

Package ‘contfrac’

May 26, 2016

Title Continued Fractions

Version 1.1-10

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Description Various utilities for evaluating continued fractions.

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License GPL-2

NeedsCompilation yes

Repository CRAN

Date/Publication 2016-05-26 09:21:11

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contfrac-package	<i>Continued fractions</i>
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Description

Various utilities for manipulating continued fractions

Details

Package: contfrac
Type: Package
Version: 1.0
Date: 2008-04-04
License: GPL

Author(s)

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References

- W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling 1992. *Numerical recipes 3rd edition: the art of scientific computing*. Cambridge University Press; section 5.2 “Evaluation of continued fractions”
- W. J. Lenz 1976. Generating Bessel functions in Mie scattering calculations using continued fractions. *Applied Optics*, 15(3):668-671

Examples

```
# approximate real numbers with continued fraction:
as_cf(pi)

as_cf(exp(1),25)    # OK up to element 21 (which should be 14)

# Some convergents of pi:
jj <- convergents(c(3,7,15,1,292))
jj$A / jj$B - pi

# An identity of Euler's:
jj <- GCF(a=seq(from=2,by=2,len=30), b=seq(from=3,by=2,len=30), b0=1)
jj - 1/(exp(0.5)-1)  # should be small

# Now a continued fraction representation of tan(z):
tan_cf <- function(z,n=14){ GCF(c(z,rep(-z^2,n-1)), seq(from=1,by=2,len=n)) }

tan_cf(1+1i) - tan(1+1i)  # should be small
```

as_cf

Approximates a real number in continued fraction form

Description

Approximates a real number in continued fraction form using a standard simple algorithm

Usage

```
as_cf(x, n = 10)
```

Arguments

x real number to be approximated in continued fraction form
n Number of partial denominators to evaluate; see Notes

Note

Has difficulties with rational values as expected

Author(s)

Robin K. S. Hankin

See Also

[CF,convergents](#)

Examples

```
phi <- (sqrt(5)+1)/2
as_cf(phi,50) # loses it after about 38 iterations ... not bad ...

as_cf(pi) # looks about right
as_cf(exp(1),20)

f <- function(x){CF(as_cf(x,30),TRUE) - x}

x <- runif(40)
plot(sapply(x,f))
```

CF

Continued fraction convergent

Description

Returns continued fraction convergent using the modified Lenz's algorithm; function CF() deals with continued fractions and GCF() deals with generalized continued fractions.

Usage

```
CF(a, finite = FALSE, tol=0)
GCF(a,b, b0=0, finite = FALSE, tol=0)
```

Arguments

a,b	In function CF(), the elements of a are the partial denominators; in GCF() the elements of a are the partial numerators and the elements of b the partial denominators
finite	Boolean, with default FALSE meaning to iterate Lenz's algorithm until convergence (a warning is given if the sequence has not converged); and TRUE meaning to evaluate the finite continued fraction
b0	In function GCF(), floor of the continued fraction
tol	tolerance, with default 0 silently replaced with <code>.Machine\$double.eps</code>

Details

Function CF() treats the first element of its argument as the integer part of the convergent.

Function CF() is a wrapper for GCF(); it includes special dispensation for infinite values (in which case the value of the appropriate finite CF is returned).

The implementation is in C; the real and complex cases are treated separately in the interests of efficiency.

The algorithm terminates when the convergence criterion is achieved irrespective of the value of finite.

Author(s)

Robin K. S. Hankin

References

- W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling 1992. *Numerical recipes 3rd edition: the art of scientific computing*. Cambridge University Press; section 5.2 "Evaluation of continued fractions"
- W. J. Lenz 1976. Generating Bessel functions in Mie scattering calculations using continued fractions. *Applied Optics*, 15(3):668-671

See Also

[convergents](#)

Examples

```
phi <- (sqrt(5)+1)/2
phi_cf <- CF(rep(1,100)) # phi = [1;1,1,1,1,1,...]
phi - phi_cf # should be small

# The tan function:

"tan_cf" <-
function(z,n=20){GCF(c(z, rep(-z^2,n-1)), seq(from=1,by=2, len=n)) }
```

```
z <- 1+1i
tan(z) - tan_cf(z) # should be small
```

convergents

Partial convergents of continued fractions

Description

Partial convergents of continued fractions or generalized continued fractions

Usage

```
convergents(a)
gconvergents(a,b, b0 = 0)
```

Arguments

a, b In function `convergents()`, the elements of `a` are the partial denominators (the first element of `a` is the integer part of the continued fraction). In `gconvergents()` the elements of `a` are the partial numerators and the elements of `b` the partial denominators

b0 The floor of the fraction

Details

Function `convergents()` returns partial convergents of the continued fraction

$$a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \frac{1}{a_4 + \frac{1}{a_5 + \ddots}}}}}$$

where $a = a_0, a_1, a_2, \dots$ (note the off-by-one issue).

Function `gconvergents()` returns partial convergents of the continued fraction

$$b_0 + \frac{a_1}{b_1 + \frac{a_2}{b_2 + \frac{a_3}{b_3 + \frac{a_4}{b_4 + \frac{a_5}{b_5 + \ddots}}}}}$$

where $a = a_1, a_2, \dots$

Value

Returns a list of two elements, A for the numerators and B for the denominators

Note

This classical algorithm generates very large partial numerators and denominators. To evaluate limits, use functions `CF()` or `GCF()`.

Author(s)

Robin K. S. Hankin

References

W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling 1992. *Numerical recipes 3rd edition: the art of scientific computing*. Cambridge University Press; section 5.2 “Evaluation of continued fractions”

See Also

[CF](#)

Examples

```
# Successive approximations to pi:

jj <- convergents(c(3,7,15,1,292))
jj$A/jj$B - pi      # should get smaller

convergents(rep(1,10))
```

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