
MDL(object, ...)

## S3 method for class 'eDist'
print(x, ...)

## S3 method for class 'eDist'
plot(x, ...)

```

**Arguments**

object,	x	An object of class eDist, usually the output of a parameter estimation function.
...		Additional parameters
k	numeric,	The penalty per parameter to be used; the default k = 2 is the classical AIC.
corr	logical;	should vcov() return correlation matrix (instead of variance-covariance matrix).
x,		A list to be returned as class eDist.
plot	logical;	if TRUE histogram, P-P and Q-Q plot of the distribution returned else only parameter estimation is returned.

**Note**

The MDL only works for parameter estimation by numerical maximum likelihood.

**Author(s)**

A. Jonathan R. Godfrey, Sarah Pirikahu, and Haizhen Wu.

**References**

Myung, I. (2000). The Importance of Complexity in Model Selection. *Journal of mathematical psychology*, 44(1), 190-204.

**Examples**

```
X <- rnorm(20)
est.par <- eNormal(X, method = "numerical.MLE")
logLik(est.par)
AIC(est.par)
AICc(est.par)
BIC(est.par)
MDL(est.par)
vcov(est.par)
vcov(est.par, corr=TRUE)
print(est.par)
plot(est.par)
```

---

eval.estimation

*Parameter Estimation Evaluation.*

---

**Description**

A function to evaluate the parameter estimation function.

**Usage**

```
eval.estimation(rdist, edist, n = 20, rep.num = 1000, params,
  method = "numerical.MLE")
```

**Arguments**

rdist	Random variable generating function.
edist	Parameter estimation function.
n	Sample size.
rep.num	Number of replicates.
params	True parameters of the distribution.
method	Estimation method.

**Value**

A list containing the mean and sd of the estimated parameters.

na.cont returns the number of "na"s that appeared in the parameter estimation.



**Author(s)**

Haizhen Wu and A. Jonathan R. Godfrey.

**Examples**

```
eval.estimation(rdist=rNormal,edist=eNormal,n = 100, rep.num = 50,
  params = list(mean = 1,sd = 5))
```

---

Exponential

*The Exponential Distribution.*

---

**Description**

Density, distribution, quantile, random number generation and parameter estimation functions for the exponential distribution. Parameter estimation can be based on a weighted or unweighted i.i.d sample and is carried out analytically.

**Usage**

```
dExp(x, scale = 1, params = list(scale = 1), ...)
pExp(q, scale = 1, params = list(scale = 1), ...)
qExp(p, scale = 1, params = list(scale = 1), ...)
rExp(n, scale = 1, params = list(scale = 1), ...)
eExp(x, w, method = "analytical.MLE", ...)
lExp(x, w, scale = 1, params = list(scale = 1), logL = TRUE, ...)
sExp(x, w, scale = 1, params = list(scale = 1), ...)
iExp(x, w, scale = 1, params = list(scale = 1), ...)
```

**Arguments**

x,q	A vector of sample values or quantiles.
scale	scale parameter, called rate in other packages.
params	A list that includes all named parameters
...	Additional parameters.
p	A vector of probabilities.
n	Number of observations.
w	An optional vector of sample weights.
method	Parameter estimation method.
logL	logical; if TRUE, lExp gives the log-likelihood, otherwise the likelihood is given.

### Details

If scale is omitted, it assumes the default value 1 giving the standard exponential distribution.

The exponential distribution is a special case of the gamma distribution where the shape parameter  $\alpha = 1$ . The `dExp()`, `pExp()`, `qExp()`, and `rExp()` functions serve as wrappers of the standard `dexp`, `pexp`, `qexp` and `rexp` functions in the `stats` package. They allow for the parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The probability density function for the exponential distribution with `scale= $\beta$`  is

$$f(x) = (1/\beta) * \exp(-x/\beta)$$

for  $\beta > 0$ , Johnson et.al (Chapter 19, p.494). Parameter estimation for the exponential distribution is carried out analytically using maximum likelihood estimation (p.506 Johnson et.al).

The likelihood function of the exponential distribution is given by

$$l(\lambda|x) = n \log \lambda - \lambda \sum x_i.$$

It follows that the score function is given by

$$dl(\lambda|x)/d\lambda = n/\lambda - \sum x_i$$

and Fisher's information given by

$$E[-d^2l(\lambda|x)/d\lambda^2] = n/\lambda^2.$$

### Value

`dExp` gives the density, `pExp` the distribution function, `qExp` the quantile function, `rExp` generates random deviates, and `eExp` estimates the distribution parameters. `lExp` provides the log-likelihood function.

### Author(s)

Jonathan R. Godfrey and Sarah Pirikahu.

### References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, volume 1, chapter 19, Wiley, New York.

Kapadia. A.S., Chan, W. and Moye, L. (2005) Mathematical Statistics with Applications, Chapter 8, Chapman & Hall/CRC.

**Examples**

```

# Parameter estimation for a distribution with known shape parameters
x <- rExp(n=500, scale=2)
est.par <- eExp(x); est.par
plot(est.par)

# Fitted density curve and histogram
den.x <- seq(min(x),max(x),length=100)
den.y <- dExp(den.x,scale=est.par$scale)
hist(x, breaks=10, probability=TRUE, ylim = c(0,1.1*max(den.y)))
lines(den.x, den.y, col="blue")
lines(density(x), lty=2)

# Extracting the scale parameter
est.par[attributes(est.par)$par.type=="scale"]

# Parameter estimation for a distribution with unknown shape parameters
# Example from Kapadia et.al(2005), pp.380-381.
# Parameter estimate as given by Kapadia et.al is scale=0.00277
cardio <- c(525, 719, 2880, 150, 30, 251, 45, 858, 15,
            47, 90, 56, 68, 6, 139, 180, 60, 60, 294, 747)
est.par <- eExp(cardio, method="analytical.MLE"); est.par
plot(est.par)

# log-likelihood, score function and Fisher's information
lExp(cardio,param = est.par)
sExp(cardio,param = est.par)
iExp(cardio,param = est.par)

```

Gamma

*The Gamma Distribution.***Description**

Density, distribution, quantile, random number generation, and parameter estimation functions for the gamma distribution with parameters shape and scale. Parameter estimation can be based on a weighted or unweighted i.i.d sample and can be carried out numerically.

**Usage**

```

dGamma(x, shape = 2, scale = 2, params = list(shape = 2, scale = 2), ...)
pGamma(q, shape = 2, scale = 2, params = list(shape = 2, scale = 2), ...)
qGamma(p, shape = 2, scale = 2, params = list(shape = 2, scale = 2), ...)
rGamma(n, shape = 2, scale = 2, params = list(shape = 2, scale = 2), ...)
eGamma(X, w, method = c("moments", "numerical.MLE"), ...)

```

```
lGamma(X, w, shape = 2, scale = 2, params = list(shape = 2, scale = 2),
       logL = TRUE, ...)
```

### Arguments

x, q	A vector of quantiles.
shape	Shape parameter.
scale	Scale parameter.
params	A list that includes all named parameters
...	Additional parameters.
p	A vector of probabilities.
n	Number of observations.
X	Sample observations.
w	An optional vector of sample weights.
method	Parameter estimation method.
logL	logical; if TRUE, lBeta_ab gives the log-likelihood, otherwise the likelihood is given.

### Details

The `dGamma()`, `pGamma()`, `qGamma()`, and `rGamma()` functions serve as wrappers of the standard `dgamma`, `pgamma`, `qgamma`, and `rgamma` functions in the `stats` package. They allow for the parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The gamma distribution with parameter  $\text{shape}=\alpha$  and  $\text{scale}=\beta$  has probability density function,

$$f(x) = (1/\beta^\alpha \Gamma(\alpha)) x^{\alpha-1} e^{-x/\beta}$$

where  $\alpha > 0$  and  $\beta > 0$ . Parameter estimation can be performed using the method of moments as given by Johnson et.al (pp.356-357).

The log-likelihood function of the gamma distribution is given by,

$$l(\alpha, \beta|x) = (\alpha - 1) \sum_i \ln(x_i) - \sum_i (x_i/\beta) - n\alpha \ln(\beta) + n \ln \Gamma(\alpha)$$

where  $\Gamma$  is the `gamma` function. The score function is provided by Rice (2007), p.270.

### Value

`dGamma` gives the density, `pGamma` the distribution function, `qGamma` the quantile function, `rGamma` generates random deviates, and `eGamma` estimates the distribution parameters. `lgamma` provides the log-likelihood function.

**Author(s)**

Haizhen Wu and A. Jonathan R. Godfrey.  
Updates and bug fixes by Sarah Pirikahu.

**References**

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, volume 1, chapter 17, Wiley, New York.

Bury, K. (1999) Statistical Distributions in Engineering, Chapter 13, pp.225-226, Cambridge University Press.

Rice, J.A. (2007) Mathematical Statistics and Data Analysis, 3rd Ed, Brookes/Cole.

**See Also**

[ExtDist](#) for other standard distributions.

**Examples**

```
# Parameter estimation for a distribution with known shape parameters
X <- rGamma(n=500, shape=1.5, scale=0.5)
est.par <- eGamma(X); est.par
plot(est.par)

# Fitted density curve and histogram
den.x <- seq(min(X),max(X),length=100)
den.y <- dGamma(den.x,shape=est.par$shape,scale=est.par$scale)
hist(X, breaks=10, probability=TRUE, ylim = c(0,1.1*max(den.y)))
lines(den.x, den.y, col="blue")
lines(density(X), lty=2)

# Extracting shape or scale parameters
est.par[attributes(est.par)$par.type=="shape"]
est.par[attributes(est.par)$par.type=="scale"]

# Parameter estimation for a distribution with unknown shape parameters
# Example from: Bury(1999) pp.225-226, parameter estimates as given by Bury are
# shape = 6.40 and scale=2.54. The log-likelihood for this data given
# Bury's parameter estimates is -656.7921.
data <- c(16, 11.6, 19.9, 18.6, 18, 13.1, 29.1, 10.3, 12.2, 15.6, 12.7, 13.1,
          19.2, 19.5, 23, 6.7, 7.1, 14.3, 20.6, 25.6, 8.2, 34.4, 16.1, 10.2, 12.3)
est.par <- eGamma(data, method="numerical.MLE"); est.par
plot(est.par)

# Estimates calculated by eGamma differ from those given by Bury(1999).
# However, eGamma's parameter estimates appear to be an improvement, due to a larger
# log-likelihood of -80.68186 (as given by lGamma below).

# log-likelihood
lGamma(data,param = est.par)
```

```
# Evaluating the precision of the parameter estimates by the Hessian matrix
H <- attributes(est.par)$nll.hessian
var <- solve(H)
se <- sqrt(diag(var));se
```

---

Gumbel

*The Gumbel distribution*


---

### Description

Density, distribution, quantile, random number generation, and parameter estimation functions for the Gumbel distribution with parameters location and scale. Parameter estimation can be based on a weighted or unweighted i.i.d sample and can be performed analytically or numerically.

### Usage

```
dGumbel(x, location = 0, scale = 1, params = list(location = 0, scale =
  1), ...)
```

```
pGumbel(q, location = 0, scale = 1, params = list(location = 0, scale =
  1), ...)
```

```
qGumbel(p, location = 0, scale = 1, params = list(location = 0, scale =
  1), ...)
```

```
rGumbel(n, location = 0, scale = 1, params = list(location = 0, scale =
  1), ...)
```

```
eGumbel(X, w, method = c("moments", "numerical.MLE"), ...)
```

```
lGumbel(X, w, location = 0, scale = 1, params = list(location = 0, scale =
  1), logL = TRUE, ...)
```

### Arguments

x,q	A vector of quantiles.
location	Location parameter.
scale	Scale parameter.
params	A list that includes all named parameters
...	Additional parameters.
p	A vector of probabilities.
n	Number of observations.
X	Sample observations.
w	An optional vector of sample weights.

method	Parameter estimation method.
logL	logical if TRUE, lGumbel gives the log-likelihood, otherwise the likelihood is given.

### Details

The `dGumbel()`, `pGumbel()`, `qGumbel()`, and `rGumbel()` functions serve as wrappers of the `dgumbel`, `pgumbel`, `qgumbel`, and `rgumbel` functions in the **VGAM** package. They allow for the parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The Gumbel distribution is a special case of the generalised extreme value (GEV) distribution and has probability density function,

$$f(x) = \exp(-\exp(-(x - \mu)/\sigma))$$

where  $\mu$  = location and  $\sigma$  = scale which has the constraint  $\sigma > 0$ . The analytical parameter estimations are as given by the **Engineering Statistics Handbook** with corresponding standard errors given by Bury (p.273).

The log-likelihood function of the Gumbel distribution is given by

$$l(\mu, \sigma|x) = \sigma^{-n} \exp(-\sum (x_i - \mu/\sigma) - \sum \exp(-(x_i - \mu/\sigma))).$$

Shi (1995) provides the score function and Fishers information matrix.

### Value

`dGumbel` gives the density, `pGumbel` the distribution function, `qGumbel` the quantile function, `rGumbel` generates random deviates, and `eGumbel` estimate the distribution parameters. `lGumbel` provides the log-likelihood function.

### Author(s)

Haizhen Wu and A. Jonathan R. Godfrey.  
Updates and bug fixes by Sarah Pirikahu.

### References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, volume 2, chapter 22, Wiley, New York.

**Engineering Statistics Handbook.**

Bury, K. (1999) Statistical Distributions in Engineering, Chapter 15, pp.283-284, Cambridge University Press.

Shi, D. (1995). Multivariate extreme value distribution and its Fisher information matrix. Acta Mathematica

**See Also**

[ExtDist](#) for other standard distributions.

**Examples**

```
# Parameter estimation for a distribution with known shape parameters
X <- rGumbel(n = 500, location = 1.5, scale = 0.5)
est.par <- eGumbel(X, method="moments"); est.par
plot(est.par)

# Extracting location and scale parameters
est.par[attributes(est.par)$par.type=="location"]
est.par[attributes(est.par)$par.type=="scale"]

# Fitted density curve and histogram
den.x <- seq(min(X),max(X),length=100)
den.y <- dGumbel(den.x, location = est.par$location, scale= est.par$scale)
hist(X, breaks=10, probability=TRUE, ylim = c(0,1.1*max(den.y)))
lines(den.x, den.y, col="blue")
lines(density(X))

# Parameter Estimation for a distribution with unknown shape parameters
# Example from; Bury(1999) pp.283-284, parameter estimates as given by Bury are location = 33.5
# and scale = 2.241
data <- c(32.7, 30.4, 31.8, 33.2, 33.8, 35.3, 34.6, 33, 32, 35.7, 35.5, 36.8, 40.8, 38.7, 36.7)
est.par <- eGumbel(X=data, method="numerical.MLE"); est.par
plot(est.par)

# log-likelihood
lGumbel(data, param = est.par)

# Evaluating the precision of the parameter estimates by the Hessian matrix
H <- attributes(est.par)$nll.hessian
var <- solve(H)
se <- sqrt(diag(var)); se
```

---

 JohnsonSB

*The Johnson SB distribution.*


---

**Description**

Density, distribution, quantile, random number generation, and parameter estimation functions for the Johnson SB (bounded support) distribution. Parameter estimation can be based on a weighted or unweighted i.i.d. sample and can be performed numerically.

**Usage**

```
dJohnsonSB(x, gamma = -0.5, delta = 2, xi = -0.5, lambda = 2,
  params = list(gamma = -0.5, delta = 2, xi = -0.5, lambda = 2), ...)
```



```

dJohnsonSB_ab(x, gamma = -0.5, delta = 2, a = -0.5, b = 1.5,
  params = list(gamma = -0.5, delta = 2, a = -0.5, b = 1.5), ...)

pJohnsonSB(q, gamma = -0.5, delta = 2, xi = -0.5, lambda = 2,
  params = list(gamma = -0.5, delta = 2, xi = -0.5, lambda = 2), ...)

qJohnsonSB(p, gamma = -0.5, delta = 2, xi = -0.5, lambda = 2,
  params = list(gamma = -0.5, delta = 2, xi = -0.5, lambda = 2), ...)

rJohnsonSB(n, gamma = -0.5, delta = 2, xi = -0.5, lambda = 2,
  params = list(gamma = -0.5, delta = 2, xi = -0.5, lambda = 2), ...)

eJohnsonSB(X, w, method = "numerical.MLE", ...)

lJohnsonSB(X, w, gamma = -0.5, delta = 2, xi = -0.5, lambda = 2,
  params = list(gamma = -0.5, delta = 2, xi = -0.5, lambda = 2),
  logL = TRUE, ...)

```

### Arguments

<code>x, q</code>	A vector of quantiles.
<code>gamma, delta</code>	Shape parameters.
<code>xi, lambda, a, b</code>	Location-scale parameters.
<code>params</code>	A list that includes all named parameters.
<code>...</code>	Additional parameters.
<code>p</code>	A vector of probabilities.
<code>n</code>	Number of observations.
<code>X</code>	Sample observations.
<code>w</code>	An optional vector of sample weights.
<code>method</code>	Parameter estimation method.
<code>logL</code>	logical, it is assumed that the log-likelihood is desired. Set to FALSE if the likelihood is wanted.

### Details

The Johnson system of distributions consists of families of distributions that, through specified transformations, can be reduced to the standard normal random variable. It provides a very flexible system for describing statistical distributions and is defined by

$$z = \gamma + \delta f(Y)$$

with  $Y = (X - xi)/lambda$ . The Johnson SB distribution arises when  $f(Y) = \ln[Y/(1 - Y)]$ , where  $0 < Y < 1$ . This is the bounded Johnson family since the range of  $Y$  is  $(0, 1)$ , Karian & Dudewicz (2011).

The `dJohnsonSB()`, `pJohnsonSB()`, `qJohnsonSB()`, and `rJohnsonSB()` functions serve as wrappers of the `dJohnson`, `pJohnson`, `qJohnson`, and `rJohnson` functions in the **SuppDists** package. They allow for the parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The JohnsonSB distribution has probability density function

$$p_X(x) = \frac{\delta \lambda}{\sqrt{2\pi}(x-xi)(1-x+xi)} \exp[-0.5(\gamma + \delta \ln((x-xi)/(1-x+xi)))^2].$$

### Value

`dJohnsonSB` gives the density, `pJohnsonSB` the distribution function, `qJohnsonSB` gives quantile function, `rJohnsonSB` generates random deviates, and `eJohnsonSB` estimate the parameters. `lJohnsonSB` provides the log-likelihood function. The `dJohnsonSB_ab` provides an alternative parameterisation of the JohnsonSB distribution.

### Author(s)

Haizhen Wu and A. Jonathan R. Godfrey.  
Updates and bug fixes by Sarah Pirikahu.

### References

- Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, volume 1, chapter 12, Wiley, New York.
- Kotz, S. and van Dorp, J. R. (2004). Beyond Beta: Other Continuous Families of Distributions with Bounded Support and Applications. Appendix B. World Scientific: Singapore.
- Z. A. Karian and E. J. Dudewicz (2011) Handbook of Fitting Statistical Distributions with R, Chapman & Hall.

### See Also

[ExtDist](#) for other standard distributions.

### Examples

```
# Parameter estimation for a distribution with known shape parameters
X <- rJohnsonSB(n=500, gamma=-0.5, delta=2, xi=-0.5, lambda=2)
est.par <- eJohnsonSB(X); est.par
plot(est.par)

# Fitted density curve and histogram
den.x <- seq(min(X),max(X),length=100)
den.y <- dJohnsonSB(den.x,params = est.par)
hist(X, breaks=10, probability=TRUE, ylim = c(0,1.2*max(den.y)))
lines(den.x, den.y, col="blue")
lines(density(X))
```

```

# Extracting location, scale and shape parameters
est.par[attributes(est.par)$par.type=="location"]
est.par[attributes(est.par)$par.type=="scale"]
est.par[attributes(est.par)$par.type=="shape"]

# Parameter Estimation for a distribution with unknown shape parameters
# Example from Karian, Z.A and Dudewicz, E.J. (2011) p.647.
# Original source of brain scan data Dudewicz, E.J et.al (1989).
# Parameter estimates as given by Karian & Dudewicz using moments are:
# gamma =-0.2081, delta=0.9167, xi = 95.1280 and lambda = 21.4607 with log-likelihood = -67.03579
brain <- c(108.7, 107.0, 110.3, 110.0, 113.6, 99.2, 109.8, 104.5, 108.1, 107.2, 112.0, 115.5, 108.4,
          107.4, 113.4, 101.2, 98.4, 100.9, 100.0, 107.1, 108.7, 102.5, 103.3)
est.par <- eJohnsonSB(brain); est.par

# Estimates calculated by eJohnsonSB differ from those given by Karian & Dudewicz (2011).
# However, eJohnsonSB's parameter estimates appear to be an improvement, due to a larger
# log-likelihood of -66.35496 (as given by lJohnsonSB below).

# log-likelihood function
lJohnsonSB(brain, param = est.par)

```

---

JohnsonSU

*The Johnson SU distribution.*


---

## Description

Density, distribution, quantile, random number generation and parameter estimation functions for the Johnson SU (unbounded support) distribution. Parameter estimation can be based on a weighted or unweighted i.i.d sample and can be carried out numerically.

## Usage

```

dJohnsonSU(x, gamma = -0.5, delta = 2, xi = -0.5, lambda = 2,
  params = list(gamma = -0.5, delta = 2, xi = -0.5, lambda = 2), ...)

pJohnsonSU(q, gamma = -0.5, delta = 2, xi = -0.5, lambda = 2,
  params = list(gamma = -0.5, delta = 2, xi = -0.5, lambda = 2), ...)

qJohnsonSU(p, gamma = -0.5, delta = 2, xi = -0.5, lambda = 2,
  params = list(gamma = -0.5, delta = 2, xi = -0.5, lambda = 2), ...)

rJohnsonSU(n, gamma = -0.5, delta = 2, xi = -0.5, lambda = 2,
  params = list(gamma = -0.5, delta = 2, xi = -0.5, lambda = 2), ...)

eJohnsonSU(X, w, method = "numerical.MLE", ...)

lJohnsonSU(X, w, gamma = -0.5, delta = 2, xi = -0.5, lambda = 2,
  params = list(gamma = -0.5, delta = 2, xi = -0.5, lambda = 2),
  logL = TRUE, ...)

```

**Arguments**

<code>x, q</code>	A vector of quantiles.
<code>gamma, delta</code>	Shape parameters.
<code>xi, lambda</code>	Location-scale parameters.
<code>params</code>	A list that includes all named parameters.
<code>...</code>	Additional parameters.
<code>p</code>	A vector of probabilities.
<code>n</code>	Number of observations.
<code>X</code>	Sample observations.
<code>w</code>	An optional vector of sample weights.
<code>method</code>	Parameter estimation method.
<code>logL</code>	logical; if TRUE, lJohnsonSU gives the log-likelihood, otherwise the likelihood is given.

**Details**

The Johnson system of distributions consists of families of distributions that, through specified transformations, can be reduced to the standard normal random variable. It provides a very flexible system for describing statistical distributions and is defined by

$$z = \gamma + \delta f(Y)$$

with  $Y = (X - xi)/lambda$ . The Johnson SB distribution arises when  $f(Y) = \operatorname{arcsinh}(Y)$ , where  $-\infty < Y < \infty$ . This is the unbounded Johnson family since the range of  $Y$  is  $(-\infty, \infty)$ , Karian & Dudewicz (2011).

The JohnsonSU distribution has probability density function

$$p_X(x) = \frac{\delta}{\sqrt{2\pi((x - xi)^2 + lambda^2)}} \exp[-0.5(\gamma + \delta \ln(\frac{x - xi + \sqrt{(x - xi)^2 + lambda^2}}{lambda}))^2].$$

Parameter estimation can only be carried out numerically.

**Value**

dJohnsonSU gives the density, pJohnsonSU the distribution function, qJohnsonSU gives the quantile function, rJohnsonSU generates random variables, and eJohnsonSU estimates the parameters. lJohnsonSU provides the log-likelihood function.

**Author(s)**

Haizhen Wu and A. Jonathan R. Godfrey.  
Updates and bug fixes by Sarah Pirikahu.

## References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, volume 1, chapter 12, Wiley, New York.

Bowman, K.O., Shenton, L.R. (1983). Johnson's system of distributions. In: Encyclopedia of Statistical Sciences, Volume 4, S. Kotz and N.L. Johnson (eds.), pp. 303-314. John Wiley and Sons, New York.

Z. A. Karian and E. J. Dudewicz (2011) Handbook of Fitting Statistical Distributions with R, Chapman & Hall.

## See Also

[ExtDist](#) for other standard distributions.

## Examples

```
# Parameter estimation for a known distribution
X <- rJohnsonSU(n=500, gamma=-0.5, delta=2, xi=-0.5, lambda=2)
est.par <- eJohnsonSU(X); est.par
plot(est.par)

# Fitted density curve and histogram
den.x <- seq(min(X),max(X),length=100)
den.y <- dJohnsonSU(den.x,params = est.par)
hist(X, breaks=10, probability=TRUE, ylim = c(0,1.2*max(den.y)))
lines(den.x, den.y, col="blue")
lines(density(X), lty=2)

# Extracting shape and boundary parameters
est.par[attributes(est.par)$par.type=="shape"]
est.par[attributes(est.par)$par.type=="boundary"]

# Parameter Estimation for a distribution with unknown shape parameters
# Example from Karian, Z.A and Dudewicz, E.J. (2011) p.657.
# Parameter estimates as given by Karian & Dudewicz are:
# gamma =-0.2823, delta=1.0592, xi = -1.4475 and lambda = 4.2592 with log-likelihood = -277.1543
data <- c(1.99, -0.424, 5.61, -3.13, -2.24, -0.14, -3.32, -0.837, -1.98, -0.120,
  7.81, -3.13, 1.20, 1.54, -0.594, 1.05, 0.192, -3.83, -0.522, 0.605,
  0.427, 0.276, 0.784, -1.30, 0.542, -0.159, -1.66, -2.46, -1.81, -0.412,
  -9.67, 6.61, -0.589, -3.42, 0.036, 0.851, -1.34, -1.22, -1.47, -0.592,
  -0.311, 3.85, -4.92, -0.112, 4.22, 1.89, -0.382, 1.20, 3.21, -0.648,
  -0.523, -0.882, 0.306, -0.882, -0.635, 13.2, 0.463, -2.60, 0.281, 1.00,
  -0.336, -1.69, -0.484, -1.68, -0.131, -0.166, -0.266, 0.511, -0.198, 1.55,
  -1.03, 2.15, 0.495, 6.37, -0.714, -1.35, -1.55, -4.79, 4.36, -1.53,
  -1.51, -0.140, -1.10, -1.87, 0.095, 48.4, -0.998, -4.05, -37.9, -0.368,
  5.25, 1.09, 0.274, 0.684, -0.105, 20.3, 0.311, 0.621, 3.28, 1.56)
est.par <- eJohnsonSU(data); est.par
plot(est.par)
```

```

# Estimates calculated by eJohnsonSU differ from those given by Karian & Dudewicz (2011).
# However, eJohnsonSU's parameter estimates appear to be an improvement, due to a larger
# log-likelihood of -250.3208 (as given by lJohnsonSU below).

# log-likelihood function
lJohnsonSU(data, param = est.par)

# Evaluation of the precision using the Hessian matrix
H <- attributes(est.par)$nll.hessian
var <- solve(H)
se <- sqrt(diag(var)); se

```

---

Laplace

*The Laplace Distribution.*


---

### Description

Density, distribution, quantile, random number generation and parameter estimation functions for the Laplace distribution with location parameter  $\mu$  and scale parameter  $b$ . Parameter estimation can for the Laplace distribution can be carried out numerically or analytically but may only be based on an unweighted i.i.d. sample.

### Usage

```

dLaplace(x, mu = 0, b = 1, params = list(mu, b), ...)

pLaplace(q, mu = 0, b = 1, params = list(mu, b), ...)

qLaplace(p, mu = 0, b = 1, params = list(mu, b), ...)

rLaplace(n, mu = 0, b = 1, params = list(mu, b), ...)

eLaplace(X, w, method = c("analytic.MLE", "numerical.MLE"), ...)

lLaplace(x, w = 1, mu = 0, b = 1, params = list(mu, b), logL = TRUE,
  ...)

```

### Arguments

<code>x, q</code>	A vector of quantiles.
<code>mu</code>	Location parameter.
<code>b</code>	Scale parameter.
<code>params</code>	A list that includes all named parameters
<code>...</code>	Additional parameters.
<code>p</code>	A vector of probabilities.
<code>n</code>	Number of observations.

<code>x</code>	Sample observations.
<code>w</code>	Optional vector of sample weights.
<code>method</code>	Parameter estimation method.
<code>logL</code>	logical; if TRUE, <code>lLaplace</code> gives the log-likelihood, otherwise the likelihood is given.

### Details

The `dLaplace()`, `pLaplace()`, `qLaplace()`, and `rLaplace()` functions allow for the parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The Laplace distribution with parameters `location =  $\mu$`  and `scale =  $b$`  has probability density function

$$f(x) = (1/2b)exp(-|x - \mu|/b)$$

where  $-\infty < x < \infty$  and  $b > 0$ . The cumulative distribution function for `pLaplace` is defined by Johnson et.al (p.166).

Parameter estimation can be carried out analytically via maximum likelihood estimation, see Johnson et.al (p.172). Where the population mean,  $\mu$ , is estimated using the sample median and  $b$  by the mean of  $|x - b|$ .

Johnson et.al (p.172) also provides the log-likelihood function for the Laplace distribution

$$l(\mu, b|x) = -n \ln(2b) - b^{-1} \sum |x_i - \mu|.$$

### Value

`dLaplace` gives the density, `pLaplace` the distribution function, `qLaplace` the quantile function, `rLaplace` generates random deviates, and `eLaplace` estimates the distribution parameters. `lLaplace` provides the log-likelihood function, `sLaplace` the score function, and `iLaplace` the observed information matrix.

### Note

The estimation of the population mean is done using the median of the sample. Unweighted samples are not yet catered for in the `eLaplace()` function.

### Author(s)

A. Jonathan R. Godfrey and Haizhen Wu.  
Updates and bug fixes by Sarah Pirikahu

### References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, volume 2, chapter 24, Wiley, New York.

Best, D.J., Rayner, J.C.W. and Thas O. (2008) Comparison of some tests of fit for the Laplace distribution, *Computational Statistics and Data Analysis*, Vol. 52, pp.5338-5343.

Gumbel, E.J., Mustafi, C.K., 1967. Some analytical properties of bivariate extremal distributions. *J. Am. Stat. Assoc.* 62, 569-588

### See Also

[ExtDist](#) for other standard distributions.

### Examples

```
# Parameter estimation for a distribution with known shape parameters
X <- rLaplace(n=500, mu=1, b=2)
est.par <- eLaplace(X, method="analytic.MLE"); est.par
plot(est.par)

# Fitted density curve and histogram
den.x <- seq(min(X),max(X),length=100)
den.y <- dLaplace(den.x, location = est.par$location, scale= est.par$scale)
hist(X, breaks=10, probability=TRUE, ylim = c(0,1.1*max(den.y)))
lines(den.x, den.y, col="blue")
lines(density(X), lty=2)

# Extracting location or scale parameters
est.par[attributes(est.par)$par.type=="location"]
est.par[attributes(est.par)$par.type=="scale"]

# Parameter estimation for a distribution with unknown shape parameters
# Example from Best et.al (2008). Original source of flood data from Gumbel & Mustafi.
# Parameter estimates as given by Best et.al mu=10.13 and b=3.36
flood <- c(1.96, 1.96, 3.60, 3.80, 4.79, 5.66, 5.76, 5.78, 6.27, 6.30, 6.76, 7.65, 7.84, 7.99,
           8.51, 9.18, 10.13, 10.24, 10.25, 10.43, 11.45, 11.48, 11.75, 11.81, 12.34, 12.78, 13.06,
           13.29, 13.98, 14.18, 14.40, 16.22, 17.06)
est.par <- eLaplace(flood, method="numerical.MLE"); est.par
plot(est.par)

#log-likelihood function
lLaplace(flood,param=est.par)

# Evaluating the precision by the Hessian matrix
H <- attributes(est.par)$nll.hessian
var <- solve(H)
se <- sqrt(diag(var));se
```



**Description**

Density, distribution, and quantile, random number generation, and parameter estimation functions for the logistic distribution with parameters location and scale. Parameter estimation can be based on a weighted or unweighted i.i.d. sample and can be carried out numerically.

**Usage**

```
dLogistic(x, location = 0, scale = 1, params = list(location = 0, scale = 1), ...)
```

```
pLogistic(q, location = 0, scale = 1, params = list(location = 0, scale = 1), ...)
```

```
qLogistic(p, location = 0, scale = 1, params = list(location = 0, scale = 1), ...)
```

```
rLogistic(n, location = 0, scale = 1, params = list(location = 0, scale = 1), ...)
```

```
eLogistic(X, w, method = "numerical.MLE", ...)
```

```
lLogistic(X, w, location = 0, scale = 1, params = list(location = 0, scale = 1), logL = TRUE, ...)
```

**Arguments**

x,q	A vector of quantiles.
location	Location parameter.
scale	Scale parameter.
params	A list that includes all named parameters.
...	Additional parameters.
p	A vector of probabilities.
n	Number of observations.
X	Sample observations.
w	An optional vector of sample weights.
method	Parameter estimation method.
logL	logical; if TRUE, lLogistic gives the log-likelihood, otherwise the likelihood is given.

**Details**

If location or scale are omitted, they assume the default values of 0 or 1 respectively.

The `dLogistic()`, `pLogistic()`, `qLogistic()`, and `rLogistic()` functions serve as wrappers of the standard `dlogis`, `plogis`, `qlogis`, and `rlogis` functions in the `stats` package. They allow for

the parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The logistic distribution with location =  $\alpha$  and scale =  $\beta$  is most simply defined in terms of its cumulative distribution function (Johnson et.al pp.115-116)

$$F(x) = 1 - [1 + \exp((x - \alpha)/\beta)]^{-1}.$$

The corresponding probability density function is given by

$$f(x) = 1/\beta[\exp(x - \alpha/\beta)[1 + \exp(x - \alpha/\beta)]^{-2}$$

Parameter estimation is only implemented numerically.

The score function and Fishers information are as given by Shi (1995) (See also Kotz & Nadarajah (2000)).

### Value

dLogistic gives the density, pLogistic the distribution function, qLogistic the quantile function, rLogistic generates random deviates, and eLogistic estimates the parameters. lLogistic provides the log-likelihood function.

### Author(s)

Haizhen Wu and A. Jonathan R. Godfrey.  
Updates and bug fixes by Sarah Pirikahu.

### References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, volume 2, chapter 23. Wiley, New York.

Shi, D. (1995) Fisher information for a multivariate extreme value distribution, Biometrika, vol 82, pp.644-649.

Kotz, S. and Nadarajah (2000) Extreme Value Distributions Theory and Applications, chapter 3, Imperial Collage Press, Singapore.

### See Also

[ExtDist](#) for other standard distributions.

### Examples

```
# Parameter estimation for a distribution with known shape parameters
X <- rLogistic(n=500, location=1.5, scale=0.5)
est.par <- eLogistic(X); est.par
plot(est.par)
# Fitted density curve and histogram
den.x <- seq(min(X),max(X),length=100)
```

```

den.y <- dLogistic(den.x,location=est.par$location,scale=est.par$scale)
hist(X, breaks=10, probability=TRUE, ylim = c(0,1.2*max(den.y)))
lines(den.x, den.y, col="blue")
lines(density(X), lty=2)

# Extracting location or scale parameters
est.par[attributes(est.par)$par.type=="location"]
est.par[attributes(est.par)$par.type=="scale"]

# log-likelihood function
lLogistic(X,param = est.par)

# Evaluation of the precision of the parameter estimates by the Hessian matrix
H <- attributes(est.par)$nll.hessian
fisher_info <- solve(H)
var <- sqrt(diag(fisher_info));var

# Example of parameter estimation for a distribution with
# unknown parameters currently been sought after.

```

---

Normal

*The Normal Distribution.*


---

### Description

Density, distribution, quantile, random number generation and parameter estimation functions for the normal distribution. Parameter estimation can be based on a weighted or unweighted i.i.d. sample and can be carried out analytically or numerically.

### Usage

```

dNormal(x, mean = 0, sd = 1, params = list(mean, sd), ...)

pNormal(q, mean = 0, sd = 1, params = list(mean, sd), ...)

qNormal(p, mean = 0, sd = 1, params = list(mean, sd), ...)

rNormal(n, mean = 0, sd = 1, params = list(mean, sd), ...)

eNormal(X, w, method = c("unbiased.MLE", "analytical.MLE", "numerical.MLE"),
  ...)

lNormal(X, w, mean = 0, sd = 1, params = list(mean, sd), logL = TRUE,
  ...)

sNormal(X, w, mean = 0, sd = 1, params = list(mean, sd), ...)

iNormal(X, w, mean = 0, sd = 1, params = list(mean, sd), ...)

```

**Arguments**

x, q	Vector of quantiles.
mean	Location parameter.
sd	Scale parameter.
params	A list that includes all named parameters.
...	Additional parameters.
p	Vector of probabilities.
n	Number of observations.
X	Sample observations.
w	Optional vector of sample weights.
method	Parameter estimation method.
logL	logical; if TRUE, lNormal gives the log-likelihood, otherwise the likelihood is given.

**Details**

If the mean or sd are not specified they assume the default values of 0 and 1, respectively.

The `dNormal()`, `pNormal()`, `qNormal()`, and `rNormal()` functions serve as wrappers of the standard `dnorm`, `pnorm`, `qnorm`, and `rnorm` functions in the **stats** package. They allow for the parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The normal distribution has probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where  $\mu$  is the mean of the distribution and  $\sigma$  is the standard deviation. The analytical unbiased parameter estimations are as given by Johnson et.al (Vol 1, pp.123-128).

The log-likelihood function of the normal distribution is given by

$$l(\mu, \sigma | x) = \sum_i [-0.5 \ln(2\pi) - \ln(\sigma) - 0.5\sigma^{-2}(x_i - \mu)^2].$$

The score function and observed information matrix are as given by Casella & Berger (2nd Ed, pp.321-322).

**Value**

`dNormal` gives the density, `pNormal` gives the distribution function, `qNormal` gives the quantiles, `rNormal` generates random deviates, and `eNormal` estimates the parameters. `lNormal` provides the log-likelihood function, `sNormal` the score function, and `iNormal` the observed information matrix.

**Author(s)**

Haizhen Wu and A. Jonathan R. Godfrey.  
Updates and bug fixes by Sarah Pirikahu.

**References**

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, volume 1, chapter 13, Wiley, New York.

Casella, G. and Berger R. L. (2002) Statistical Inference, 2nd Ed, pp.321-322, Duxbury.

Bury, K. (1999) Statistical Distributions in Engineering, Chapter 10, p.143, Cambridge University Press.

**See Also**

[ExtDist](#) for other standard distributions.

**Examples**

```
# Parameter estimation for a distribution with known shape parameters
x <- rNormal(n=500, params=list(mean=1, sd=2))
est.par <- eNormal(X=x, method="unbiased.MLE"); est.par
plot(est.par)

# Fitted density curve and histogram
den.x <- seq(min(x),max(x),length=100)
den.y <- dNormal(den.x, mean = est.par$mean, sd = est.par$sd)
hist(x, breaks=10, probability=TRUE, ylim = c(0,1.2*max(den.y)))
lines(lines(den.x, den.y, col="blue")) # Original data
lines(density(x), col="red")          # Fitted curve

# Extracting location and scale parameters
est.par[attributes(est.par)$par.type=="location"]
est.par[attributes(est.par)$par.type=="scale"]

# Parameter Estimation for a distribution with unknown shape parameters
# Example from: Bury(1999) p.143, parameter estimates as given by Bury are
# mu = 11.984 and sigma = 0.067
data <- c(12.065, 11.992, 11.992, 11.921, 11.954, 11.945, 12.029, 11.948, 11.885, 11.997,
          11.982, 12.109, 11.966, 12.081, 11.846, 12.007, 12.011)
est.par <- eNormal(X=data, method="numerical.MLE"); est.par
plot(est.par)

# log-likelihood, score function and observed information matrix
lNormal(data, param = est.par)
sNormal(data, param = est.par)
iNormal(data, param = est.par)

# Evaluating the precision of the parameter estimates by the Hessian matrix
H <- attributes(est.par)$nll.hessian; H
```

```
var <- solve(H)
se <- sqrt(diag(var)); se
```

---

Normal\_sym\_trunc\_ab    *The symmetric truncated normal distribution.*

---

## Description

Density, distribution, quantile, random number generation and parameter estimation functions for the symmetric truncated normal distribution with parameters,  $\sigma$ ,  $a$  and  $b$  which represent the lower and upper truncation points respectively. Parameter estimation can be based on a weighted or unweighted i.i.d sample and can be carried out numerically.

## Usage

```
dNormal_sym_trunc_ab(x, sigma = 0.3, a = 0, b = 1, params = list(sigma,
  a, b), ...)
```

```
pNormal_sym_trunc_ab(q, sigma = 0.3, a = 0, b = 1, params = list(mu = 2,
  sigma = 5, a = 0, b = 1), ...)
```

```
qNormal_sym_trunc_ab(p, sigma = 0.3, a = 0, b = 1, params = list(mu = 2,
  sigma = 5, a = 0, b = 1), ...)
```

```
rNormal_sym_trunc_ab(n, mu = 2, sigma = 3, a = 0, b = 1,
  params = list(sigma, a, b), ...)
```

```
eNormal_sym_trunc_ab(X, w, method = "numerical.MLE", ...)
```

```
lNormal_sym_trunc_ab(X, w, mu = 2, sigma = 3, a = 0, b = 1,
  params = list(sigma, a, b), logL = TRUE, ...)
```

## Arguments

$x, q$	A vector of quantiles.
$a, b$	Boundary parameters.
$params$	A list that includes all named parameters.
$\dots$	Additional parameters
$p$	A vector of probabilities.
$n$	Number of observations.
$\mu, \sigma$	Shape parameters.
$X$	Sample observations.
$w$	An optional vector of sample weights.
$method$	Parameter estimation method.
$logL$	logical; if TRUE, <code>lNormal_sym_trunc_ab</code> gives the log-likelihood, otherwise the likelihood is given.

**Details**

The normal symmetric truncated distribution is a special case of the truncated normal distribution. See [Normal\\_trunc\\_ab](#).

**Value**

dNormal\_sym\_trunc\_ab gives the density, pNormal\_sym\_trunc\_ab the distribution function, qNormal\_sym\_trunc\_ab the quantile function, rNormal\_sym\_trunc\_ab generates random deviates, and eNormal\_sym\_trunc\_ab estimates the parameters. lNormal\_sym\_trunc\_ab provides the log-likelihood function.

**Author(s)**

Haizhen Wu and A. Jonathan R. Godfrey.

**See Also**

[ExtDist](#) for other standard distributions.

---

Normal_trunc_ab	<i>The truncated normal distribution.</i>
-----------------	---

---

**Description**

Density, distribution, quantile, random number generation and parameter estimation functions for the truncated normal distribution with parameters mean, sd and a and b which represent the lower and upper truncation points respectively. Parameter estimation can be based on a weighted or unweighted i.i.d. sample and is performed numerically.

**Usage**

```
dNormal_trunc_ab(x, mu = 0, sigma = 1, a = 0, b = 1, params = list(mu,
  sigma, a, b), ...)
```

```
pNormal_trunc_ab(q, mu = 0, sigma = 1, a = 0, b = 1, params = list(mu
  = 2, sigma = 5, a = 0, b = 1), ...)
```

```
qNormal_trunc_ab(p, mu = 0, sigma = 1, a = 0, b = 1, params = list(mu
  = 2, sigma = 5, a = 0, b = 1), ...)
```

```
rNormal_trunc_ab(n, mu = 0, sigma = 1, a = 0, b = 1, params = list(mu,
  sigma, a, b), ...)
```

```
eNormal_trunc_ab(X, w, method = "numerical.MLE", ...)
```

```
lNormal_trunc_ab(X, w, mu = 0, sigma = 1, a = 0, b = 1,
  params = list(mu, sigma, a, b), logL = TRUE, ...)
```

**Arguments**

<code>x, q</code>	A vector of quantiles.
<code>mu, sigma</code>	Shape parameters.
<code>a, b</code>	Boundary parameters.
<code>params</code>	A list that includes all named parameters.
<code>...</code>	Additional parameters.
<code>p</code>	A vector of probabilities.
<code>n</code>	Number of observations.
<code>X</code>	Sample observations.
<code>w</code>	An optional vector of sample weights.
<code>method</code>	Parameter estimation method.
<code>logL</code>	logical; if TRUE, <code>lNormal_trunc_ab</code> gives the log-likelihood, otherwise the likelihood is given.

**Details**

If the mean, `sd`, `a` or `b` are not specified they assume the default values of 0, 1, 0, 1 respectively.

The `dNormal_trunc_ab()`, `pNormal_trunc_ab()`, `qNormal_trunc_ab()`, and `rNormal_trunc_ab()` functions serve as wrappers of the `dtrunc`, `ptrunc`, `qtrunc`, and `rtrunc` functions in the **truncdist** package. They allow for the parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The probability density function of the doubly truncated normal distribution is given by

$$f(x) = \sigma^{-1} Z(x - \mu/\sigma) [\Phi(b - \mu/\sigma) - \Phi(a - \mu/\sigma)]^{-1}$$

where  $-\infty < a \leq x \leq b < \infty$ . The degrees of truncation are  $\Phi((a - \mu)/\sigma)$  from below and  $1 - \Phi((a - \mu)/\sigma)$  from above. If `a` is replaced by  $-\infty$ , or `b` by  $\infty$ , the distribution is singly truncated, (Johnson et.al, p.156). The upper and lower limits of truncation `a` and `b` are normally known parameters whereas  $\mu$  and  $\sigma$  may be unknown. Crain (1979) discusses parameter estimation for the truncated normal distribution and the method of numerical maximum likelihood estimation is used for parameter estimation in `eNormal_trunc_ab`.

**Value**

`dNormal_trunc_ab` gives the density, `pNormal_trunc_ab` the distribution function, `qNormal_trunc_ab` the quantile function, `rNormal_trunc_ab` generates random variables, and `eNormal_trunc_ab` estimates the parameters. `lNormal_trunc_ab` provides the log-likelihood function.

**Author(s)**

Haizhen Wu and A. Jonathan R. Godfrey.  
Updates and bug fixes by Sarah Pirikahu.



## References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1994) Continuous Univariate Distributions, volume 1, chapter 13, Wiley, New York.

Crain, B.R (1979). Estimating the parameters of a truncated normal distribution, Applied Mathematics and Computation, vol 4, pp. 149-156

## See Also

[ExtDist](#) for other standard distributions.

## Examples

```
# Parameter estimation for a distribution with known shape parameters
X <- rNormal_trunc_ab(n= 500, mu= 2, sigma = 5, a = 1, b = 2)
est.par <- eNormal_trunc_ab(X); est.par
plot(est.par)

# Fitted density curve and histogram
den.x <- seq(min(X),max(X),length=100)
den.y <- dNormal_trunc_ab(den.x,params = est.par)
hist(X, breaks=10, probability=TRUE, ylim = c(0,1.2*max(den.y)))
lines(den.x, den.y, col="blue")
lines(density(X), lty = 2)

# Extracting boundary and shape parameters
est.par[attributes(est.par)$par.type=="boundary"]
est.par[attributes(est.par)$par.type=="shape"]

# log-likelihood function
lNormal_trunc_ab(X,param = est.par)
```

---

SRTB\_ab

*The symmetric-reflected truncated beta (SRTB) distribution.*

---

## Description

Density, distribution, quantile, random number generation and parameter estimation functions for the SRTB distribution. Parameter estimation can be based on a weighted or unweighted i.i.d. sample and can be carried out numerically.

## Usage

```
dSRTB_ab(x, shape1 = 2, shape2 = 3, a = 0, b = 1,
  params = list(shape1, shape2, a, b), ...)

pSRTB_ab(q, shape1 = 2, shape2 = 3, a = 0, b = 1, params = list(shape1
  = 2, shape2 = 5, a = 0, b = 1), ...)
```

```
qSRTB_ab(p, shape1 = 2, shape2 = 3, a = 0, b = 1, params = list(shape1
= 2, shape2 = 5, a = 0, b = 1), ...)
```

```
rSRTB_ab(n, shape1 = 2, shape2 = 3, a = 0, b = 1,
params = list(shape1, shape2, a, b), ...)
```

```
eSRTB_ab(X, w, method = "numerical.MLE", ...)
```

```
lSRTB_ab(X, w, shape1 = 2, shape2 = 3, a = 0, b = 1,
params = list(shape1, shape2, a, b), logL = TRUE, ...)
```

### Arguments

x, q	A vector of quantiles.
shape1, shape2	Shape parameters.
a, b	Boundary parameters.
params	A list that includes all named parameters.
...	Additional parameters.
p	A vector of probabilities.
n	Number of observations.
X	Sample observations.
w	An optional vector of sample weights.
method	Parameter estimation method.
logL	logical; if TRUE, lSRTB_ab gives the log-likelihood, otherwise the likelihood is given.

### Details

No details as of yet.

### Value

dSRTB\_ab gives the density, pSRTB\_ab the distribution function, qSRTB\_ab gives the quantile function, rSRTB\_ab generates random variables, and eSRTB\_ab estimates the parameters. lSRTB\_ab provides the log-likelihood function and sSRTB\_ab the score function.

### Author(s)

Haizhen Wu.

### See Also

[ExtDist](#) for other standard distributions.

**Examples**

```
# Parameter estimation for a distribution with known shape parameters
X <- rSRTB_ab(n=500, shape1=2, shape2=10, a=1, b=2)
est.par <- eSRTB_ab(X)
plot(est.par)

# Extracting boundary and shape parameters
est.par[attributes(est.par)$par.type=="boundary"]
est.par[attributes(est.par)$par.type=="shape"]

# log-likelihood function
lSRTB_ab(X,param = est.par)
```

---

SSRTB

*The standard symmetric-reflected truncated beta (SSRTB) distribution.*


---

**Description**

Density, distribution, quantile, random number generation and parameter estimation functions for the SSRTB distribution. Parameter estimation can be based on a weighted or unweighted i.i.d sample and can be carried out numerically.

**Usage**

```
dSSRTB(x, shape1 = 2, shape2 = 3, params = list(shape1, shape2), ...)
pSSRTB(q, shape1 = 2, shape2 = 3, params = list(shape1, shape2), ...)
qSSRTB(p, shape1 = 2, shape2 = 3, params = list(shape1, shape2), ...)
rSSRTB(n, shape1 = 2, shape2 = 3, params = list(shape1, shape2), ...)
eSSRTB(X, w, method = "numerical.MLE", ...)
lSSRTB(X, w, shape1 = 2, shape2 = 3, params = list(shape1, shape2),
logL = TRUE, ...)
```

**Arguments**

x, q	A vector of quantiles.
shape1, shape2	Shape parameters.
params	A list that includes all named parameters.
...	Additional parameters.
p	A vector of probabilities.
n	Number of observations.
X	Sample observations.

w	An optional vector of sample weights.
method	Parameter estimation method.
logL	logical; if TRUE, lSSRTB gives the log-likelihood, otherwise the likelihood is given.

### Details

No details as of yet.

### Value

dSSRTB gives the density, pSSRTB the distribution function, qSSRTB the quantile function, rSSRTB generates random variables, eSSRTB estimates the parameters and lSSRTB provides the log-likelihood.

### Author(s)

Haizhen Wu.

### See Also

[ExtDist](#) for other standard distributions.

### Examples

```
# Parameter estimation for a distribution with known shape parameters
X <- rSSRTB(n=500, shape1=2, shape2=10)
est.par <- eSSRTB(X); est.par
plot(est.par)

# Fitted density curve and histogram
den.x <- seq(min(X),max(X),length=100)
den.y <- dSSRTB(den.x,shape1=est.par$shape1,shape2=est.par$shape2)
hist(X, breaks=10, probability=TRUE, ylim = c(0,1.2*max(den.y)))
lines(den.x, den.y, col="blue")
lines(density(X), lty=2)

# Extracting shape parameters
est.par[attributes(est.par)$par.type=="shape"]

# log-likelihood function
lSSRTB(X,param = est.par)
```

Triangular

*The Triangular Distribution.***Description**

Density, distribution, quantile, random number generation and parameter estimation functions for the triangular distribution with support  $[a, b]$  and shape parameter  $\theta$ . Parameter estimation can be based on a weighted or unweighted i.i.d. sample and can be performed numerically.

**Usage**

```
dTriangular(x, a = 0, b = 1, theta = 0.5, params = list(a, b, theta),
  ...)
```

```
pTriangular(q, a = 0, b = 1, theta = 0.5, params = list(a, b, theta),
  ...)
```

```
qTriangular(p, a = 0, b = 1, theta = 0.5, params = list(a, b, theta),
  ...)
```

```
rTriangular(n, a = 0, b = 1, theta = 0.5, params = list(a, b, theta),
  ...)
```

```
eTriangular(X, w, method = "numerical.MLE", ...)
```

```
lTriangular(X, w, a = 0, b = 1, theta = 0.5, params = list(a, b, theta),
  logL = TRUE, ...)
```

**Arguments**

<code>x, q</code>	A vector of quantiles.
<code>a, b</code>	Boundary parameters.
<code>theta</code>	Shape parameters.
<code>params</code>	A list that includes all named parameters.
<code>...</code>	Additional parameters.
<code>p</code>	A vector of probabilities.
<code>n</code>	Number of observations.
<code>X</code>	Sample observations.
<code>w</code>	An optional vector of sample weights.
<code>method</code>	Parameter estimation method.
<code>logL</code>	logical, it is assumed that the log-likelihood is desired. Set to FALSE if the likelihood is wanted.

### Details

If  $a$ ,  $b$  or  $\theta$  are not specified they assume the default values of 0, 1 and 0.5 respectively.

The `dTriangle()`, `pTriangle()`, `qTriangle()`, and `rTriangle()` functions serve as wrappers of the `dtriangle`, `ptriangle`, `qtriangle`, and `rtriangle` functions in the **VGAM** package. They allow for the parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The triangular distribution has a probability density function, defined in Forbes et.al (2010), that consists of two lines joined at  $\theta$ , where  $\theta$  is the location of the mode.

### Value

`dTriangular` gives the density, `pTriangular` the distribution function, `qTriangular` the quantile function, `rTriangular` generates random variables, and `eTriangular` estimates the parameters. `lTriangular` provides the log-likelihood function.

### Author(s)

Haizhen Wu and A. Jonathan R. Godfrey.  
Updates and bug fixes by Sarah Pirikahu.

### References

Kotz, S. and van Dorp, J. R. (2004). Beyond Beta: Other Continuous Families of Distributions with Bounded Support and Applications. Chapter 1. World Scientific: Singapore.

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2010) Triangular Distribution, in Statistical Distributions, Fourth Edition, John Wiley & Sons, Inc., Hoboken, NJ, USA.

### See Also

[ExtDist](#) for other standard distributions.

---

Uniform

*The Uniform Distribution.*

---

### Description

Density, distribution, quantile, random number generation and parameter estimation functions for the uniform distribution on the interval  $[a, b]$ . Parameter estimation can be based on an unweighted i.i.d. sample only and can be performed analytically or numerically.

**Usage**

```

dUniform(x, a = 0, b = 1, params = list(a, b), ...)

pUniform(q, a = 0, b = 1, params = list(a, b), ...)

qUniform(p, a = 0, b = 1, params = list(a, b), ...)

rUniform(n, a = 0, b = 1, params = list(a, b), ...)

eUniform(X, w, method = c("analytic.MLE", "moments", "numerical.MLE"), ...)

lUniform(X, w, a = 0, b = 1, params = list(a, b), logL = TRUE, ...)

```

**Arguments**

x,q	A vector of quantiles.
a,b	Boundary parameters.
params	A list that includes all named parameters.
...	Additional parameters.
p	A vector of probabilities.
n	Number of observations.
X	Sample observations.
w	An optional vector of sample weights.
method	Parameter estimation method.
logL	logical;if TRUE, lUniform gives the log-likelihood, otherwise the likelihood is given.

**Details**

If a or b are not specified they assume the default values of 0 and 1, respectively.

The dUniform(), pUniform(), qUniform(),and rUniform() functions serve as wrappers of the standard `dunif`, `punif`, `qunif`, and `runif` functions in the `stats` package. They allow for the parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The uniform distribution has probability density function

$$p_x(x) = 1/(b - a)$$

for  $a \leq x \leq b$ . The analytic maximum likelihood parameter estimates are as given by [Engineering Statistics Handbook](#). The method of moments parameter estimation option is also available and the estimates are as given by Forbes et.al (2011), p.179.

The log-likelihood function for the uniform distribution is given by

$$l(a, b|x) = -n \log(b - a)$$

**Value**

dUniform gives the density, pUniform the distribution function, qUniform the quantile function, rUniform generates random deviates, and eUniform estimates the parameters. lUniform provides the log-likelihood function.

**Note**

The analytical maximum likelihood estimation of the parameters  $a$  and  $b$  is calculated using the range and mid-range of the sample. Therefore, only unweighted samples are catered for in the eUniform distribution when the method `analytic.MLE` is selected.

**Author(s)**

Haizhen Wu and A. Jonathan R. Godfrey.  
Updates and bugfixes by Sarah Pirikahu.

**References**

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, volume 2, chapter 26, Wiley, New York.

**Engineering Statistics Handbook**

Forbes, C. Evans, M. Hastings, N. & Peacock, B. (2011) Statistical Distributions, 4th Ed, chapter 40, Wiley, New Jersey.

**See Also**

[ExtDist](#) for other standard distributions.

**Examples**

```
# Parameter estimation for a distribution with known shape parameters
X <- rUniform(n=500, a=0, b=1)
est.par <- eUniform(X, method="analytic.MLE"); est.par
plot(est.par)

# Histogram and fitted density
den.x <- seq(min(X),max(X),length=100)
den.y <- dUniform(den.x,a=est.par$a,b=est.par$b)
hist(X, breaks=10, probability=TRUE, ylim = c(0,1.2*max(den.y)))
lines(den.x, den.y, col="blue") # Original data
lines(density(X), lty=2)      # Fitted curve

# Extracting boundary parameters
est.par[attributes(est.par)$par.type=="boundary"]

# log-likelihood
lUniform(X,param = est.par)
```



```
# Example of parameter estimation for a distribution with
# unknown parameters currently been sought after.
```

---

Weibull

*The Weibull Distribution.*


---

### Description

Density, distribution, quantile, random number generation, and parameter estimation functions for the Weibull distribution with parameters shape and scale. Parameter estimation can be based on a weighted or unweighted i.i.d sample and can be carried out analytically or numerically.

### Usage

```
dWeibull(x, shape = 2, scale = 2, params = list(shape = 2, scale = 2))
pWeibull(q, shape = 2, scale = 2, params = list(shape = 2, scale = 2))
qWeibull(p, shape = 2, scale = 2, params = list(shape = 2, scale = 2))
rWeibull(n, shape = 2, scale = 2, params = list(shape = 2, scale = 2))
eWeibull(X, w, method = c("numerical.MLE", "moments"), ...)
lWeibull(X, w, shape = 2, scale = 2, params = list(shape = 2, scale = 2),
logL = TRUE)
```

### Arguments

x, q	A vector of quantiles.
shape	Shape parameter.
scale	Scale parameter.
params	A list that includes all named parameters
p	A vector of probabilities.
n	Number of observations.
X	Sample observations.
w	An optional vector of sample weights.
method	Parameter estimation method.
...	Additional parameters.
logL	logical; if TRUE, lWeibull gives the log-likelihood, otherwise the likelihood is given.

### Details

The Weibull distribution is a special case of the generalised gamma distribution. The `dWeibull()`, `pWeibull()`, `qWeibull()`, and `rWeibull()` functions serve as wrappers of the standard `dgamma`, `pgamma`, `qgamma`, and `rgamma` functions with in the **stats** package. They allow for the parameters to be declared not only as individual numerical values, but also as a list so parameter estimation can be carried out.

The Weibull distribution with parameters  $\text{shape}=a$  and  $\text{scale}=b$  has probability density function,

$$f(x) = (a/b)(x/b)^{a-1} \exp(-(x/b)^a)$$

for  $x > 0$ . Parameter estimation can be carried out using the method of moments as done by Winston (2003) or by numerical maximum likelihood estimation.

The log-likelihood function of the Weibull distribution is given by

$$l(a, b|x) = n(\ln a - \ln b) + (a - 1) \sum \ln(xi/b) - \sum (xi/b)^a$$

The score function and information matrix are as given by Rinne (p.412).

### Value

`dWeibull` gives the density, `pWeibull` the distribution function, `qWeibull` the quantile function, `rWeibull` generates random deviates, and `eWeibull` estimates the distribution parameters. `lWeibull` provides the log-likelihood function.

### Author(s)

Haizhen Wu and A. Jonathan R. Godfrey.  
Updates and bug fixes by Sarah Pirikahu.

### References

- Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, volume 1, chapter 21, Wiley, New York.
- Rinne, H. (2009) The Weibull Distribution A Handbook, chapter 11, Chapman & Hall/CRC.
- Winston, W.L (2003) Operations Research: Applications and algorithms, 4th Ed, Duxbury.

### See Also

[ExtDist](#) for other standard distributions.

### Examples

```
# Parameter estimation for a distribution with known shape parameters
X <- rWeibull(n=1000, params=list(shape=1.5, scale=0.5))
est.par <- eWeibull(X=X, method="numerical.MLE"); est.par
plot(est.par)
```

```

# Fitted density curve and histogram
den.x <- seq(min(X),max(X),length=100)
den.y <- dWeibull(den.x,shape=est.par$shape,scale=est.par$scale)
hist(X, breaks=10, col="red", probability=TRUE, ylim = c(0,1.1*max(den.y)))
lines(den.x, den.y, col="blue", lwd=2) # Original data
lines(density(X), lty=2) # Fitted curve

# Extracting shape and scale parameters
est.par[attributes(est.par)$par.type=="shape"]
est.par[attributes(est.par)$par.type=="scale"]

# Parameter Estimation for a distribution with unknown shape parameters
# Example from: Rinne (2009) Dataset p.338 and example pp.418-419
# Parameter estimates are given as shape = 99.2079 and scale = 2.5957. The log-likelihood
# for this data and Rinne's parameter estimates is -1163.278.
data <- c(35,38,42,56,58,61,63,76,81,83,86,90,99,104,113,114,117,119,141,183)
est.par <- eWeibull(X=data, method="numerical.MLE"); est.par
plot(est.par)

# Estimates calculated by eWeibull differ from those given by Rinne(2009).
# However, eWeibull's parameter estimates appear to be an improvement, due to a larger
# log-likelihood of -99.09037 (as given by lWeibull below).

# log-likelihood function
lWeibull(data, param = est.par)

# evaluate the precision of estimation by Hessian matrix
H <- attributes(est.par)$nll.hessian
var <- solve(H)
se <- sqrt(diag(var));se

```

---

wmlc

*Weighted Maximum Likelihood Estimation.*


---

## Description

A general weighted maximum likelihood estimation function.

## Usage

```
wmlc(X, w, distname, initial, lower, upper, loglik.fn, score.fn, obs.info.fn)
```

## Arguments

X	Sample observations.
w	Frequency (or weights) of observation.
distname	Name of distribution to be estimated.
initial	Initial value of the parameters.

lower	The lower bound of the parameters.
upper	The upper bound of the parameters.
loglik.fn	Function to compute (weighted) log likelihood.
score.fn	Function to compute (weighted) score.
obs.info.fn	Function to compute observed information matrix.

**Details**

Weighted Maximum Likelihood Estimation

**Value**

weighted mle estimates.

**Author(s)**

Haizhen Wu and A. Jonathan R. Godfrey

# Index

AIC.eDist (eDist), 14  
AICc (eDist), 14

bestDist, 3  
Beta, 5  
beta, 6, 8  
Beta\_ab, 7  
BIC (eDist), 14  
Burr, 10

compareDist, 13

dBeta (Beta), 5  
dbeta, 6, 8  
dBeta\_ab (Beta\_ab), 7  
dBurr (Burr), 10  
dExp (Exponential), 17  
dexp, 18  
dGamma (Gamma), 19  
dgamma, 20, 50  
dGumbel (Gumbel), 22  
dgumbel, 23  
DistSelCriteria, 13  
dJohnson, 26  
dJohnsonSB (JohnsonSB), 24  
dJohnsonSB\_ab (JohnsonSB), 24  
dJohnsonSU (JohnsonSU), 27  
dLaplace (Laplace), 30  
dlogis, 33  
dLogistic (Logistic), 32  
dnorm, 36  
dNormal (Normal), 35  
dNormal\_sym\_trunc\_ab  
    (Normal\_sym\_trunc\_ab), 38  
dNormal\_trunc\_ab (Normal\_trunc\_ab), 39  
dparetoIV, 11  
dSRTB\_ab (SRTB\_ab), 41  
dSSRTB (SSRTB), 43  
dtriangle, 46  
dTriangular (Triangular), 45

dtrunc, 40  
dunif, 47  
dUniform (Uniform), 46  
dWeibull (Weibull), 49

eBeta (Beta), 5  
eBeta\_ab (Beta\_ab), 7  
eBurr (Burr), 10  
eDist, 14  
eExp (Exponential), 17  
eGamma (Gamma), 19  
eGumbel (Gumbel), 22  
eJohnsonSB (JohnsonSB), 24  
eJohnsonSU (JohnsonSU), 27  
eLaplace (Laplace), 30  
eLogistic (Logistic), 32  
eNormal (Normal), 35  
eNormal\_sym\_trunc\_ab  
    (Normal\_sym\_trunc\_ab), 38  
eNormal\_trunc\_ab (Normal\_trunc\_ab), 39  
eSRTB\_ab (SRTB\_ab), 41  
eSSRTB (SSRTB), 43  
eTriangular (Triangular), 45  
eUniform (Uniform), 46  
eval.estimation, 16  
eWeibull (Weibull), 49  
Exponential, 17  
ExtDist, 6, 9, 11, 21, 24, 26, 29, 32, 34, 37,  
    39, 41, 42, 44, 46, 48, 50  
ExtDist (ExtDist-package), 2  
ExtDist-package, 2

Gamma, 19  
gamma, 20  
Gumbel, 22

iBeta (Beta), 5  
iExp (Exponential), 17  
iNormal (Normal), 35  
JohnsonSB, 24

- JohnsonSU, 27
- Laplace, 30
- lBeta (Beta), 5
- lBeta\_ab (Beta\_ab), 7
- lBurr (Burr), 10
- lExp (Exponential), 17
- lGamma (Gamma), 19
- lGumbel (Gumbel), 22
- lJohnsonSB (JohnsonSB), 24
- lJohnsonSU (JohnsonSU), 27
- lLaplace (Laplace), 30
- lLogistic (Logistic), 32
- lNormal (Normal), 35
- lNormal\_sym\_trunc\_ab  
(Normal\_sym\_trunc\_ab), 38
- lNormal\_trunc\_ab (Normal\_trunc\_ab), 39
- Logistic, 32
- logLik.eDist (eDist), 14
- lSRTB\_ab (SRTB\_ab), 41
- lSSRTB (SSRTB), 43
- lTriangular (Triangular), 45
- lUniform (Uniform), 46
- lWeibull (Weibull), 49
- MDL (eDist), 14
- Normal, 35
- Normal\_sym\_trunc\_ab, 38
- Normal\_trunc\_ab, 39, 39
- pBeta (Beta), 5
- pbeta, 6, 8
- pBeta\_ab (Beta\_ab), 7
- pBurr (Burr), 10
- pExp (Exponential), 17
- pexp, 18
- pGamma (Gamma), 19
- pgamma, 20, 50
- pGumbel (Gumbel), 22
- pgumbel, 23
- pJohnson, 26
- pJohnsonSB (JohnsonSB), 24
- pJohnsonSU (JohnsonSU), 27
- pLaplace (Laplace), 30
- plogis, 33
- pLogistic (Logistic), 32
- pnorm, 36
- pNormal (Normal), 35
- pNormal\_sym\_trunc\_ab  
(Normal\_sym\_trunc\_ab), 38
- pNormal\_trunc\_ab (Normal\_trunc\_ab), 39
- pparetoIV, 11
- print.eDist (eDist), 14
- pSRTB\_ab (SRTB\_ab), 41
- pSSRTB (SSRTB), 43
- ptriangle, 46
- pTriangular (Triangular), 45
- ptrunc, 40
- punif, 47
- pUniform (Uniform), 46
- pWeibull (Weibull), 49
- qBeta (Beta), 5
- qbeta, 6, 8
- qBeta\_ab (Beta\_ab), 7
- qBurr (Burr), 10
- qExp (Exponential), 17
- qexp, 18
- qGamma (Gamma), 19
- qgamma, 20, 50
- qGumbel (Gumbel), 22
- qgumbel, 23
- qJohnson, 26
- qJohnsonSB (JohnsonSB), 24
- qJohnsonSU (JohnsonSU), 27
- qLaplace (Laplace), 30
- qlogis, 33
- qLogistic (Logistic), 32
- qnorm, 36
- qNormal (Normal), 35
- qNormal\_sym\_trunc\_ab  
(Normal\_sym\_trunc\_ab), 38
- qNormal\_trunc\_ab (Normal\_trunc\_ab), 39
- qparetoIV, 11
- qSRTB\_ab (SRTB\_ab), 41
- qSSRTB (SSRTB), 43
- qtriangle, 46
- qTriangular (Triangular), 45
- qtrunc, 40
- qunif, 47
- qUniform (Uniform), 46
- qWeibull (Weibull), 49
- rBeta (Beta), 5
- rbeta, 6, 8
- rBeta\_ab (Beta\_ab), 7

rBurr (Burr), 10  
rExp (Exponential), 17  
rexp, 18  
rGamma (Gamma), 19  
rgamma, 20, 50  
rGumbel (Gumbel), 22  
rgumbel, 23  
rJohnson, 26  
rJohnsonSB (JohnsonSB), 24  
rJohnsonSU (JohnsonSU), 27  
rLaplace (Laplace), 30  
rlogis, 33  
rLogistic (Logistic), 32  
rnorm, 36  
rNormal (Normal), 35  
rNormal\_sym\_trunc\_ab  
    (Normal\_sym\_trunc\_ab), 38  
rNormal\_trunc\_ab (Normal\_trunc\_ab), 39  
rparetoIV, 11  
rSRTB\_ab (SRTB\_ab), 41  
rSSRTB (SSRTB), 43  
rtriangle, 46  
rTriangular (Triangular), 45  
rtrunc, 40  
runif, 47  
rUniform (Uniform), 46  
rWeibull (Weibull), 49

sBeta (Beta), 5  
sBeta\_ab (Beta\_ab), 7  
sExp (Exponential), 17  
sNormal (Normal), 35  
SRTB\_ab, 41  
SSRTB, 43  
stats, 6, 8, 18, 20, 33, 36, 47, 50

Triangular, 45

Uniform, 46

vcov.eDist (eDist), 14

Weibull, 49  
wml, 51