

Package ‘Directional’

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Description A collection of functions for directional data analysis. Hypothesis testing, discriminant and regression analysis, MLE of distributions and more are included. The standard textbook for such data is the “Directional Statistics” by Mardia, K. V. and Jupp, P. E. (2000).

License GPL-2

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Directional-package *This is an R package that provides methods for the statistical analysis of Directional data.*

Description

Circular-linear regression, spherical-spherical regression, discriminant analysis, ANOVA for circular and (hyper-)spherical data, tests for equality of concentration parameters, fitting distributions, random values generation, contour plots and many more are included in this package.

Details

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Maintainers

Michail Tsagris <mtsagris@yahoo.gr>

Note

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Author(s)

Michail Tsagris <mtsagris@yahoo.gr>, Giorgos Athineou <athineou@csd.uoc.gr> and Anamul Sajib <pmxahsa@nottingham.ac.uk>

References

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chichester: John Wiley and Sons.

A test for testing the equality of the concentration parameters for circular data
A test for testing the equality of the concentration parameter among g samples, where $g \geq 2$ for circular data

Description

A test for testing the equality of the concentration parameter among g samples, where $g \geq 2$ for circular data. It is a tangential approach.

Usage

```
tang.conc(u, ina, rads = FALSE)
```

Arguments

u	A numeric vector containing the values of all samples.
ina	A numerical variable or factor indicating the groups of each value.
rads	If the data are in radians this should be TRUE and FALSE otherwise.

Details

This test works for circular data.

Value

A vector including:

test	The value of the test statistic.
p-value	The p-value of the test.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chicester: John Wiley & Sons. Fisher, N. I. (1995). Statistical analysis of circular data. Cambridge University Press.

See Also

[embed.circaov](#), [hcf.circaov](#), [lr.circaov](#), [het.circaov](#), [conc.test](#)

Examples

```
x <- rvonmises(100, 2.4, 15)
ina <- rep(1:4,each = 25)
tang.conc(x, ina, rads = TRUE)
```

Angular central Gaussian random values simulation

Angular central Gaussian random values simulation

Description

Angular central Gaussian random values simulation.

Usage

```
racg(n, sigma)
```

Arguments

n	The sample size, a numerical value.
sigma	The covariance matrix in R^d .

Details

The algorithm uses univariate normal random values and transforms them to multivariate via a spectral decomposition. The vectors are then scaled to have unit length.

Value

A matrix with the simulated data.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr>

References

Tyler D. E. (1987). Statistical analysis for the angular central Gaussian distribution on the sphere. *Biometrika* 74(3): 579-589.

See Also

[acg](#), [rvmf](#), [rvonmises](#)

Examples

```
s <- cov( iris[, 1:4] )
x <- racg(100, s)
acg(x)
vmf(x) ## the concentration parameter, kappa, is very low, close to zero, as expected.
```

Anova for (hyper-)spherical data
Analysis of variance for (hyper-)spherical data

Description

Analysis of variance for (hyper-)spherical data.

Usage

```
hcf.aov(x, ina, fc = TRUE)
```

```
lr.aov(x, ina)
```

```
embed.aov(x, ina)
```

```
het.aov(x, ina)
```

Arguments

x	A matrix with the data in Euclidean coordinates, i.e. unit vectors.
ina	A numerical variable or a factor indicating the group of each vector.
fc	A boolean that indicates whether a corrected F test should be used or not.

Details

The high concentration (hcf.aov), log-likelihood ratio (lr.aov), embedding approach (embed.aov) or the non equal concentration parameters approach (het.aov) is used.

Value

A vector including:

test	The test statistic value.
p-value	The p-value of the F test.
kappa	The common concentration parameter kappa based on all the data.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chichester: John Wiley & Sons.

See Also

[lr.aov](#), [embed.aov](#), [het.aov](#), [spherconc.test](#), [conc.test](#)

Examples

```
x <- rvmf(60, rnorm(3), 15)
ina <- rep(1:3, each = 20)
hcf.aov(x, ina)
hcf.aov(x, ina, fc = FALSE)
lr.aov(x, ina)
embed.aov(x, ina)
het.aov(x, ina)
```

Anova for circular data

Analysis of variance for circular data

Description

Analysis of variance for circular data.

Usage

```
hcf.circaov(u, ina, rads = FALSE)

lr.circaov(u, ina, rads = FALSE)

het.circaov(u, ina, rads = FALSE)

embed.circaov(u, ina, rads = FALSE)
```

Arguments

<code>u</code>	A numeric vector containing the data.
<code>ina</code>	A numerical or factor variable indicating the group of each value.
<code>rads</code>	If the data are in radians, this should be TRUE and FALSE otherwise.

Details

The high concentration (`hcf.circaov`), log-likelihood ratio (`lr.circaov`), embedding approach (`embed.circaov`) or the non equal concentration parameters approach (`het.circaov`) is used.

Value

A vector including:

test	The value of the test statistic.
p-value	The p-value of the test.
kappa	The concentration parameter based on all the data. If the <code>het.circaov</code> is used this argument is not returned.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chichester: John Wiley & Sons.

See Also

[conc.test](#), [hcf.aov](#), [lr.aov](#), [het.aov](#), [embed.aov](#)

Examples

```
x <- rvonmises(100, 2.4, 15)
ina <- rep(1:4,each = 25)
hcf.circaov(x, ina, rads = TRUE)
lr.circaov(x, ina, rads = TRUE)
het.circaov(x, ina, rads = TRUE)
embed.circaov(x, ina, rads = TRUE)
```

BIC for the model based clustering using mixtures of von Mises-Fisher distributions
*BIC to choose the number of components in a model based clustering
using mixtures of von Mises-Fisher distributions*

Description

BIC to choose the number of components in a model based clustering using mixtures of von Mises-Fisher distributions

Usage

```
bic.mixvmf(x, A)
```

Arguments

x	A matrix containing directional data.
A	The maximum number of clusters to be tested. Default value is 3.

Details

If the data are not unit vectors, they are transformed into unit vectors.

Value

A list including:

BIC	The BIC values for all the models tested.
A plot	A plot of the BIC values.
runtime	The run time of the algorithm. A numeric vector. The first element is the user time, the second element is the system time and the third element is the elapsed time.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Hornik, K. and Grun, B. (2014). movMF: An R package for fitting mixtures of von Mises-Fisher distributions. *Journal of Statistical Software*, 58(10):1–31.

See Also

[mix.vmf](#), [rmixvmf](#), [mixvmf.contour](#)

Examples

```
x <- as.matrix( iris[, 1:4] )
x <- x / sqrt( rowSums(x^2) )
bic.mixvmf(x, 5)
```

Check visually whether matrix Fisher samples is correctly generated or not
Check visually whether matrix Fisher samples is correctly generated or not.

Description

It plots the log probability trace of matrix Fisher distribution which should close to the maximum value of the logarithm of matrix Fisher distribution, if samples are correctly generated.

Usage

```
visual.check(x, Fa)
```

Arguments

`x` The simulated data. An array with at least 2 3x3 matrices.
`Fa` An arbitrary 3x3 matrix represents the parameter matrix of this distribution.

Details

For a given parameter matrix Fa , maximum value of the logarithm of matrix Fisher distribution is calculated via the form of singular value decomposition of $Fa = U\Lambda V^T$ which is $tr(\Lambda)$. Multiply the last column of U by -1 and replace small eigenvalue, say, λ_3 by $-\lambda_3$ if $|UV^T| = -1$.

Value

A plot which shows log probability trace of matrix Fisher distribution. The values are also returned.

Author(s)

Anamul Sajib<pmxahsa@nottingham.ac.uk>

R implementation and documentation: Anamul Sajib<pmxahsa@nottingham.ac.uk>

References

Habeck M. (2009). Generation of three-dimensional random rotations in fitting and matching problems. *Computational Statistics*, 24(4):719–731.

Examples

```
Fa <- matrix( c(85, 11, 41, 78, 39, 60, 43, 64, 48), ncol = 3) / 10  
x <- rmatrixfisher(1000, Fa)  
a <- visual.check(x, Fa)
```

Circular or angular regression
Circular or angular regression

Description

Regression with circular dependent variable and Euclidean or categorical independent variables.

Usage

```
spml.reg(y, x, rads = TRUE, xnew = NULL, seb = TRUE, tol = 1e-07)
```

Arguments

y	The dependent variable, a numerical vector, it can be in radians or degrees.
x	The independent variable(s). Can be Euclidean or categorical (factor variables).
rads	If the dependent variable is expressed in rads, this should be TRUE and FALSE otherwise.
xnew	The new values of some independent variable(s) whose circular values you want to predict. Can be Euclidean or categorical. If they are categorical, the user must provide them as dummy variables. It does not accept factor variables. If you have no new x values, leave it NULL (default).
seb	a boolean variable. If TRUE, the standard error of the coefficients will be returned. Set to FALSE in case of simulation studies or in other cases such as a forward regression setting for example. In these cases, it can save some time.
tol	The tolerance value to terminate the Newton-Raphson algorithm.

Details

The Newton-Raphson algorithm is fitted in this regression as described in Presnell et al. (1998).

Value

A list including:

runtime	The runtime of the procedure.
iters	The number of iterations required until convergence of the EM algorithm.
beta	The regression coefficients.
seb	The standard errors of the coefficients.
loglik	The value of the maximised log-likelihood.
est	The fitted values expressed in radians if the observed data are in radians and in degrees otherwise. If xnew is not NULL, i.e. if you have new x values, then the predicted values of y will be returned.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Presnell Brett, Morrison Scott P. and Littell Ramon C. (1998). Projected multivariate linear models for directional data. *Journal of the American Statistical Association*, 93(443): 1068-1077.

See Also

[circlin.cor](#), [circ.cor1](#), [circ.cor2](#), [spher.cor](#), [spher.reg](#)

Examples

```
x <- rnorm(100)
z <- cbind(3 + 2 * x, 1 -3 * x)
y <- cbind( rnorm(100,z[,1], 1), rnorm(100, z[,2], 1) )
y <- y / sqrt( rowSums(y^2) )
y <- ( atan( y[, 2] / y[, 1] ) + pi * I(y[, 1] < 0) ) %%( 2 * pi)
spml.reg(y, x, rads = TRUE)
```

Circular-linear correlation

Circular-linear correlation

Description

It calculates the squared correlation between a circular and one or more linear variables.

Usage

```
circlin.cor(theta, x, rads = FALSE)
```

Arguments

theta	The circular variable.
x	The linear variable or a matrix containing many linear variables.
rads	If the circular variable is in rads, this should be TRUE and FALSE otherwise.

Details

The squared correlation between a circular and one or more linear variables is calculated.

Value

A matrix with as many rows as linear variables including:

R-squared	The value of the squared correlation.
p-value	The p-value of the zero correlation hypothesis testing.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chichester: John Wiley & Sons.

See Also

[circ.cor1](#), [circ.cor2](#), [spml.reg](#)

Examples

```
phi <- rvonmises(50, 2, 20, rads = TRUE)
x <- 2 * phi + rnorm(50)
y <- matrix(rnorm(50 * 5), ncol = 5)
circlin.cor(phi, x, rads = TRUE)
circlin.cor(phi, y, rads = TRUE)
```

Circular correlations between two circular variables

Circular correlations between two circular variables

Description

Circular correlations between two circular variables.

Usage

```
circ.cor1(theta, phi, rads = FALSE)
```

```
circ.cor2(theta, phi, rads = FALSE)
```

Arguments

theta	The first circular variable.
phi	The other circular variable.
rads	If the data are expressed in rads, then this should be TRUE. If the data are in degrees, then this is FALSE.

Details

circ.cor1: Correlation for circular variables using the cosinus and sinus formula of Jammaladaka and SenGupta (1988).

circ.cor2: Correlation for circular variables using the cosinus and sinus formula of Mardia and Jupp (2000).

Value

A vector including:

rho The value of the correlation coefficient.

p-value The p-value of the zero correlation hypothesis testing.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Jammalamadaka, R. S. and Sengupta, A. (2001). Topics in circular statistics. World Scientific.

Jammalamadaka, S. R. and Sarma, Y. R. (1988) . A correlation coefficient for angular variables. Statistical Theory and Data Analysis, 2:349–364.

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chicester: John Wiley & Sons.

See Also

[circlin.cor](#), [circ.cor2](#), [spml.reg](#)

Examples

```
y <- runif(50, 0, 2 * pi)
x <- runif(50, 0, 2 * pi)
circ.cor1(x, y, rads = TRUE)
circ.cor2(x, y, rads = TRUE)
```

Contour plot of a mixture of von Mises-Fisher distributions model

Contour plot of a mixture of von Mises-Fisher distributions model for spherical data only.

Description

Contour lines are produced of mixture model for spherical data only.

Usage

```
mixvmf.contour(u, mod)
```

Arguments

u	A two column matrix. The first column is the longitude and the second is the latitude.
mod	This is mix.vmf object. Run a mixture model and save it as mod for example, mod = mix.vmf(x, 3).

Details

The contour plot is displayed with latitude and longitude in the axes. No Lambert projection is used here. This works for spherical data only which are given as longitude and latitude.

Value

A plot including: The points and the contour lines.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kurt Hornik and Bettina Grun (2014). movMF: An R Package for Fitting Mixtures of von Mises-Fisher Distributions <http://cran.r-project.org/web/packages/movMF/vignettes/movMF.pdf>

See Also

[vmf.kerncontour](#), [vmf.contour](#), [mix.vmf](#)

Examples

```
k <- runif(2, 4, 20)
prob <- c(0.4, 0.6)
mu <- matrix( rnorm(6), ncol = 3 )
mu <- mu / sqrt( rowSums(mu^2) )
x <- rmixvmf(200, prob, mu, k)$x
mod <- mix.vmf(x, 2)
y <- euclid.inv(x)
mixvmf.contour(y, mod)
```

Contour plot of spherical data using a von Mises-Fisher kernel density estimate
Contour plot of spherical data using a von Mises-Fisher kernel density estimate

Description

Contour plot of spherical data using a von Mises-Fisher kernel density estimate.

Usage

```
vmf.kerncontour(u, thumb = "none")
```

Arguments

u	A two column matrix. The first column is the latitude and the second is the longitude.
thumb	This is either 'none' (default), or 'rot' for the rule of thumb suggested by Garcia-Portugues (2013). If it is "none" it is estimated via cross validation, with the fast function "vmfkde.tune_2".

Details

It calculates the contour plot using a von Mises-Fisher kernel for spherical data only.

Value

A plot with the data in latitude and longitude along with their kernel density estimate contour lines.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Garcia Portugues, E. (2013). Exact risk improvement of bandwidth selectors for kernel density estimation with directional data. *Electronic Journal of Statistics*, 7, 1655–1685.

See Also

[vmf.kde](#), [vmfkde.tune](#), [vmf.contour](#), [kent.datacontour](#)

Examples

```
x <- rvmf(100, rnorm(3), 15)
x <- euclid.inv(x)
par( mfrow = c(1, 2) )
vmf.kerncontour(x, "rot")
vmf.kerncontour(x, "none")
```

Contour plot of the Kent distribution for some data

Contour plot of the Kent distribution for some data

Description

The contour plot of the Kent distribution on the sphere for some data is produced.

Usage

```
kent.datacontour(x)
```

Arguments

x A two column matrix, where the first column is the latitude and the second column is the longitude. If the matrix has two columns, it is assumed to have unit vectors and in this case it is turned into latitude and longitude.

Details

MLE of the parameters of the Kent distribution are calculated, then the contour plot is plotted using these estimates and finally the data are also plotted.

Value

A plot containing the contours of the distribution along with the data.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kent John (1982). The Fisher-Bingham distribution on the sphere. Journal of the Royal Statistical Society, Series B, 44(1): 71-80.

See Also

[kent.contour](#), [kent.mle](#), [vmf.kerncontour](#)

Examples

```
x <- rvmf(100, rnorm(3), 10)
kent.mle(x)
y <- euclid.inv(x)
kent.datacontour(y)
vmf.kerncontour(y, thumb = "none")
```

Contour plot of the Kent distribution without any data

Contour plot of the Kent distribution without any data

Description

The contour plot of the Kent distribution on the sphere is produced. The user can see how the shape and ovalness change as he/she changes the ovalness parameter.

Usage

```
kent.contour(k, b)
```

Arguments

k	The concentration parameter.
b	The ovalness parameter. It has to be less than $k/2$ in order for the distribution to be unimodal. Otherwise it is bimodal.

Details

The goal of this function is for the user to see how the Kent distribution looks like.

Value

A plot containing the contours of the distribution.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kent John (1982). The Fisher-Bingham distribution on the sphere. *Journal of the Royal Statistical Society, Series B*, 44(1): 71-80.

See Also

[kent.datacontour](#), [kent.mle](#), [vmf.contour](#), [vmf.kerncontour](#)

Examples

```
par( mfrow = c(1, 2) )
kent.contour(10, 2)
kent.contour(10, 4)
```

Contour plots of the von Mises-Fisher distribution

Contour plots of the von Mises-Fisher distribution on the sphere

Description

Contour plots of the von Mises-Fisher distribution on the sphere.

Usage

```
vmf.contour(k)
```

Arguments

k The concentration parameter.

Details

The user specifies the concentration parameter only and not the mean direction or data. This is for illustration purposes only. The graph will always contain circles, as the von Mises-Fisher distribution is the analogue of a bivariate normal in two dimensions with a zero covariance.

Value

A contour plot of the von Mises-Fisher distribution.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

See Also

[rvmf](#), [vmf](#), [vmf.kerncontour](#), [kent.contour](#)

Examples

```
par(mfrow = c(1,3) )
vmf.contour(1)
vmf.contour(5)
vmf.contour(10)
```

Converting a rotation matrix on $SO(3)$ to an unsigned unit quaternion

Converting a rotation matrix on $SO(3)$ to an unsigned unit quaternion

Description

It returns an unsigned unit quaternion in S^3 (the four-dimensional sphere) from a 3×3 rotation matrix on $SO(3)$.

Usage

```
rot2quat(X)
```

Arguments

X A rotation matrix in $SO(3)$.

Details

Firstly construct a system of linear equations by equating the corresponding components of the theoretical rotation matrix proposed by Prentice (1986), and given a rotation matrix. Finally, the system of linear equations are solved by following the tricks mentioned in second reference here in order to achieve numerical accuracy to get quaternion values.

Value

A unsigned unit quaternion.

Author(s)

Anamul Sajib

R implementation and documentation: Anamul Sajib <pmxahsa@nottingham.ac.uk>

References

Prentice, M. J. (1986). Orientation statistics without parametric assumptions. *Journal of the Royal Statistical Society. Series B: Methodological* 48(2). //http://www.euclideanspace.com/maths/geometry/rotations/conversions/

See Also

[quat2rot](#), [rotation](#), [Arotation](#) \ `link{rot.matrix}`

Examples

```
x <- rnorm(4)
x <- x/sqrt( sum(x^2) ) ## an unit quaternion in R4 ##
R <- quat2rot(x)
R
x
rot2quat(R) ## sign is not exact as you can see
```

Converting an unsigned unit quaternion to rotation matrix on $SO(3)$

Converting an unsigned unit quaternion to rotation matrix on $SO(3)$

Description

It forms a (3 x 3) rotation matrix on $SO(3)$ from an unsigned unit quaternion in S^3 (the four-dimensional sphere).

Usage

```
quat2rot(x)
```

Arguments

x An unsigned unit quaternion in S^3 .

Details

Given an unsigned unit quaternion in S^3 it forms a rotation matrix on $SO(3)$, according to the transformation proposed by Prentice (1986).

Value

A rotation matrix.

Author(s)

Anamul Sajib

R implementation and documentation: Anamul Sajib <pmxahsa@nottingham.ac.uk>

References

Prentice, M. J. (1986). Orientation statistics without parametric assumptions. *Journal of the Royal Statistical Society. Series B: Methodological* 48(2).

See Also

[rot2quat](#), [rotation](#), [Arotation](#) \ [link{rot.matrix}](#)

Examples

```
x <- rnorm(4)
x <- x/sqrt( sum(x^2) )
x                                     ## an unit quaternion in R4 ##
quat2rot(x)
```

Cross validation in von Mises-Fisher discriminant analysis

Cross validation for estimating the classification rate of a discriminant analysis for directional data assuming a von Mises-Fisher distribution

Description

Cross validation for estimating the classification rate of a discriminant analysis for directional data assuming a von Mises-Fisher distribution.

Usage

```
vmf.da(x, ina, fraction = 0.2, R = 1000, seed = FALSE)
```

Arguments

x	A matrix with the dataset in Eulclidean coordinates, i.e. unit vectors.
ina	A variable indicating the groupings.
fraction	The fraction of data to be used as test set.
R	The number of repetitions.
seed	If seed is TRUE, the results will always be the same.

Details

A repeated cross validation procedure is performed to estimate the rate of correct classification.

Value

A list including:

percent	The estimated percent of correct classification and two estimated standard deviations. The one is the standard deviation of the rates and the other is assuming a binomial distribution.
ci	Three types of confidence intervals, the standard one, another one based on the binomial distribution and the third one is the empirical one, which calculates the upper and lower 2.5% of the rates.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Morris, J. E., & Laycock, P. J. (1974). Discriminant analysis of directional data. *Biometrika*, 61(2): 335-341.

See Also

[vmfda.pred](#), [mix.vmf](#), [vmf](#), [dirknn](#)

Examples

```
x <- rvmf(100, rnorm(4), 15)
ina <- rep(1:2, each = 50)
vmf.da(x, ina, fraction = 0.2, R = 200, seed = FALSE)
```

Density of the ESAG distribution

Density of the ESAG distribution

Description

Density of the ESAG distribution.

Usage

```
ESAGdensity(y, param, logden = FALSE)
```

Arguments

y	A matrix or a vector with the data expressed in Euclidean coordinates, i.e. unit vectors.
param	The parameters of the ESAG distribution, the first three are the mean vector in R^3 and the next two are the two gammas.
logden	If you the logarithm of the density values set this to TRUE.

Details

MLE of the projected normal distribution, on the sphere, is implemented.

Value

A vector with the density values of y.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr>

References

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chichester: John Wiley & Sons.

Paine P.J., Preston S.P., Tsagris M and Wood A.T.A. (2017). An Elliptically Symmetric Angular Gaussian Distribution. Statistics and Computing (Accepted).

See Also

[ESAGmle](#), [ESAGsim](#), [spml.mle](#), [acg](#), [circ.summary](#)

Examples

```
m <- colMeans( as.matrix( iris[,1:3] ) )
y <- ESAGsim(1000, c(m, 1,0.5) )
mod <- ESAGmle(y)
ESAGdensity( y, c(mod$mu, mod$gam) )
```

Euclidean transformation

Euclidean transformation

Description

It transforms the data from the spherical coordinates to Euclidean coordinates.

Usage

```
euclid(u)
```

Arguments

u A two column matrix or even one single vector, where the first column (or element) is the latitude and the second is the longitude. The order is important.

Details

It takes the matrix of unit vectors of latitude and longitude and transforms it to unit vectors.

Value

A three column matrix:

U The Euclidean coordinates of the latitude and longitude.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

See Also

[euclid.inv](#), [Arotation](#), [lambert](#)

Examples

```
x <- rvmf(10, rnorm(3), 10)
u <- euclid.inv(x)
euclid(u)
x
```

Euler angles from a rotation matrix on $SO(3)$

Compute the Euler angles from a rotation matrix on $SO(3)$.

Description

It calculates three euler angles $(\theta_{12}, \theta_{13}, \theta_{23})$ from a (3×3) rotation matrix X , where X is defined as $X = R_z(\theta_{12}) \times R_y(\theta_{13}) \times R_x(\theta_{23})$. Here $R_x(\theta_{23})$ means a rotation of θ_{23} radians about the x axis.

Usage

```
rot2eul(X)
```

Arguments

X A rotation matrix which is defined as a product of three elementary rotations mentioned above. Here $\theta_{12}, \theta_{23} \in (-\pi, \pi)$ and $\theta_{13} \in (-\pi/2, \pi/2)$.

Details

Given a rotation matrix X , euler angles are computed by equating each element in X with the corresponding element in the matrix product defined above. This results in nine equations that can be used to find the euler angles.

Value

For a given rotation matrix, there are two equivalent sets of euler angles.

Author(s)

Anamul Sajib<pmxahsa@nottingham.ac.uk>

R implementation and documentation: Anamul Sajib<pmxahsa@nottingham.ac.uk>

References

Green, P. J. & Mardia, K. V. (2006). Bayesian alignment using hierarchical models, with applications in proteins bioinformatics. *Biometrika*, 93(2):235–254.

<http://www.staff.city.ac.uk/~sbbh653/publications/euler.pdf>

See Also

[eul2rot](#)

Examples

```
# three euler angles

theta.12 <- sample( seq(-3, 3, 0.3), 1 )
theta.23 <- sample( seq(-3, 3, 0.3), 1 )
theta.13 <- sample( seq(-1.4, 1.4, 0.3), 1 )

theta.12 ; theta.23 ; theta.13

X <- eul2rot(theta.12, theta.23, theta.13)
X ## A rotation matrix

e <- rot2eul(X)$v1

theta.12 <- e[3]
theta.23 <- e[2]
theta.13 <- e[1]

theta.12 ; theta.23 ; theta.13
```

Goodness of fit test for grouped data

Goodness of fit test for grouped data

Description

Goodness of fit test for grouped data.

Usage

```
group.gof(g, ni, m, k, dist = "vm", rads = FALSE, R = 999, ncores = 1)
```

Arguments

<code>g</code>	A vector with the group points, either in radians or in degrees.
<code>ni</code>	The frequency of each or group class.
<code>m</code>	The mean direction in radians or in degrees.
<code>k</code>	The concentration parameter, κ .
<code>dist</code>	The distribution to be tested, it can be either "vm" or "uniform".
<code>rads</code>	If the data are in radians, this should be TRUE and FALSE otherwise.
<code>R</code>	The number of bootstrap simulations to perform, set to 999 by default.
<code>ncores</code>	The number of cores to use.

Details

When you have grouped data, you can test whether the data come from the von Mises-Fisher distribution or from a uniform distribution.

Value

A list including:

<code>info</code>	A vector with two elements, the test statistic value and the bootstrap p-value.
<code>runtime</code>	The runtime of the procedure.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Arthur Pewsey, Markus Neuhauser, and Graeme D. Ruxton (2013). Circular Statistics in R.

See Also

[pvm](#), [circ.summary](#), [rvonmises](#)

Examples

```
x <- rvonmises(100, 2, 10)
g <- seq(min(x) - 0.1, max(x) + 0.1, length = 6)
ni <- as.vector( table( cut(x, g) ) )
group.gof(g, ni, 2, 10, dist = "vm", rads = TRUE, R = 299, ncores = 1)
group.gof(g, ni, 2, 5, dist = "vm", rads = TRUE, R = 299, ncores = 1)
```

Habeck's rotation matrix generation

Generation of three-dimensional random rotations using Habeck's algorithm.

Description

It generates random rotations in three-dimensional space that follow a probability distribution, matrix Fisher distribution, arising in fitting and matching problem.

Usage

```
habeck.rot(F)
```

Arguments

F An arbitrary 3 x 3 matrix represents the parameter matrix of this distribution.

Details

Firstly rotation matrices \mathbf{X} are chosen which are the closest to F , and then parameterized using euler angles. Then a Gibbs sampling algorithm is implemented to generate rotation matrices from the resulting distribution of the euler angles.

Value

A simulated rotation matrix.

Author(s)

Anamul Sajib<pmxahsa@nottingham.ac.uk>

R implementation and documentation: Anamul Sajib<pmxahsa@nottingham.ac.uk>

References

Habeck M (2009). Generation of three-dimensional random rotations in fitting and matching problems. *Computational Statistics*, 24, 719–731.

Examples

```
F <- 10^(-1) * matrix( c(85, 11, 41, 78, 39, 60, 43, 64, 48), ncol = 3 ) ## Arbitrary F matrix
X <- habeck.rot(F)
det(X)
```

Hypothesis test for von Mises-Fisher distribution over Kent distribution
Hypothesis test for von Mises-Fisher distribution over Kent distribution

Description

The null hypothesis is whether a von Mises-Fisher distribution fits the data well, where the alternative is that Kent distribution is more suitable.

Usage

```
fishkent(x, B = 999)
```

Arguments

x	A numeric matrix containing the data as unit vectors in Euclidean coordinates.
B	The number of bootstrap re-samples. By default is set to 999. If it is equal to 1, no bootstrap is performed and the p-value is obtained through the asymptotic distribution.

Details

Essentially it is a test of rotational symmetry, whether Kent's ovalness parameter (beta) is equal to zero. This works for spherical data only.

Value

A vector including:

test	The value of the test statistic
p-value or Bootstrap p-value	The p-value of the test.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Rivest, L. P. (1986). Modified Kent's statistics for testing goodness of fit for the Fisher distribution in small concentrated samples. *Statistics & probability letters*, 4(1): 1-4.

See Also

[vmf](#), [kent.mle](#), [rkent](#)

Examples

```
x <- rvmf(100, rnorm(3), 15)
fishkent(x)
fishkent(x, B = 1)
```

Inverse of Lambert's equal area projection
Inverse of Lambert's equal area projection

Description

It takes some points from the cartesian coordinates and maps them onto the sphere. The inverse of the Lambert's equal area projection.

Usage

```
lambert.inv(z, mu)
```

Arguments

<code>z</code>	A two- column matrix containing the Lambert's equal area projected data.
<code>mu</code>	The mean direction of the data on the sphere.

Details

The data are first mapped on the sphere with mean direction equal to the north pole. Then, they are rotated to have the given mean direction. It is the inverse of the Lambert's equal area projection.

Value

A matrix containing spherical data (unit vectors).

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kent, John T. (1982). The Fisher-Bingham distribution on the sphere. Journal of the Royal Statistical Society. Series B (Methodological) 44(1):71-80.

See Also

[lambert](#)

Examples

```
m <- rnorm(3)
m <- m / sqrt( sum(m^2) )
x <- rvmf(20, m, 19)
mu <- vmf(x)$mu
y <- lambert( euclid.inv(x) )
lambert.inv(y, mu)
euclid.inv(x)
```

Inverse of the Euclidean transformation

Inverse of the Euclidean transformation

Description

It transforms the data from the Euclidan coordinates to latitude dn longitude.

Usage

```
euclid.inv(U)
```

Arguments

U A matrix of unit vectors, or even one single unit vector in three dimensions.

Details

It takes the matrix of unit vectors and back transforms it to latitude and longitude.

Value

A two column matrix:

u The first column is the latitude and the second is the longitude, both expressed in degrees.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

See Also

[euclid](#), [Arotation](#), [lambert](#)

Examples

```
x <- rvmf(10, rnorm(3), 10)
euclid.inv(x)
euclid( euclid.inv(x) )
x
```

k-NN algorithm using the arc cosinus distance

k-NN algorithm using the arc cosinus distance

Description

It classifies new observations to some known groups via the k-NN algorithm.

Usage

```
dirknn(x, xnew, k = 5, ina, type = "S", mesos = TRUE)
```

Arguments

x	The data, a numeric matrix with unit vectors.
xnew	The new data whose membership is to be predicted, a numeric matrix with unit vectors.
k	The number of nearest neighbours, set to 5 by default.
ina	A variable indicating the groups of the data x.
type	If type is "S", the standard k-NN algorithm is to be used, else "NS" for the non standard one. See below (details) for more information.
mesos	A boolean variable used only in the case of the non standard algorithm (type="NS"). Should the average of the distances be calculated (TRUE) or not (FALSE)? If it is FALSE, the harmonic mean is calculated.

Details

The standard algorithm is to keep the k nearest observations and see the groups of these observations. The new observation is allocated to the most frequent seen group. The non standard algorithm is to calculate the classical mean or the harmonic mean of the k nearest observations for each group. The new observation is allocated to the group with the smallest mean distance.

Value

A vector including:

g	The predicted group.
---	----------------------

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

course webpage of Howard E. Haber. http://scipp.ucsc.edu/~haber/ph216/rotation_12.pdf

See Also

[dirknn.tune](#), [vmfda.pred](#), [mix.vmf](#)

Examples

```
k <- runif(4, 4, 20)
prob <- c(0.2, 0.4, 0.3, 0.1)
mu <- matrix(rnorm(16), ncol = 4)
mu <- mu / sqrt( rowSums(mu^2) )
da <- rmixvmf(200, prob, mu, k)
nu <- sample(1:200, 180)
x <- da$x[nu, ]
ina <- da$id[nu]
xx <- da$x[-nu, ]
id <- da$id[-nu]
a1 <- dirknn(x, xx, k = 5, ina, type = "S", mesos = TRUE)
a2 <- dirknn(x, xx, k = 5, ina, type = "NS", mesos = TRUE)
a3 <- dirknn(x, xx, k = 5, ina, type = "S", mesos = FALSE)
a4 <- dirknn(x, xx, k = 5, ina, type = "NS", mesos = FALSE)
b <- vmfda.pred(xx, x, ina)
table(id, a1)
table(id, a2)
table(id, a3)
table(id, a4)
```

k-NN regression

k-NN regression with Euclidean or (hyper-)spherical response and or predictor variables

Description

k-NN regression with Euclidean or (hyper-)spherical response and or predictor variables.

Usage

```
knn.reg(xnew, y, x, k = 5, res = "eucl", type = "euclidean", estim = "arithmetic")
```

Arguments

xnew	The new data, new predictor variables values. A matrix with either euclidean (univariate or multivariate) or (hyper-)spherical data. If you have a circular response, say u, transform it to a unit vector via $(\cos(u), \sin(u))$. If $x_{\text{new}} = x$, you will get the fitted values.
y	The currently available data, the response variables values. A matrix with either euclidean (univariate or multivariate) or (hyper-)spherical data. If you have a circular response, say u, transform it to a unit vector via $(\cos(u), \sin(u))$.
x	The currently available data, the predictor variables values. A matrix with either euclidean (univariate or multivariate) or (hyper-)spherical data. If you have a circular response, say u, transform it to a unit vector via $(\cos(u), \sin(u))$.
k	The number of nearest neighbours, set to 5 by default.
res	The type of the response variable. If it is Euclidean, set this argument equal to "res". If it is a unit vector set it to $\text{res}=\text{"spher"}$.
type	The type of distance to be used. This determines the nature of the predictor variables. This is actually the argument "method" of the distance function in R. The default value is "euclidean". If you use the Euclidean distance, the package "Rfast" is used. The "dista" function of that package is about 3 times faster than the standard built-in "dist". R has several options the type of the distance. Just type <code>?Rfast::Dist</code> in R and see the methods. Any method can be given here. If you have unit vectors in general, you should put $\text{type}=\text{"spher"}$, so that the cosinus distance is calculated.
estim	Once the k observations with the smallest distance are discovered, what should the prediction be? The arithmetic average of the corresponding y values be used $\text{estim}=\text{"arithmetic"}$ or their harmonic average $\text{estim}=\text{"harmonic"}$.

Details

This function covers a broad range of data, Euclidean and spherical, along with their combinations.

Value

A matrix (or a vector in the case of Euclidean data) with the predicted response values.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

See Also

[knnreg.tune](#), [spher.reg](#), [spml.reg](#)

Examples

```
y <- iris[, 1]
x <- iris[, 2:4]
x <- x / sqrt( rowSums(x^2) ) ## Euclidean response and spherical predictors
a <- knn.reg(x, y, x, k = 5, res = "eucl", type = "spher", estim = "arithmetic")

y <- iris[, 2:4]
y <- y / sqrt( rowSums(y^2) ) ## Spherical response and Euclidean predictor
x <- iris[, 1]
a <- knn.reg(x, y, x, k = 5, res = "spher", type = "euclidean", estim = "arithmetic")
```

Lambert's equal area projection

Lambert's equal area projection

Description

It calculates the Lambert's equal area projection.

Usage

```
lambert(y)
```

Arguments

y A two column matrix with the data. The first column is the altitude and the second is the longitude.

Details

The spherical data are first rotated so that their mean direction is the north pole and then are projected on the plane tangent to the sphere at the north pole.

Value

A two-column matrix with the projected points.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kent, John T. (1982). The Fisher-Bingham distribution on the sphere. Journal of the Royal Statistical Society. Series B (Methodological) 44(1):71-80.

See Also

[euclid](#), [lambert.inv](#)

Examples

```
x <- rvmf(100, rnorm(3), 20)
x <- euclid.inv(x)
a <- lambert(x)
plot(a)
```

Logarithm of the Kent distribution normalizing constant

Logarithm of the Kent distribution normalizing constant

Description

Logarithm of the Kent distribution normalizing constant.

Usage

```
kent.logcon(k, b, j = 100)
```

Arguments

k	The concentration parameter, κ .
b	The ovalness parameter, β .
j	The number of the terms in the sum to use. By default this is 100.

Details

It calculates logarithm of the normalising constant of the Kent distribution.

Value

The value of the logarithm of the normalising constant of the Kent distribution.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kent John (1982). The Fisher-Bingham distribution on the sphere. Journal of the Royal Statistical Society, Series B, 44(1): 71-80.

See Also

[fb.saddle](#), [kent.mle](#)

Examples

```
kent.logcon(10, 2)
fb.saddle( c(0, 10, 0), c(0, -2, 2) )
```

Mixtures of Von Mises-Fisher distributions

Mixtures of Von Mises-Fisher distributions

Description

It performs model based clustering for circular, spherical and hyperspherical data assuming von Mises-Fisher distributions.

Usage

```
mix.vmf(x, g)
```

Arguments

x	A matrix with the data expressed as unit vectors.
g	The number of groups to fit. It must be greater than or equal to 2.

Details

The initial step of the algorithm is not based on a spherical k-means, but on simple k-means. The results are comparable to the package movMF.

Value

A list including:

param	A matrix with the mean direction, the concentrations parameter and mixing probability of each group.
loglik	The value of the maximised log-likelihood.
pred	The predicted group of each observation.
runtime	The run time of the algorithm. A numeric vector. The first element is the user time, the second element is the system time and the third element is the elapsed time.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kurt Hornik and Bettina Grün (2014). movMF: An R Package for Fitting Mixtures of von Mises-Fisher Distributions <http://cran.r-project.org/web/packages/movMF/vignettes/movMF.pdf>

See Also

[rmixvmf](#), [bic.mixvmf](#), [mixvmf.contour](#)

Examples

```
k <- runif(4, 4, 20)
prob <- c(0.2, 0.4, 0.3, 0.1)
mu <- matrix(rnorm(16), ncol = 4)
mu <- mu / sqrt( rowSums(mu^2) )
x <- rmixvmf(200, prob, mu, k)$x
mix.vmf(x, 3)
mix.vmf(x, 4)
mix.vmf(x, 5)
```

MLE of some circular distributions

MLE of some circular distributions

Description

MLE of some circular distributions.

Usage

```
spml.mle(x, rads = FALSE, tol = 1e-07)
wrapcauchy(x, rads = FALSE, tol = 1e-07)
```

Arguments

x	A numerical vector with the circular data. They must be expressed in radians.
rads	If the data are in radians set this to TRUE.
tol	The tolerance level to stop the iterative process of finding the MLEs.

Details

The parameters of the bivariate angular Gaussian and wrapped Cauchy distributions are estimated. For the Wrapped Cauchy, the iterative procedure described by Kent and Tyler (1988) is used. The Newton-Raphson algorithm for the angular Gaussian is described in the regression setting in Presnell et al. (1998).

Value

A list including:

<code>iters</code>	The iterations required until convergence.
<code>loglik</code>	The value of the maximised log-likelihood.
<code>param</code>	A vector consisting of the estimates of the two parameters, the mean direction for both distributions and the concentration parameter κ and the ρ for the von Mises and wrapped Cauchy respectively.
<code>gamma</code>	The norm of the mean vector of the angular Gaussian distribution.
<code>mu</code>	The mean vector of the angular Gaussian distribution.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr>

References

- Mardia K. V. and Jupp P. E. (2000). Directional statistics. Chicester: John Wiley & Sons.
- Sra S. (2012). A short note on parameter approximation for von Mises-Fisher distributions: and a fast implementation of $Is(x)$. Computational Statistics, 27(1): 177-190.
- Presnell Brett, Morrison Scott P. and Littell Ramon C. (1998). Projected multivariate linear models for directional data. Journal of the American Statistical Association, 93(443): 1068-1077.
- Kent J. and Tyler D. (1988). Maximum likelihood estimation for the wrapped Cauchy distribution. Journal of Applied Statistics, 15(2): 247-254.

See Also

[vmf](#), [rvonmises](#), [rvmf](#)

Examples

```
y <- rcauchy(100, 3, 1)
x <- y
spml.mle(x)
wrapcauchy(x)
```

MLE of the angular central Gaussian distribution
MLE of the angular central Gaussian distribution

Description

MLE of the angular central Gaussian distribution.

Usage

```
acg(x, tol = 1e-07)
```

Arguments

x	A matrix with directional data, i.e. unit vectors.
tol	The tolerance value at which to terminate the iterations.

Details

There is a constraint on the estimated covariance matrix; its trace is equal to the number of variables. An iterative algorithm takes place and convergence is guaranteed.

Value

A list including:

iter	The number if iterations required by the algorithm to converge to the solution.
cova	The estimated covatriance matrix.

Author(s)

Michail Tsagris R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Tyler D. E. (1987). Statistical analysis for the angular central Gaussian distribution on the sphere. *Biometrika* 74(3): 579-589.

See Also

[rbingham](#), [rfb](#), [f.rbing](#), [fb.saddle](#)

Examples

```

m <- c(0, 0, 0, 0)
s <- cov(iris[, 1:4])
x <- MASS::mvrnorm(500, m, s)
x <- x / sqrt( Rfast::rowsums(x^2) )
mod <- acg(x)
mod
cov2cor(mod$cova) ## estimated covariance matrix turned into a correlation matrix
cov2cor(s) ## true covariance matrix turned into a correlation matrix

```

MLE of the ESAG distribution

MLE of the ESAG distribution

Description

MLE of the ESAG distribution.

Usage

```
ESAGmle(y, tol = 1e-07)
```

Arguments

<code>y</code>	A matrix with the data expressed in Euclidean coordinates, i.e. unit vectors.
<code>tol</code>	The tolerance to accept that the E-M algorithm used to estimate the concentration parameter has converged.

Details

MLE of the MLE of the ESAG distribution, on the sphere, is implemented. ESAG stands for Elliptically Symmetric Angular Gaussian and it was suggested by Paine et al. (2017). Unlike the projected normal distribution this is rotationally symmetric and is a competitor of the spherical Kent distribution (which is also non rotational symmetric).

Value

A list including:

<code>mu</code>	The mean vector in R^3 .
<code>gam</code>	The two gamma parameters.
<code>loglik</code>	The log-likelihood value.
<code>iag.loglik</code>	The log-likelihood value of the isotropic angular Gaussian distribution. That is, the projected normal distribution which is rotational symmetric.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr>

References

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chichester: John Wiley & Sons.

Paine P.J., Preston S.P., Tsagris M and Wood A.T.A. (2017). An Elliptically Symmetric Angular Gaussian Distribution. Statistics and Computing (To appear).

See Also

[ESAGdensity](#), [ESAGsim](#), [iag.mle](#), [kent.mle](#), [acg](#), [circ.summary](#)

Examples

```
m <- colMeans( as.matrix( iris[,1:3] ) )
y <- ESAGsim(1000, c(m, 1,0.5) )
ESAGmle(y)
```

MLE of the generalised von Mises distribution

MLE of the geometrically generalised von Mises distribution

Description

MLE of the geometrically generalised von Mises distribution.

Usage

```
ggvm(phi, rads = FALSE)
```

Arguments

phi	A numerical vector with the circular data.
rads	Whether the data are in rads (TRUE) or not (FALSE).

Details

The generalised von Mises distribution (Dietrich and Richter, 2016) is fitted to some data and its parameters are estimated.

Value

A list including:

loglik	The value of the maximised log-likelihood.
param	A vector consisting of the ζ , κ , μ and α parameters of the generalised von Mises distribution as describe din Equation (2.7) of Dietrich and Richter (2016).

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr>

References

Dietrich, T., & Richter, W. D. (2016). Classes of geometrically generalized von Mises distributions. *Sankhya B*, 1-39.

See Also

[circ.summary](#), [rvonmises](#)

Examples

```
x <- rvonmises(100, 2, 25, rads = TRUE)
circ.summary(x, rads = TRUE)
ggvm(x, rads = TRUE)
```

MLE of the Kent distribution

MLE of the Kent distribution

Description

It estimates the concentration and the ovalness parameter of some directional data assuming the Kent distribution. The mean direction and major and minor axes are also estimated.

Usage

```
kent.mle(x)
```

Arguments

`x` A matrix containing spherical data in Euclidean coordinates.

Details

The Kent distribution is fitted to some data and its parameters are estimated.

Value

A list including:

runtime	The run time of the procedure.
G	A 3 x 3 matrix whose first column is the mean direction. The second and third columns are the major and minor axes respectively.
para	The concentration κ and ovalness β parameters are stored here.
logcon	The logarithm of the normalising constant, using the third type approximation.
loglik	The value of the log-likelihood.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kent John (1982). The Fisher-Bingham distribution on the sphere. Journal of the Royal Statistical Society, Series B, 44(1): 71-80.

See Also

[kent.mle](#), [fb.saddle](#), [vmf](#), [wood.mle](#)

Examples

```
x <- rvmf(200, rnorm(3), 15)
kent.mle(x)
vmf(x)
A <- diag( c(-5, 0, 5) )
x <- rfb(200, 15, rnorm(3), A)
kent.mle(x)
vmf(x)
```

Mle of the Matrix Fisher distribution on $SO(3)$

Mle of the Matrix Fisher distribution on $SO(3)$

Description

It returns the maximum likelihood estimate of the Matrix Fisher parameter $F(3 \times 3)$.

Usage

```
matrixfisher.mle(X)
```

Arguments

`X` An array containing rotation matrices in $SO(3)$.

Value

The components of $svd(\bar{X})$.

Author(s)

Anamul Sajib & Chris Fallaize.

R implementation and documentation: Anamul Sajib <pmxahsa@nottingham.ac.uk> & Chris Fallaize.

References

Prentice M. J. (1986). Orientation statistics without parametric assumptions. *Journal of the Royal Statistical Society. Series B: Methodological* 48(2).

See Also

[rmatrixfisher](#)

Examples

```
F <- 10^(-1) * matrix( c(85, 11, 41, 78, 39, 60, 43, 64, 48), ncol = 3 ) ### An arbitrary F matrix
X <- rmatrixfisher(3000, F)
matrixfisher.mle(X)
svd(F)
## Bigger sample is needed to obtain better results
## X <- rmatrixfisher(30000, F)
## matrixfisher.mle(X)
## svd(F)
```

MLE of the spherical projected normal distribution

MLE of the spherical projected normal distribution

Description

MLE of the spherical projected normal distribution.

Usage

```
iag.mle(y, tol = 1e-07)
```

Arguments

<code>y</code>	A matrix with the data expressed in Euclidean coordinates, i.e. unit vectors.
<code>tol</code>	The tolerance to accept that the E-M algorithm used to estimate the concentration parameter has converged.

Details

MLE of the projected normal distribution, on the sphere, is implemented.

Value

A list including:

<code>iters</code>	The number of iteration required by the Newton-Raphson.
<code>mesi</code>	A matrix with two rows. The first row is the mean direction and the second is the mean vector. The first comes from the second by normalising to have unit length.
<code>param</code>	A vector with the elements, the norm of mean vector, the log-likelihood and the log-likelihood of the spherical uniform distribution. The third value helps in case you want to do a log-likelihood ratio test for uniformity.
<code>kappa</code>	The concentration parameter.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr>

References

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chichester: John Wiley & Sons.

Paine P.J., Preston S.P., Tsagris M and Wood A.T.A. (2017). An Elliptically Symmetric Angular Gaussian Distribution. Statistics and Computing (Accepted).

See Also

[ESAGmle](#), [spm1.mle](#), [acg](#), [circ.summary](#)

Examples

```
x <- as.matrix(iris[,1:3])
x <- x / sqrt( rowSums(x^2) )
iag.mle(x)
```

MLE of the von Mises-Fisher distribution
MLE of von Mises-Fisher distribution

Description

MLE of the von Mises-Fisher distribution.

Usage

```
vmf(x, fast = FALSE, tol = 1e-07)
```

Arguments

x	A matrix with the data expressed in Euclidean coordinates, i.e. unit vectors.
fast	A boolean variable to do a faster implementation.
tol	The tolerance to accept that the E-M algorithm used to estimate the concentration parameter has converged.

Details

The mean direction and concentration of a fitted von Mises-Fisher distribution are estimated.

Value

If fast = FALSE a list including all the following. If fast = TRUE less items are returned.

mu	The mean direction.
kappa	The concentration parameter.
MRL	The mean resultant length.
vark	The variance of the concentration parameter.
loglik	The maximum log-likelihood value.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chichester: John Wiley & Sons.
 Sra, S. (2012). A short note on parameter approximation for von Mises-Fisher distributions: and a fast implementation of $I_s(x)$. Computational Statistics, 27(1): 177–190.

See Also

[iag.mle](#), [rvmf](#), [kent.mle](#), [vmf.kde](#), [wood.mle](#)

Examples

```
m <- rnorm(3)
m <- m/sqrt(sum(m^2))
m
x <- rvmf(100, m, 7)
vmf(x)
x <- rvmf(500, m, 7)
vmf(x)
```

MLE of the Wood bimodal distribution on the sphere

MLE of the Wood bimodal distribution on the sphere

Description

It estimates the parameters of the Wood bimodal distribution.

Usage

```
wood.mle(y)
```

Arguments

`y` A matrix containing two columns. The first one is the latitude and the second is the longitude, both expressed in degrees.

Details

The Wood distribution is fitted to some data and its parameters are estimated. It is a bimodal distribution which contains 5 parameters, just like the Kent distribution.

Value

A list including:

<code>info</code>	A 5 x 3 matrix containing the 5 parameters, gamma, delta, alpha, beta and kappa along with their corresponding 95% confidence intervals all expressed in degrees.
<code>modes</code>	The two axis of the modes of the distribution expressed in degrees.
<code>unitvectors</code>	A 3 x 3 matrix with the 3 unitvectors associated with the gamma and delta parameters.
<code>loglik</code>	The value of the log-likelihood.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Wood A.T.A. (1982). A bimodal distribution on the sphere. Journal of the Royal Statistical Society, Series C, 31(1): 52-58.

See Also

[kent.mle](#), [vmf](#), [rvmf](#)

Examples

```
x <- rvmf(100, rnorm(3), 15)
x <- euclid.inv(x)
wood.mle(x)
```

Prediction in discriminant analysis based on von Mises-Fisher distributions
*Prediction of a new observation using discriminant analysis based on
von Mises-Fisher distributions*

Description

Prediction of a new observation using discriminant analysis based on von Mises-Fisher distributions.

Usage

```
vmfda.pred(xnew, x, ina)
```

Arguments

xnew	The new observation(s) (unit vector(s)) whose group is to be predicted.
x	A data matrix with unit vectors, i.e. directional data.
ina	A vector indicating the groups of the data x.

Details

Discriminant analysis assuming von Mises-Fisher distributions.

Value

A vector with the predicted group of each new observation.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Morris J. E. & Laycock P. J. (1974). Discriminant analysis of directional data. *Biometrika*, 61(2): 335-341.

See Also

[vmf.da](#), [mix.vmf](#), [mix.vmf](#), [dirknn](#), [knn.reg](#)

Examples

```
m1 <- rnorm(5)
m2 <- rnorm(5)
x <- rbind( rvmf(100, m1, 5), rvmf(80, m2, 10) )
ina <- c( rep(1,100), rep(2, 80) )
y <- rbind(rvmf(10, m1, 10), rvmf(10, m2, 5))
id <- rep(1:2, each = 10)
g <- vmfda.pred(y, x, ina)
table(id, g)
```

Probability density function of the von Mises-Fisher distribution

Probability density function of the von Mises-Fisher distribution

Description

Probability density function of the von Mises-Fisher distribution.

Usage

```
pvm(theta, m, k, rads = FALSE)
```

Arguments

theta	A numerical value, either in radians or in degrees.
m	The mean direction in radians or in degrees.
k	The concentration parameter, κ .
rads	If the data are in radians, this should be TRUE and FALSE otherwise.

Details

This value calculates the probability of x being less than theta and is used by [group.gof](#).

Value

The probability that of x being less than θ , where x follows the von Mises-Fisher distribution.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Arthur Pewsey, Markus Neuhauser, and Graeme D. Ruxton (2013). Circular Statistics in R.

See Also

[group.gof](#), [circ.summary](#), [rvonmises](#)

Examples

```
pvm(1, 2, 10, rads = TRUE)
pvm(2, 2, 10, rads = TRUE)
```

Random sample of matrices in $SO(p)$

Random sample of matrices in $SO(p)$

Description

Random sample of matrices in $SO(p)$.

Usage

```
rsop(n, p)
```

Arguments

n	The sample size, the number of matrices you want to generate.
p	The dimensionality of the matrices.

Details

The idea is very simple. Start with a unit vector pointing at the north pole $(1,0,\dots,0)$. Then generate random numbers from a standard normal and scale them so that they have a unit length. To put it differently, a sample of n values from the uniform distribution on the sphere is generated. Then calculate the rotation matrix required to go from the north pole to each of a generated vector.

Value

If $n = 1$ one matrix is returned. If n is greater than 1, an array with n matrices inside.

Author(s)

Michail Tsagris R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

G. J. A. Amaral, I. L. Dryden & Andrew T. A. Wood (2007). Pivotal Bootstrap Methods for k-Sample Problems in Directional Statistics and Shape Analysis. *Journal of the American Statistical Association*, 102(478): 695-707.

See Also

[rotation](#), [Arotation](#), [rot.matrix](#)

Examples

```
x1 <- rsop(1, 3)
x2 <- rsop(10, 3)
x3 <- rsop(100, 10)
```

Random values generation from the ESAG distribution
Random values generation from the ESAG distribution

Description

Random values generation from the ESAG distribution.

Usage

```
ESAGsim(n, param)
```

Arguments

n	A matrix with the data expressed in Euclidean coordinates, i.e. unit vectors.
param	The parameters of the ESAG distribution, the first three are the mean vector in R^3 and the next two are the two gammas.

Details

MLE of the projected normal distribution, on the sphere, is implemented.

Value

An nx3 matrix with the simulated unit vectors.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr>

References

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chichester: John Wiley & Sons.

Paine P.J., Preston S.P., Tsagris M and Wood A.T.A. (2017). An Elliptically Symmetric Angular Gaussian Distribution. Statistics and Computing (Accepted).

See Also

[ESAGmle](#), [ESAGdensity](#), [spml.mle](#), [acg](#), [circ.summary](#)

Examples

```
m <- colMeans( as.matrix( iris[,1:3] ) )
y <- ESAGsim(1000, c(m, 1, 0.5) )
ESAGmle(y)
```

Random values simulation from a von Mises distribution

Random values simulation from a von Mises distribution

Description

It generates random vectors following the von Mises distribution. The data can be spherical or hyper-spherical.

Usage

```
rvonmises(n, m, k, rads = TRUE)
```

Arguments

n	The sample size.
m	The mean angle expressed in radians or degrees.
k	The concentration parameter. If k is zero the sample will be generated from the uniform distribution over $(0, 2\pi)$.
rads	If the mean angle is expressed in radians, this should be TRUE and FALSE otherwise. The simulated data will be expressed in radians or degrees depending on what the mean angle is expressed.

Details

The mean direction is transformed to the Euclidean coordinates (i.e. unit vector) and then the `fvmf` function is employed. It uses a rejection sampling as suggested by Andrew Wood in 1994. I have mentioned the description of the algorithm as I found it in Dhillon and Sra in 2003. Finally, the data are transformed to radians or degrees.

Value

A vector with the simulated data.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Wood, A. T. (1994). Simulation of the von Mises Fisher distribution. *Communications in statistics-simulation and computation*, 23(1): 157-164. Dhillon, I. S., & Sra, S. (2003). Modeling data using directional distributions. Technical Report TR-03-06, Department of Computer Sciences, The University of Texas at Austin. <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.75.4122&rep=rep1&type=pdf>

See Also

[circ.summary](#), [rvmf](#), [racg](#)

Examples

```
x <- rvonmises(100, 2, 25, rads = TRUE)
circ.summary(x, rads = TRUE)
```

Rayleigh's test of uniformity

Rayleigh's test of uniformity

Description

It checks whether the data are uniformly distributed on the sphere or hypersphere.

Usage

```
rayleigh(x, modif = TRUE, B = 999)
```

Arguments

x	A matrix containing the data, unit vectors.
modif	If modif is TRUE, the modification as suggested by Jupp (2001) is used.
B	If B is greater than 1, bootstrap calibration is performed. If it is equal to 1, classical theory is used.

Details

The Rayleigh test of uniformity is not the best, when there are two antipodal mean directions. In this case it will fail. It is good to test whether there is one mean direction or not. To put it differently, it tests whether the concentration parameter of the Fisher distribution is zero or not.

Value

A vector including:

test	The value of the test statistic.
p-value or Bootstrap p-value	The (bootstrap) p-value of the test.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

- Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chichester: John Wiley & Sons.
- Jupp, P. E. (2001). Modifications of the rayleigh and bingham tests for uniformity of directions. *Journal of Multivariate Analysis*, 77(2):1-20.
- Rayleigh, L. (1919). On the problem of random vibrations, and of random flights in one, two, or three dimensions. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 37(220):321-347.

See Also

[vmf](#), [meandir.test](#), [acg](#)

Examples

```
x <- rvmf(100, rnorm(5), 1) ## Fisher distribution with low concentration
rayleigh(x)
```

Rotation axis and angle of rotation given a rotation matrix

Rotation axis and angle of rotation given a rotation matrix

Description

Given a 3 x 3 rotation matrix, the angle and the axis of rotation are calculated.

Usage

```
Arotation(A)
```

Arguments

A A 3 x 3 rotation matrix.

Details

If the user does not supply a rotation matrix a message will appear.

Value

A list including:

angle	The angle of rotation expressed in degrees.
axis	The axis of rotation. A vector of two components, latitude and longitude, expressed in degrees.

Author(s)

Michail Tsagris R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

course webpage of Howard E. Haber. http://scipp.ucsc.edu/~haber/ph216/rotation_12.pdf

See Also

[rot.matrix](#), [rotation](#), [rsop](#)

Examples

```
ksi <- c(25.31, 24.29)
theta <- 2.38
A <- rot.matrix(ksi, theta, rads = FALSE)
A
Arotation(A)
```

Rotation matrix from a rotation axis and angle of rotation
Rotation matrix from a rotation axis and angle of rotation

Description

It calculates a rotation matrix from a rotation axis and angle of rotation.

Usage

```
rot.matrix(ksi, theta, rads = FALSE)
```

Arguments

ksi	The rotation axis, a vector with two elements, the first is the latitude and the second is the longitude.
theta	The angle of rotation.
rads	If both the ksi and theta are in rads, this should be TRUE. If both the ksi and theta are in degrees, this should be FALSE.

Details

The function accepts as arguments the rotation axis and the angle of rotation and it calculates the requested rotation matrix.

Value

A 3 x 3 rotation matrix.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Ted Chang (1986). Spherical Regression. *Annals of Statistics*, 14(3): 907-924.

See Also

[Arotation](#), [rotation](#), [rsop](#)

Examples

```
ksi <- c(25.31, 24.29)
theta <- 2.38
A <- rot.matrix(ksi, theta, rads = FALSE)
A
Arotation(A)
```

Rotation matrix on $SO(3)$ from three Euler angles

Construct a rotation matrix on $SO(3)$ from the Euler angles.

Description

It forms a rotation matrix X on $SO(3)$ by using three Euler angles $(\theta_{12}, \theta_{13}, \theta_{23})$, where X is defined as $X = R_z(\theta_{12}) \times R_y(\theta_{13}) \times R_x(\theta_{23})$. Here $R_x(\theta_{23})$ means a rotation of θ_{23} radians about the x axis.

Usage

```
eul2rot(theta.12, theta.23, theta.13)
```

Arguments

theta.12	An Euler angle, a number which must lie in $(-\pi, \pi)$.
theta.23	An Euler angle, a number which must lie in $(-\pi, \pi)$.
theta.13	An Euler angle, a number which must lie in $(-\pi/2, \pi/2)$.

Details

Given three euler angles a rotation matrix X on $SO(3)$ is formed using the transformation according to Green and Mardia (2006) which is defined above.

Value

A rotation matrix.

Author(s)

Anamul Sajib<pmxahsa@nottingham.ac.uk>

R implementation and documentation: Anamul Sajib<pmxahsa@nottingham.ac.uk>

References

Green, P. J. & Mardia, K. V. (2006). Bayesian alignment using hierarchical models, with applications in proteins bioinformatics. *Biometrika*, 93(2):235–254.

See Also[rot2eul](#)**Examples**

```
# three euler angles

theta.12 <- sample( seq(-3, 3, 0.3), 1 )
theta.23 <- sample( seq(-3, 3, 0.3), 1 )
theta.13 <- sample( seq(-1.4, 1.4, 0.3), 1 )

theta.12 ; theta.23 ; theta.13

X <- eul2rot(theta.12, theta.23, theta.13)
X # A rotation matrix
det(X)

e <- rot2eul(X)$v1

theta.12 <- e[3]
theta.23 <- e[2]
theta.13 <- e[1]

theta.12 ; theta.23 ; theta.13
```

Rotation matrix to rotate a spherical vector along the direction of another
Rotation matrix to rotate a spherical vector along the direction of another

Description

A rotation matrix is calculated to rotate a unit vector along the direction of another.

Usage

```
rotation(a, b)
```

Arguments

a	The initial unit vector.
b	The target unit vector.

Details

The function calculates a rotation matrix given two vectors. This rotation matrix is the connection between the two spherical only, vectors.

Value

A rotation matrix whose dimension is equal to the length of the unit vectors.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

G. J. A. Amaral, I. L. Dryden & Andrew T. A. Wood (2007). Pivotal Bootstrap Methods for k-Sample Problems in Directional Statistics and Shape Analysis. *Journal of the American Statistical Association*, 102(478): 695-707.

See Also

[Arotation](#), [rot.matrix](#), [lambert](#), [lambert.inv](#), [rsop](#)

Examples

```
a <- rnorm(3)
a <- a/sqrt(sum(a^2))
b <- rnorm(3)
b <- b/sqrt(sum(b^2))
A <- rotation(a, b)
A
a ; b
a %*% t(A)
```

```
a <- rnorm(7)
a <- a/sqrt(sum(a^2))
b <- rnorm(7)
b <- b/sqrt(sum(b^2))
A <- rotation(a, b)
A
a ; b
a %*% t(A)
```

Saddlepoint approximations of the Fisher-Bingham distributions
Saddlepoint approximations of the Fisher-Bingham distributions

Description

It calculates the logarithm of the normalising constant of the Fisher-Bingham distribution.

Usage

```
fb.saddle(gam, lam)
```

Arguments

gam	A numeric vector containing the parameters of the Fisher part.
lam	All the eigenvalues of the Bingham part. Not just the non zero ones.

Details

It calculate the three approximations given by Kume and Wood (2005) and it uses the Fisher-Bingham parametrization of that paper.

Value

A list including:

first oder	The first order approximation
second oder	The second order approximation
third oder	The third order approximation

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kume Alfred and Wood Andrew T.A. (2005). Saddlepoint approximations for the Bingham and Fisher-Bingham normalizing constants. *Biometrika*, 92(2):465-476

See Also

[kent.logcon](#), [rfb](#), [kent.mle](#), [rbingham](#)

Examples

```
p <- 3 ; k <- 1
0.5 * p * log(2 * pi) - (p/2 - 1) * log(k) + log( bessell(k, p/2 - 1, expon.scaled = TRUE) ) + k
## normalising constant of the
## von Mises-Fisher distribution
fb.saddle( c(0, k, 0), c(0, 0, 0) ) ## saddlepoint approximation

## Normalising constant of the Kent distribution
fb.saddle( c(0, 10, 0), c(0, -2, 2) )
kent.logcon(10, 2)
```

Simulation from a Bingham distribution using any symmetric matrix A
Simulation from a Bingham distribution using any symmetric matrix A

Description

It simulates random values from a Bingham distribution with any given symmetric matrix.

Usage

```
rbingham(n, A)
```

Arguments

n	The sample size.
A	A symmetric matrix.

Details

The eigenvalues are first calculated and then Chris Fallaize and Theo Kypraios's code (`f.rbing`) is used. The resulting simulated data are then right multiplied by the eigenvectors of the matrix A.

Value

A matrix with the simulated data.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kent J.T., Ganeiber A.M. and Mardia K.V. (2013). A new method to simulate the Bingham and related distributions in directional data analysis with applications <http://arxiv.org/pdf/1310.8110v1.pdf>
C.J. Fallaize and T. Kypraios (2014). Exact Bayesian Inference for the Bingham Distribution. Statistics and Computing (To appear). <http://arxiv.org/pdf/1401.2894v1.pdf>

See Also

[f.rbing](#), [rfb](#), [rvmf](#), [rkent](#)

Examples

```
A <- cov(iris[, 1:3])  
x <- rbingham(100, A)
```

Simulation from a Matrix Fisher distribution on $SO(3)$

Simulation from a Matrix Fisher distribution on $SO(3)$

Description

It simulates random samples (rotation matrices) from a Matrix Fisher distribution with any given parameter matrix, F (3×3).

Usage

```
rmatrixfisher(n, F)
```

Arguments

n	the sample size.
F	An arbitrary 3×3 matrix.

Details

Firstly corresponding Bingham parameter A is determined for a given Matrix Fisher parameter F using John Kent (2013) algorithm and then Bingham samples for parameter A are generated using `rbingham` code. Finally convert Bingham samples to Matrix Fisher samples according to the Kent (2013) transformation.

Value

An array with simulated rotation matrices.

Author(s)

Anamul Sajib & Chris Fallaize

R implementation and documentation: Anamul Sajib <pmxahsa@nottingham.ac.uk> & Chris Fallaize

References

Kent J.T., Ganeiber A.M. and Mardia K.V. (2013). A new method to simulate the Bingham and related distributions in directional data analysis with applications. <http://arxiv.org/pdf/1310.8110v1.pdf>

Examples

```
F <- matrix( c(85, 11, 41, 78, 39, 60, 43, 64, 48), ncol = 3) / 10 ### An arbitrary F matrix
a <- rmatrixfisher(10, F)
```

Simulation of random values from a Bingham distribution
Simulating from a Bingham distribution

Description

It simulates from a Bingham distribution using the code suggested by Kent et al. (2013).

Usage

```
f.rbing(n, lam)
```

Arguments

n	Sample size.
lam	Eigenvalues of the diagonal symmetric matrix of the Bingham distribution.

Details

The user must have calculated the eigenvalues of the diagonal symmetric matrix of the Bingham distribution. The function accepts the $q-1$ eigenvalues only. This means, that the user must have subtracted the lowest eigenvalue from the rest and give the non zero ones. The function uses rejection sampling and it was written by Chris Fallaize and Theo Kypraios (University of Nottingham) and kindly offered. Any questions on the code can be addressed to one of the two aforementioned people. It is slightly different than the one Kent et al. (2013) suggests.

Value

A list including:

x	The simulated data.
avtry	The estimate of M in the rejection sampling. The average number of simulated values before a value is accepted.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kent J.T., Ganeiber A.M. and Mardia K.V. (2013). A new method to simulate the Bingham and related distributions in directional data analysis with applications. <http://arxiv.org/pdf/1310.8110v1.pdf>

C.J. Fallaize and T. Kypraios (2014). Exact Bayesian Inference for the Bingham Distribution. Statistics and Computing (No volum assigned yet). <http://arxiv.org/pdf/1401.2894v1.pdf>

See Also

[rfb](#), [rvmf](#), [rbingham](#), [rkent](#), [link{rsop}](#)

Examples

```
x <- f.rbing( 100, c(1, 0.6, 0.1) )  
x
```

Simulation of random values from a mixture of von Mises-Fisher distributions
Random values simulation from a mixture of von Mises-Fisher distributions

Description

The function simulates random values simulation from a given mixture of von Mises-Fisher distributions.

Usage

```
rmixvmf(n, prob, mu, k)
```

Arguments

n	The sample size.
prob	This is a vector with the mixing probability of each group.
mu	A matrix with the mean direction of each group.
k	A vector with the concentration parameter of each group.

Details

The function simulates random values simulation from a given mixture of von Mises-Fisher distributions using the `rvmf` function.

Value

A list including:

id	An indicator of the group of each simulated vector.
x	A matrix with the simulated data.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kurt Hornik and Bettina Grün (2014). movMF: An R Package for Fitting Mixtures of von Mises-Fisher Distributions <http://cran.r-project.org/web/packages/movMF/vignettes/movMF.pdf>

See Also

[mix.vmf](#), [rvmf](#), [bic.mixvmf](#)

Examples

```
k <- runif(4, 4, 20)
prob <- c(0.2, 0.4, 0.3, 0.1)
mu <- matrix(rnorm(16), ncol = 4)
mu <- mu / sqrt( rowSums(mu^2) )
x <- rmixvmf(200, prob, mu, k)$x
bic.mixvmf(x, 5)
```

Simulation of random values from a spherical Fisher-Bingham distribution
Simulation of random values from a spherical Fisher-Bingham distribution

Description

Simulation of random values from a spherical Fisher-Bingham distribution.

Usage

```
rfb(n, k, m, A)
```

Arguments

n	The sample size.
k	The concentration parameter (Fisher part). It has to be greater than 0.
m	The mean direction (Fisher part).
A	A symmetric matrix (Bingham part).

Details

Random values from a spherical Fisher-Bingham distribution are generated. This function included the option of simulating from a Kent distribution also.

Value

A matrix with the simulated data.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kent J.T., Ganeiber A.M. and Mardia K.V. (2013). A new method to simulate the Bingham and related distributions in directional data analysis with applications. <http://arxiv.org/pdf/1310.8110v1.pdf>

See Also

[rbingham](#), [rvmf](#), [rkent](#), [f.rbing](#)

Examples

```
k <- 15
mu <- rnorm(3)
mu <- mu / sqrt( sum(mu^2) )
A <- cov(iris[, 1:3])
x <- rfb(50, k, mu, A)
vmf(x) ## fits a von Mises-Fisher distribution to the simulated data
## Next we simulate from a Kent distribution
A <- diag( c(-5, 0, 5) )
n <- 100
x <- rfb(n, k, mu, A) ## data follow a Kent distribution
kent.mle(x) ## fits a Kent distribution
vmf(x) ## fits a von Mises-Fisher distribution
A <- diag( c(5, 0, -5) )
n <- 100
x <- rfb(n, k, mu, A) ## data follow a Kent distribution
kent.mle(x) ## fits a Kent distribution
vmf(x) ## fits a von Mises-Fisher distribution
```

Simulation of random values from a spherical Kent distribution

Simulation of random values from a spherical Kent distribution

Description

Simulation of random values from a spherical Kent distribution.

Usage

```
rkent(n, k, m, b)
```

Arguments

n	The sample size.
k	The concentraion parameter κ . It has to be greater than 0.
m	The mean direction (Fisher part).
b	The ovalness parameter, β .

Details

Random values from a Kent distribution on the sphere are generated. The function generates from a spherical Kent distribution using `rfb` with an arbitrary mean direction and then rotates the data to have the desired mean direction.

Value

A matrix with the simulated data.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kent J.T., Ganeiber A.M. and Mardia K.V. (2013). A new method to simulate the Bingham and related distributions in directional data analysis with applications. <http://arxiv.org/pdf/1310.8110v1.pdf>

See Also

`rfb`, `rbingham`, `rvmf`, `f.rbing`

Examples

```
k <- 15
mu <- rnorm(3)
mu <- mu / sqrt( sum(mu^2) )
A <- diag( c(-5, 0, 5) )
x <- rfb(500, k, mu, A)
kent.mle(x)
y <- rkent(500, k, mu, A[3, 3])
kent.mle(y)
```

Simulation of random values from a von Mises-Fisher distribution

Random values simulation from a von Mises-Fisher distribution

Description

It generates random vectors following the von Mises-Fisher distribution. The data can be spherical or hyper-spherical.

Usage

```
rvmf(n, mu, k)
```

Arguments

n	The sample size.
mu	The mean direction.
k	The concentration parameter. If $k = 0$, random values from the spherical uniform will be drawn. Values from a multivariate normal distribution with zero mean vector and the identity matrix as the covariance matrix. Then each vector becomes a unit vector.

Details

It uses a rejection sampling as suggested by Andrew Wood (1994).

Value

A matrix with the simulated data.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Wood A. T. A. (1994). Simulation of the von Mises Fisher distribution. *Communications in statistics-simulation and computation*, 23(1): 157–164.

Dhillon I. S. & Sra S. (2003). Modeling data using directional distributions. Technical Report TR-03-06, Department of Computer Sciences, The University of Texas at Austin. <http://citeseerx.ist.psu.edu/viewdoc/download?>

See Also

[vmf](#), [rfb](#), [racg](#), [rvonmises](#), [rmixvmf](#)

Examples

```
m <- rnorm(4)
m <- m/sqrt(sum(m^2))
x <- rvmf(100, m, 25)
m
vmf(x)
```

Spherical and hyperspherical median

Fast calculation of the spherical and hyperspherical median

Description

It calculates, very fast, the (hyper-)spherical median of a sample.

Usage

```
mediandir(x)
mediandir_2(x)
```

Arguments

x The data, a numeric matrix with unit vectors.

Details

The "mediandir" employs a fixed point iterative algorithm stemming from the first derivative (Cabrera and Watson, 1990) to find the median direction as described by Fisher (1985) and Fisher, Lewis and Embleton (1987). In the big samples this is much much faster than "mediandir_2", since the search is based on iterations.

Value

The median direction.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

- Fisher N. I. (1985). Spherical medians. *Journal of the Royal Statistical Society. Series B*, 47(2): 342-348.
- Fisher N. I., Lewis T. and Embleton B. J. (1987). *Statistical analysis of spherical data*. Cambridge university press.
- Cabrera J. and Watson G. S. (1990). On a spherical median related distribution. *Communications in Statistics-Theory and Methods*, 19(6): 1973-1986.

See Also

[vmf](#), [kent.mle](#)

Examples

```
m <- rnorm(3)
m <- m / sqrt( sum(m^2) )
x <- rvmf(100, m, 10)
mediandir(x)
mediandir_2(x)
```

Spherical-spherical correlation

Spherical-spherical correlation

Description

Correlation between two spherical variables.

Usage

```
spher.cor(x, y)
```

Arguments

x	A spherical variable. A matrix with three columns, each row is a unit vector.
y	A spherical variable. A matrix with three columns, each row is a unit vector.

Details

A very similar to the classical correlation is calculated. In addition, a hypothesis test for no correlation is performed. Note, that this is a squared correlation actually, so negative values will never be returned.

Value

A vector including:

R-squared	The value of the squared correlation.
p-value	The p-value of the no correlation hypothesis testing.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kanti V. Mardia and Peter E. Jupp. Directional statistics, pg. 254–255.

See Also

[spher.reg](#), [vmf](#), [circ.cor1](#), [circ.cor2](#)

Examples

```
x <- rvmf(100, rnorm(3), 10)
y <- rvmf(100, rnorm(3), 10)
spher.cor(x, y)
```

Spherical-spherical regression

Spherical-Spherical regression

Description

It performs regression when both the dependent and independent variables are spherical.

Usage

```
spher.reg(y, x, rads = FALSE)
```

Arguments

y	The dependent variable; a matrix with either two columns, latitude and longitude, either in radians or in degrees. Alternatively it is a matrix with three columns, unit vectors.
x	The dependent variable; a matrix with either two columns, latitude and longitude, either in radians or in degrees. Alternatively it is a matrix with three columns, unit vectors. The two matrices must agree in the scale and dimensions.
rads	If the data are expressed in latitude and longitude then it matter to know if they are in radians or degrees. If they are in radians, then this should be TRUE and FALSE otherwise. If the previous argument, euclidean, is TRUE, this one does not matter what its value is.

Details

Spherical regression as proposed by Chang (1986) is implemented. If the estimated rotation matrix has a determinant equal to -1, singular value decomposition is performed and the last unit vector of the second matrix is multiplied by -1.

Value

A list including:

A	The estimated rotation matrix.
fitted	The fitted values in Euclidean coordinates (unit vectors).

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Ted Chang (1986). Spherical Regression. *Annals of Statistics*, 14(3): 907–924.

See Also

[spher.cor](#), [spml.reg](#), [circ.cor1](#), [circ.cor2](#)

Examples

```
mx <- rnorm(3)
mx <- mx/sqrt( sum(mx^2) )
my <- rnorm(3)
my <- my/sqrt( sum(my^2) )
x <- rvmf(100, mx, 15)
A <- rotation(mx, my)
y <- x %*% t(A)
mod <- spher.reg(y, x)
A
mod$A ## exact match, no noise
y <- x %*% t(A)
y <- y + rvmf(100, colMeans(y), 40)
mod <- spher.reg(y, x)
A
mod$A ## noise added, more realistic example
```

Summary statistics for circular data

Summary statistics for circular data

Description

It produces a few summary measures for circular data.

Usage

```
circ.summary(u, rads = FALSE, fast = FALSE, tol = 1e-09, plot = TRUE)
```

Arguments

<code>u</code>	A vector with circular data.
<code>rads</code>	If the data are in rads, then this should be TRUE, otherwise FALSE.
<code>fast</code>	A boolean variable to do a faster implementation.
<code>tol</code>	The tolerance level to stop the Newton-Raphson algorithm for finding kappa.
<code>plot</code>	If you want to see the data plotted on a circle make this TRUE.

Details

It returns the circular mean, mean resultant length, variance, standard deviation and concentration parameter. So, basically it returns the estimated values of the parameters of the von Mises distribution.

Value

If `fast = FALSE` a list including all the following. If `fast = TRUE` less items are returned.

<code>mesos</code>	The circular mean direction.
<code>confint</code>	The 95% confidence interval for the circular mean direction.
<code>kappa</code>	The concentration parameter.
<code>MRL</code>	The mean resultant length.
<code>circvariance</code>	The circular variance.
<code>circstd</code>	The circular standard deviation.
<code>loglik</code>	The log-likelihood of the fitted von Mises distribution.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chichester: John Wiley & Sons.

See Also

[rvonmises](#), [vm.kde](#), [vmf](#), [group.vm](#), [hcf.circaov](#)

Examples

```
x <- rvonmises(50, 2.5, 15, rads = TRUE)
circ.summary(x, rads = TRUE, plot = TRUE)
```

Summary statistics for grouped circular data

Summary statistics for grouped circular data

Description

It produces a few summary measures for grouped circular data.

Usage

```
group.vm(group, fi, rads = FALSE)
```

Arguments

group	A matrix denoting the classes. Each row consists of two numbers, the lower and upper points of each class.
fi	The frequency of each class of data.
rads	If the data are in rads, then this should be TRUE, otherwise FALSE.

Details

It returns the circular mean, mean resultant length, variance, standard deviation and concentration parameter. So, basically it returns the estimated values of the parameters of the von Mises distribution. The mean resultant length though is group corrected.

Value

A list including:

mesos	The circular mean direction.
confint	The 95% confidence interval for the circular mean direction.
kappa	The concentration parameter.
MRL	The mean resultant length.
circvariance	The circular variance.
circstd	The circular standard deviation.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Pewsey Arthur, Markus Neuhauser and Graeme D. Ruxton (2013). Circular statistics in R. Oxford University Press.

Mardia K. V. and Jupp P. E. (2000). Directional statistics. Chicester: John Wiley & Sons.

See Also

[circ.summary](#), [rvonmises](#), [vm.kde](#)

Examples

```
x <- rvonmises(200, 3, 10)
a <- circ.summary(x, rads = TRUE, plot = FALSE)
group <- seq(min(x) - 0.1, max(x) + 0.1, length = 6)
y <- cut(x, breaks = group, length = 6)
group <- matrix( c( group[1], rep(group[2:5], each = 2), group[6]), ncol = 2, byrow = TRUE)
fi <- as.vector( table(y) )
b <- group.vm(group, fi, rads = TRUE)
a
b
```

Test for a given mean direction

Test for a given mean direction

Description

A log-likelihood ratio test for testing whether the sample mena direction is equal to some predefined one.

Usage

```
meandir.test(x, mu, B = 999)
```

Arguments

x	A matrix with the data, unit vectors.
mu	A unit vector with the hypothesized mean direction.
B	A number either 1, so no bootstrap calibration is performed or more than 1, so bootstrap calibration is performed.

Details

The log-likelihood ratio test is performed.

Value

A list including:

<code>mean.dir</code>	The sample mean direction
<code>pvalue</code>	The p-value of the test.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chichester: John Wiley & Sons.

See Also

[vmf](#), [kent.mle](#), [rayleigh](#)

Examples

```
mu <- rnorm(5)
mu <- mu / sqrt( sum(mu^2) )
x <- rvmf(100, mu, 10)
meandir.test(x, mu, 1)
meandir.test(x, mu, 499)
```

Test for equality of concentration parameters for spherical data

Test for equality of concentration parameters for spherical data

Description

This tests the equality of concentration parameters for spherical data only.

Usage

```
spherconc.test(x, ina)
```

Arguments

<code>x</code>	A matrix with the data in Euclidean coordinates, i.e. unit vectors
<code>ina</code>	A variable indicating the groupings of the observations.

Details

The test is designed for spherical data only.

Value

A list including:

<code>mess</code>	A message stating the value of the mean resultant and which test statistic was used, U1, U2 or U3.
<code>res</code>	A vector containing the test statistic and its p-value.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Kanti V. Mardia and Peter E. Jupp. Directional statistics, pg. 226-227.

See Also

[het.aov](#), [lr.aov](#), [embed.aov](#), [hcf.aov](#), [conc.test](#)

Examples

```
x <- rvmf(100, rnorm(3), 15)
ina <- rep(1:4, each = 25)
spherconc.test(x, ina)
```

Test of equality of the concentration parameters for circular data

A test for testing the equality of the concentration parameter among g samples, where $g \geq 2$ for circular data

Description

A test for testing the equality of the concentration parameter among g samples, where $g \geq 2$ for circular data.

Usage

```
conc.test(u, ina, rads = FALSE)
```

Arguments

<code>u</code>	A numeric vector containing the values of all samples.
<code>ina</code>	A numerical variable or factor indicating the groups of each value.
<code>rads</code>	If the data are in radians this should be TRUE and FALSE otherwise.

Details

This test works for circular data.

Value

A list including:

<code>mess</code>	A message informing the use of the test statistic used.
<code>res</code>	A numeric vector containing the value of the test statistic and its associated p-value.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Mardia, K. V. and Jupp, P. E. (2000). Directional statistics. Chichester: John Wiley & Sons.

See Also

[embed.circaov](#), [hcf.circaov](#), [lr.circaov](#), [het.circaov](#)

Examples

```
x <- rvonmises(100, 2.4, 15)
ina <- rep(1:4, each = 25)
conc.test(x, ina, rads = TRUE)
```

Tuning of the bandwidth parameter in the von Mises kernel

Tuning of the bandwidth parameter in the von Mises kernel for circular data

Description

Tuning of the bandwidth parameter in the von Mises kernel for circular data. Cross validation is used.

Usage

```
vmkde.tune(u, low = 0.1, up = 1, rads = TRUE)
```


Arguments

u	The data, a numerical vector.
low	The lower value of h to search.
up	The lower value of h to search.
rads	If the data are in radians this should be TRUE and FALSE otherwise.

Details

Tuning of the bandwidth parameter in the von Mises kernel for circular data via cross validation. The procedure is fast because an optimiser is used.

Value

A vector including two elements:

Optimal h	The best H found.
cv	The value of the maximised pseudo-likelihood.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Taylor C. C. (2008). Automatic bandwidth selection for circular density estimation. *Computational Statistics & Data Analysis*, 52(7), 3493–3500.

Wand M. P., & Jones M. C. (1994). *Kernel smoothing*. CrC Press.

See Also

[vm.kde](#), [vmfkde.tune](#), [vmf.kde](#)

Examples

```
u <- rvonmises(100, 2.4, 10, rads = TRUE)
vmkde.tune(u)
```

Tuning of the bandwidth parameter in the von Mises-Fisher kernel
*Tuning of the bandwidth parameter in the von Mises-Fisher kernel for
(hyper-)spherical data*

Description

Tuning of the bandwidth parameter in the von Mises-Fisher kernel for (hyper-)spherical data with cross validation.

Usage

```
vmfkde.tune(x, low = 0.1, up = 1)
```

Arguments

x	A matrix with the data in Euclidean coordinates, i.e. unit vectors.
low	The lower value of the bandwidth to search.
up	The upper value of the bandwidth to search.

Details

Fast tuning of the bandwidth parameter in the von Mises-Fisher kernel for (hyper-)spherical data via cross validation.

Value

A vector including two elements:

Optimal h	The best H found.
cv	The value of the maximised pseudo-likelihood.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Garcia Portugues E. (2013). Exact risk improvement of bandwidth selectors for kernel density estimation with directional data. *Electronic Journal of Statistics*, 7, 1655–1685.

Wand M. P., and Jones M. C. (1994). *Kernel smoothing*. Crc Press.

See Also

[vmf.kde](#), [vmf.kerncontour](#), [vm.kde](#), [vmkde.tune](#)

Examples

```
x <- rvmf(100, rnorm(3), 15)
vmfkde.tune(x)
```

Tuning of the k-NN algorithm using the arc cosinus distance

k-NN algorithm using the arc cosinus distance. Tuning the k neighbours

Description

It estimates the percentage of correct classification via an m-fold cross validation. The bias is estimated as well using the algorithm suggested by Tibshirani and Tibshirani (2009) and is subtracted.

Usage

```
dirknn.tune(z, M = 10, A = 5, ina, type = "S", mesos = TRUE, mat = NULL)
```

Arguments

<code>z</code>	The data, a numeric matrix with unit vectors.
<code>M</code>	The number of folds for the m-fold cross validation, set to 10 by default.
<code>A</code>	The maximum number of nearest neighbours, set to 5 by default. The 1 nearest neighbour is not used.
<code>ina</code>	A variable indicating the groups of the data <code>x</code> .
<code>type</code>	If type is "S", the standard k-NN algorithm is to be used, else "NS" for the non standard one. See below (details) for more information.
<code>mesos</code>	A boolean variable used only in the case of the non standard algorithm (type="NS"). Should the average of the distances be calculated (TRUE) or not (FALSE)? If it is FALSE, the harmonic mean is calculated.
<code>mat</code>	You can specify your own folds by giving a <code>mat</code> , where each column is a fold. Each column contains indices of the observations. You can also leave it NULL and it will create folds.

Details

The standard algorithm is to keep the `k` nearest observations and see the groups of these observations. The new observation is allocated to the most frequent seen group. The non standard algorithm is to calculate the classical mean or the harmonic mean of the `k` nearest observations for each group. The new observation is allocated to the group with the smallest mean distance. The estimated bias is calculated as Tibshirani and Tibshirani (2009) suggested.

We have made an efficient (not very much efficient though) memory allocation. Even if you have hundreds of thousands of observations, the computer will not clush, it will only take longer. Instead of calculating the distance matrix once in the beginning we calculate the distances of the out-of-sample observations from the rest. If we calculated the distance matrix in the beginning, once, the resulting matrix could have dimensions thousands by thousands. This would not fit into the memory. If you have a few hundred of observations, the runtime is about the same (maybe less, maybe more) as calculating the distance matrix in the first place.

Value

A list including:

per	The average percent of correct classification across the neighbours.
percent	The bias corrected percent of correct classification.
runtime	The run time of the algorithm. A numeric vector. The first element is the user time, the second element is the system time and the third element is the elapsed time.

Author(s)

Michail Tsagris R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Tibshirani, Ryan J., and Robert Tibshirani. A bias correction for the minimum error rate in cross-validation. *The Annals of Applied Statistics* (2009), 3(2): 822-829.

See Also

[dirknn](#), [vmf.da](#), [mix.vmf](#)

Examples

```
k <- runif(4, 4, 20)
prob <- c(0.2, 0.4, 0.3, 0.1)
mu <- matrix(rnorm(16), ncol = 4)
mu <- mu / sqrt( rowSums(mu^2) )
da <- rmixvmf(200, prob, mu, k)
x <- da$x
ina <- da$id
dirknn.tune(x, M = 5, A = 10, ina, type = "S", mesos = TRUE)
dirknn.tune(x, M = 10, A = 5, ina, type = "S", mesos = TRUE)
```

Tuning of the k-NN regression

Tuning of the k-NN regression with Euclidean or (hyper-)spherical response and or predictor variables

Description

Tuning of the k-NN regression with Euclidean or (hyper-)spherical response and or predictor variables. It estimates the percentage of correct classification via an m-fold cross validation. The bias is estimated as well using the algorithm suggested by Tibshirani and Tibshirani (2009) and is subtracted.

Usage

```
knnreg.tune(y, x, M = 10, A = 10, ncores = 1, res = "eucl", type = "euclidean",
  estim = "arithmetic", mat = NULL, graph = FALSE)
```

Arguments

y	The currently available data, the response variables values. A matrix with either euclidean (univariate or multivariate) or (hyper-)spherical data. If you have a circular response, say u, transform it to a unit vector via (cos(u), sin(u)).
x	The currently available data, the predictor variables values. A matrix with either euclidean (univariate or multivariate) or (hyper-)spherical data. If you have a circular response, say u, transform it to a unit vector via (cos(u), sin(u)).
M	The number of folds for the m-fold cross validation, set to 10 by default.
A	The maximum number of nearest neighbours, set to 5 by default. The maximum is actually A + 1, since 1 nearest neighbour is not used.
ncores	How many cores to use. This is taken into account only when the predictor variables are spherical.
res	The type of the response variable. If it is Euclidean, set this argument equal to "res". If it is a unit vector set it to res="spher".
type	The type of distance to be used. This determines the nature of the predictor variables. This is actually the argument "method" of the distance function in R. The default value is "euclidean". R has several options the type of the distance. Just type ?dist in R and see the methods. Any method can be given here. If you have unit vectors in general, you should put type="spher", so that the cosinus distance is calculated.
estim	Once the k observations with the smallest distance are discovered, what should the prediction be? The arithmetic average of the corresponding y values be used estim="arithmetic" or their harmonic average estim="harmonic".
mat	You can specify your own folds by giving a mat, where each column is a fold. Each column contains indices of the observations. You can also leave it NULL and it will create folds.
graph	If this is TRUE a graph with the results will appear.

Details

Tuning of the k-NN regression with Euclidean or (hyper-)spherical response and or predictor variables. It estimates the percentage of correct classification via an m-fold cross validation. The bias is estimated as well using the algorithm suggested by Tibshirani and Tibshirani (2009) and is subtracted. The sum of squares of prediction is used in the case of Euclidean responses. In the case of spherical responses the $\sum \hat{y}_i^T y_i$ is calculated.

Value

A list including:

crit	The value of the criterion to minimise/maximise for all values of the nearest neighbours.
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best_k	The best value of the nearest neighbours.
performance	The bias corrected optimal value of the criterion, along with the estimated bias. For the case of Euclidean response this will be higher than the crit and for the case of spherical responses it will be lower than crit.
runtime	The run time of the algorithm. A numeric vector. The first element is the user time, the second element is the system time and the third element is the elapsed time.

Author(s)

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References

Tibshirani, Ryan J., and Robert Tibshirani. A bias correction for the minimum error rate in cross-validation. *The Annals of Applied Statistics* (2009), 3(2): 822-829.

See Also

[knn.reg](#), [spher.reg](#), [dirknn.tune](#)

Examples

```

y <- iris[, 1]
x <- iris[, 2:4]
x <- x/ sqrt( rowSums(x^2) ) ## Euclidean response and spherical predictors
knnreg.tune(y, x, A = 5, res = "eucl", type = "spher", estim = "arithmetic",
mat = NULL, graph = TRUE)

y <- iris[, 1:3]
y <- y/ sqrt( rowSums(y^2) ) ## Spherical response and Euclidean predictor
x <- iris[, 2]
knnreg.tune(y, x, A = 5, res = "spher", type = "euclidean", estim = "arithmetic",
mat = NULL, graph = TRUE)

```

Uniformity test for circular data

Uniformity tests for circular data.

Description

Hypothesis tests of uniformity for circular data.

Usage

```
kuiper(u, rads = FALSE, R = 1)
```

```
watson(u, rads = FALSE, R = 1)
```

Arguments

u	A numeric vector containing the circular data, which can be expressed in degrees or radians.
rads	A boolean variable. If the data are in radians, put this TRUE. If the data are expressed in degrees make this FALSE.
R	If R = 1 the asymptotic p-value will be calculated. If R is greater than 1 the bootstrap p-value is returned.

Details

The high concentration (`hcf.circaov`), log-likelihood ratio (`lr.circaov`), embedding approach (`embed.circaov`) or the non equal concentration parameters approach (`het.circaov`) is used.

Value

A vector including:

Test	The value of the test statistic.
p-value	The p-value of the test (bootstrap or asymptotic depends upon the value of the argument R).

Author(s)

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References

Jammalamadaka, S. Rao and SenGupta, A. (2001). Topics in Circular Statistics, pg. 153-55 (Kuiper's test) & 156-157 (Watson's test).

See Also

[rayleigh](#), [vmf](#), [rvonmises](#)

Examples

```
x <- rvonmises(n = 40, m = 2, k = 10)
kuiper(x, rads = TRUE)
watson(x, rads = TRUE)
x <- rvonmises(40, m = 2, k = 0)
kuiper(x, rads = TRUE)
watson(x, rads = TRUE)
```

Unit vector(s) with a given angle

Unit vector(s) with a given angle

Description

Unit vector with a given angle from a given unit vector.

Usage

```
vec(x, n = 1, deg = 90)
```

Arguments

x	A unit vector. If it is not a unit vector it becomes one.
n	The number of unit vectors to return.
deg	The angle between the given vector and the n vectors to be returned. This must be in degrees and it has to be between 0 and 180 degrees. If the angle is 0, the same unit vector will be returned. If the angle is 180, the same unit vector with the signs changed will be returned.

Details

The user provides a unit vector and the degrees. The function will return n unit vectors whose angle with the given unit vector equals the degrees given. For example, if you want 10 unit vectors perpendicular to the x put `vec(x, 10, 90)`.

Value

A list including:

runtime	The runtime of the procedure.
crit	The calculated angle between the given unit vector and each of the generated unit vectors.
mat	A matrix with the n unit vectors.

Author(s)

Michail Tsagris R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

See Also

[rvmf](#), [rbingham](#), [rfb](#)

Examples

```
x <- rnorm(10)
x <- x / sqrt( sum(x^2) )
a <- vec(x, 20, 90)
```

von Mises kernel density estimation

Kernel density estimation of circular data with a von Mises kernel

Description

Kernel density estimation of circular data with a von Mises kernel.

Usage

```
vm.kde(u, h, thumb = "none", rads = TRUE)
```

Arguments

u	A numeric vector containing the data.
h	The bandwidth.
thumb	It can be either "none", so the bandwidth the user has set will be used, "tay" for the method of Taylor (2008) or "rot" for the method of Garcia-Portugues (2013).
rads	If the data are in radians, this should be TRUE and FALSE otherwise.

Details

The user has the option to use a bandwidth he/she has found in some way (cross-validation) or estimate it as Taylor (2008) or Garcia-Portugues (2013).

Value

A list including:

h	The bandwidth. If the user chose one of "tay" or "rot" the estimated bandwidth will be returned.
f	The kernel density estimate at the observed points.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Taylor, C. C. (2008). Automatic bandwidth selection for circular density estimation. *Computational Statistics & Data Analysis*, 52(7): 3493-3500.

Garcia Portugues, E. (2013). Exact risk improvement of bandwidth selectors for kernel density estimation with directional data. *Electronic Journal of Statistics*, 7, 1655-1685.

See Also

[vmkde.tune](#), [vmfkde.tune](#), [vmf.kde](#)

Examples

```
x <- rvonmises(100, 2.4, 10, rads = TRUE)
hist(x, freq = FALSE)
f1 <- vm.kde(x, h = 0.1, thumb = "rot", rads = TRUE)$f
f2 <- vm.kde(x, h = 0.1, thumb = "tay", rads = TRUE)$f
h <- vmkde.tune(x)[1]
f3 <- vm.kde(x, h = h, thumb = "none", rads = TRUE)$f
points(x, f1, col = 1)
points(x, f2, col = 2)
points(x, f3, col = 3)
```

von Mises-Fisher kernel density estimation for (hyper-)spherical data
Kernel density estimation for (hyper-)spherical data using a von Mises-Fisher kernel

Description

A von Mises-Fisher kernel is used for the non parametric density estimation.

Usage

```
vmf.kde(x, h, thumb = "none")
```

Arguments

x	A matrix with unit vectors, i.e. the data being expressed in Euclidean coordinates.
h	The bandwidth to be used.
thumb	If this is "none", the given bandwidth is used. If it is "rot" the rule of thumb suggested by Garcia-Portugues (2013) is used.

Details

A von Mises-Fisher kernel is used for the non parametric density estimation.

Value

A list including:

- h The bandwidth used.
- f A vector with the kernel density estimate calculated for each of the data points.

Author(s)

Michail Tsagris

R implementation and documentation: Michail Tsagris <mtsagris@yahoo.gr> and Giorgos Athineou <athineou@csd.uoc.gr>

References

Garcia Portugues, E. (2013). Exact risk improvement of bandwidth selectors for kernel density estimation with directional data. *Electronic Journal of Statistics*, 7, 1655-1685.

See Also

[vmfkde.tune](#), [vm.kde](#), [vmf](#), [vmkde.tune](#)

Examples

```
x <- rvmf(100, rnorm(5), 15)
h <- vmfkde.tune(x)[1]
f1 <- vm.kde(x, h = h, thumb = "none")
f2 <- vm.kde(x, h = h, thumb = "rot")
f1$h ; f2$h
```

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