Package 'ftsa'

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LazyData yes

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Description

Functions for visualizing, modeling, forecasting and hypothesis testing of functional time series.

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`ftsa-package` *Functional Time Series Analysis*

Description

This package presents descriptive statistics of functional data; implements principal component regression and partial least squares regression to provide point and distributional forecasts for functional data; utilizes functional linear regression, ordinary least squares, penalized least squares, ridge regression, and moving block approaches to dynamically update point and distributional forecasts when partial data points in the most recent curve are observed; performs stationarity test for a functional time series; estimates a long-run covariance function by kernel sandwich estimator.

Author(s)

Rob J Hyndman and Han Lin Shang

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References

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H. L. Shang (2011) "rainbow: an R package for visualizing functional time series, *The R Journal*, **3**(2), 54-59.

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H. L. Shang (2013) "ftsa: An R package for analyzing functional time series", *The R Journal*, **5**(1), 64-72.

H. L. Shang, A. Wisniowski, J. Bijak, P. W. F. Smith and J. Raymer (2014) "Bayesian functional models for population forecasting", in M. Marsili and G. Capacci (eds), Proceedings of the Sixth Eurostat/UNECE Work Session on Demographic Projections, Istituto nazionale di statistica, Rome, pp. 313-325.

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centre *Mean function, variance function, median function, trim mean function of functional data*

Description

Mean function, variance function, median function, trim mean function of functional data

Usage

```
centre(x, type)
```

Arguments

x	An object of class matrix.
type	Mean, variance, median or trim mean?

Value

Return mean function, variance function, median function or trim mean function.

Author(s)

Han Lin Shang

See Also

pcscorebootstrapdata, mean.fts, median.fts, sd.fts, var.fts

Examples

```
# mean function is often removed in the functional principal component analysis.
# trimmed mean function is sometimes employed for robustness in the presence of outliers.
# In calculating trimmed mean function, several functional depth measures were employed.
centre(x = ElNino$y, type = "mean")
```

diff.fts *Differences of a functional time series*

Description

Computes differences of a fts object at each variable.

Usage

```
## S3 method for class 'fts'
diff(x, lag = 1, differences = 1, ...)
```

Arguments

x	An object of class fts.
lag	An integer indicating which lag to use.
differences	An integer indicating the order of the difference.
...	Other arguments.

Value

An object of class fts.

Author(s)

Rob J Hyndman

Examples

```
# ElNino is an object of sliced functional time series.
# Differencing is sometimes used to achieve stationarity.
diff(x = ElNino)
```

---

dynamic_FLR *Dynamic updates via functional linear regression*

Description

A functional linear regression is used to address the problem of dynamic updating, when partial data in the most recent curve are observed.

Usage

```
dynamic_FLR(dat, newdata, holdoutdata, order_k_percent = 0.9, order_m_percent = 0.9,
    pcd_method = c("classical", "M"), robust_lambda = 2.33, bootrep = 100,
     pointfore, level = 80)
```

Arguments

dat	An object of class sfts.
newdata	A data vector of newly arrived observations.
holdoutdata	A data vector of holdout sample to evaluate point forecast accuracy.
order_k_percent	Select the number of components that explains at least 90 percent of the total variation.
order_m_percent	Select the number of components that explains at least 90 percent of the total variation.

pcd_method	Method to use for principal components decomposition. Possibilities are "M", "rapca" and "classical".
robust_lambda	Tuning parameter in the two-step robust functional principal component analysis, when pcdmethod = "M".
bootrep	Number of bootstrap samples.
pointfore	If pointfore = TRUE, point forecasts are produced.
level	Nominal coverage probability.

Details

This function is designed to dynamically update point and interval forecasts, when partial data in the most recent curve are observed.

Value

update_forecast	Updated forecasts.
holdoutdata	Holdout sample.
err	Forecast errors.
order_k	Number of principal components in the first block of functions.
order_m	Number of principal components in the second block of functions.
update_comb	Bootstrapped forecasts for the dynamically updating time period.
update_comb_lb_ub	By taking corresponding quantiles, obtain lower and upper prediction bounds.
err_boot	Bootstrapped in-sample forecast error for the dynamically updating time period.

Author(s)

Han Lin Shang

References

J-M. Chiou (2012) "Dynamical functional prediction and classification with application to traffic flow prediction", *Annals of Applied Statistics*, **6**(4), 1588-1614.

H. L. Shang (2015) "Forecasting Intraday S&P 500 Index Returns: A Functional Time Series Approach", Working paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2647233.

See Also

dynupdate

Examples

```
dynamic_FLR_point = dynamic_FLR(dat = ElNino$y[,1:56], newdata = ElNino$y[1:4,57],
holdoutdata = ElNino$y[5:12,57], pointfore = TRUE)

dynamic_FLR_interval = dynamic_FLR(dat = ElNino$y[,1:56], newdata = ElNino$y[1:4,57],
holdoutdata = ElNino$y[5:12,57], pointfore = FALSE)
```

dynupdate *Dynamic updates via BM, OLS, RR and PLS methods*

Description

Four methods, namely block moving (BM), ordinary least squares (OLS) regression, ridge regression (RR), penalized least squares (PLS) regression, were proposed to address the problem of dynamic updating, when partial data in the most recent curve are observed.

Usage

```
dynupdate(data, newdata = NULL, holdoutdata, method = c("ts", "block",
 "ols", "pls", "ridge"), fmethod = c("arima", "ar", "ets", "ets.na",
  "rwdrift", "rw"), pcdmethod = c("classical", "M", "rapca"),
   ngrid = max(1000, ncol(data$y)), order = 6,
    robust_lambda = 2.33, lambda = 0.01, value = FALSE,
     interval = FALSE, level = 80,
      pimethod = c("parametric", "nonparametric"), B = 1000)
```

Arguments

data	An object of class sfts.
newdata	A data vector of newly arrived observations.
holdoutdata	A data vector of holdout sample to evaluate point forecast accuracy.
method	Forecasting methods. The latter four can dynamically update point forecasts.
fmethod	Univariate time series forecasting methods used in method = "ts" or method = "block".
pcdmethod	Method to use for principal components decomposition. Possibilities are "M", "rapca" and "classical".
ngrid	Number of grid points to use in calculations. Set to maximum of 1000 and ncol(data$y).
order	Number of principal components to fit.
robust_lambda	Tuning parameter in the two-step robust functional principal component analysis, when pcdmethod = "M".
lambda	Penalty parameter used in method = "pls" or method = "ridge".
value	When value = TRUE, returns forecasts or when value = FALSE, returns forecast errors.
interval	When interval = TRUE, produces distributional forecasts.
level	Nominal coverage probability.
pimethod	Parametric or nonparametric method to construct prediction intervals.
B	Number of bootstrap samples.

Details

This function is designed to dynamically update point and interval forecasts, when partial data in the most recent curve are observed.

If `method = "classical"`, then standard functional principal component decomposition is used, as described by Ramsay and Dalzell (1991).

If `method = "rapca"`, then the robust principal component algorithm of Hubert, Rousseeuw and Verboven (2002) is used.

If `method = "M"`, then the hybrid algorithm of Hyndman and Ullah (2005) is used.

Value

`forecasts`	An object of class `fts` containing the dynamic updated point forecasts.
`bootsamp`	An object of class `fts` containing the bootstrapped point forecasts, which are updated by the PLS method.
`low`	An object of class `fts` containing the lower bound of prediction intervals.
`up`	An object of class `fts` containing the upper bound of prediction intervals.

Author(s)

Han Lin Shang

References

J. O. Ramsay and C. J. Dalzell (1991) "Some tools for functional data analysis (with discussion)", *Journal of the Royal Statistical Society: Series B*, **53**(3), 539-572.

M. Hubert and P. J. Rousseeuw and S. Verboven (2002) "A fast robust method for principal components with applications to chemometrics", *Chemometrics and Intelligent Laboratory Systems*, **60**(1-2), 101-111.

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H. L. Shang (2015) "Forecasting Intraday S&P 500 Index Returns: A Functional Time Series Approach", Working paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2647233.

See Also

`ftsm`, `forecast.ftsm`, `plot.fm`, `residuals.fm`, `summary.fm`

Examples

```
# ElNino is an object of sliced functional time series, constructed from a univariate time series.
# When we observe some newly arrived information in the most recent time period, this function
# allows us to update the point and interval forecasts for the remaining time period.
dynupdate(data = ElNino, newdata = ElNino$y[1:4,57], holdoutdata = ElNino$y[5:12,57],
    method = "block", interval = FALSE)
```

---

error *Forecast error measure*

Description

Computes the forecast error measure.

Usage

```
error(forecast, forecastbench, true, insampletrue, method = c("me", "mpe", "mae",
 "mse", "sse", "rmse", "mdae", "mdse", "mape", "mdape", "smape",
  "smdape", "rmspe", "rmdspe", "mrae", "mdrae", "gmrae",
   "relmae", "relmse", "mase", "mdase", "rmsse"), giveall = FALSE)
```

Arguments

`forecast`	Out-of-sample forecasted values.
`forecastbench`	Forecasted values using a benchmark method, such as random walk.
`true`	Out-of-sample holdout values.
`insampletrue`	Insample values.
`method`	Method of forecast error measure.
`giveall`	If `giveall = TRUE`, all error measures are provided.

Details

Bias measure:

If `method = "me"`, the forecast error measure is mean error.

If `method = "mpe"`, the forecast error measure is mean percentage error.

Forecast accuracy error measure:

If `method = "mae"`, the forecast error measure is mean absolute error.

If `method = "mse"`, the forecast error measure is mean square error.

If `method = "sse"`, the forecast error measure is sum square error.

If `method = "rmse"`, the forecast error measure is root mean square error.

If `method = "mdae"`, the forecast error measure is median absolute error.

If `method = "mape"`, the forecast error measure is mean absolute percentage error.

If `method = "mdape"`, the forecast error measure is median absolute percentage error.

If `method = "rmspe"`, the forecast error measure is root mean square percentage error.

If `method = "rmdspe"`, the forecast error measure is root median square percentage error.

Forecast accuracy symmetric error measure:

If `method = "smape"`, the forecast error measure is symmetric mean absolute percentage error.

If `method = "smdape"`, the forecast error measure is symmetric median absolute percentage error.

Forecast accuracy relative error measure:

If `method = "mrae"`, the forecast error measure is mean relative absolute error.

If `method = "mdrae"`, the forecast error measure is median relative absolute error.

If `method = "gmrae"`, the forecast error measure is geometric mean relative absolute error.

If `method = "relmae"`, the forecast error measure is relative mean absolute error.

If `method = "relmse"`, the forecast error measure is relative mean square error.

Forecast accuracy scaled error measure:

If `method = "mase"`, the forecast error measure is mean absolute scaled error.

If `method = "mdase"`, the forecast error measure is median absolute scaled error.

If `method = "rmsse"`, the forecast error measure is root mean square scaled error.

Value

A numeric value.

Author(s)

Han Lin Shang

References

P. A. Thompson (1990) "An MSE statistic for comparing forecast accuracy across series", *International Journal of Forecasting*, **6**(2), 219-227.

C. Chatfield (1992) "A commentary on error measures", *International Journal of Forecasting*, **8**(1), 100-102.

S. Makridakis (1993) "Accuracy measures: theoretical and practical concerns", *International Journal of Forecasting*, **9**(4), 527-529.

R. J. Hyndman and A. Koehler (2006) "Another look at measures of forecast accuracy", *International Journal of Forecasting*, **22**(3), 443-473.

Examples

```
# Forecast error measures can be categorized into three groups: (1) scale-dependent,
# (2) scale-independent but with possible zero denominator,
# (3) scale-independent with non-zero denominator.
error(forecast = 1:2, true = 3:4, method = "mae")
error(forecast = 1:5, forecastbench = 6:10, true = 11:15, method = "mrae")
error(forecast = 1:5, forecastbench = 6:10, true = 11:15, insampletrue = 16:20,
giveall = TRUE)
```

`extract` *Extract variables or observations*

Description

Creates subsets of a `fts` object.

Usage

```
extract(data, direction = c("time", "x"), timeorder, xorder)
```

Arguments

`data`	An object of `fts`.
`direction`	In time direction or x variable direction?
`timeorder`	Indexes of time order.
`xorder`	Indexes of x variable order.

Value

When `xorder` is specified, it returns a `fts` object with same argument as data but with a subset of `x` variables.

When `timeorder` is specified, it returns a `fts` object with same argument as data but with a subset of `time` variables.

Author(s)

Han Lin Shang

Examples

```
# ElNino is an object of class sliced functional time series.
# This function truncates the data series rowwise or columnwise.
extract(data = ElNino, direction = "time", timeorder = 1980:2006) # Last 27 curves
extract(data = ElNino, direction = "x", xorder = 1:8) # First 8 x variables
```

`farforecast` *Functional data forecasting through functional principal component autoregression*

Description

The coefficients from the fitted object are forecasted using a multivariate time-series forecasting method. The forecast coefficients are then multiplied by the functional principal components to obtain a forecast curve.

Usage

```
farforecast(object, h = 10, var_type = "const", level = 80, PI = FALSE)
```

Arguments

`object`	An object of `ftsm`.
`h`	Forecast horizon.
`var_type`	Type of multivariate time series forecasting method; see `VAR` for details.
`level`	Nominal coverage probability of prediction error bands.
`PI`	When `PI = TRUE`, a prediction interval will be given along with the point forecast.

Details

1. Decompose the smooth curves via a functional principal component analysis (FPCA).

2. Fit a multivariate time-series model to the principal component score matrix.

3. Forecast the principal component scores using the fitted multivariate time-series models. The order of VAR is selected optimally via an information criterion.

4. Multiply the forecast principal component scores by estimated principal components to obtain forecasts of $f_{n+h}(x)$.

5. Prediction intervals are constructed by taking quantiles of the one-step-ahead forecast errors.

Value

`point_fore`	Point forecast
`PI_lb`	Lower bound of a prediction interval
`PI_ub`	Upper bound of a prediction interval

Author(s)

Han Lin Shang

References

A. Aue, D. D. Norinho and S. Hormann (2015) "On the prediction of stationary functional time series", *Journal of the American Statistical Association*, **110**(509), 378-392.

J. Klepsch, C. Kl\"uppelberg and T. Wei (2016) "Prediction of functional ARMA processes with an application to traffic data", URL: http://arxiv.org/abs/1603.02049.

See Also

forecast.ftsm, forecastfplsr

Examples

```
ElNino_subset = extract(data = ElNino, direction = "time",timeorder = 1967:2006)
ex_PI = farforecast(ftsm(y = ElNino_subset), PI = TRUE)
plot(extract(ex_PI$point_fore, direction = "time", 1), ylim = c(14, 31))
lines(extract(ex_PI$PI_lb, direction = "time", 1), lty = 2, col = 2)
lines(extract(ex_PI$PI_ub, direction = "time", 1), lty = 2, col = 2)
```

---

fbootstrap *Bootstrap independent and identically distributed functional data*

Description

Computes bootstrap or smoothed bootstrap samples based on independent and identically distributed functional data.

Usage

```
fbootstrap(data, estad = func.mean, alpha = 0.05, nb = 200, suav = 0,
 media.dist = FALSE, graph = FALSE, ...)
```

Arguments

data	An object of class fds or fts.
estad	Estimate function of interest. Default is to estimate the mean function. Other options are func.mode or func.var.
alpha	Significance level used in the smooth bootstrapping.
nb	Number of bootstrap samples.
suav	Smoothing parameter.
media.dist	Estimate mean function.
graph	Graphical output.
...	Other arguments.

Value

A list containing the following components is returned.

`estimate`	Estimate function.
`max.dist`	Max distance of bootstrap samples.
`rep.dist`	Distances of bootstrap samples.
`resamples`	Bootstrap samples.
`center`	Functional mean.

Author(s)

Han Lin Shang

References

A. Cuevas and M. Febrero and R. Fraiman (2006), "On the use of the bootstrap for estimating functions with functional data", *Computational Statistics and Data Analysis*, **51**(2), 1063-1074.

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D. Gervini (2012) "Outlier detection and trimmed estimation in general functional spaces", *Statistica Sinica*, **22**(4), 1639-1660.

H. L. Shang (2015) "Re-sampling techniques for estimating the distribution of descriptive statistics of functional data", *Communication in Statistics–Simulation and Computation*, 44(3), 614-635.

See Also

`pcscorebootstrapdata`

Examples

```
# Bootstrapping the distribution of a summary statistics of functional data.
fbootstrap(data = ElNino)
```

forecast.ftsm *Forecast functional time series*

Description

The coefficients from the fitted object are forecasted using either an ARIMA model (`method = "arima"`), an AR model (`method = "ar"`), an exponential smoothing method (`method = "ets"`), a linear exponential smoothing method allowing missing values (`method = "ets.na"`), or a random walk with drift model (`method = "rwdrift"`). The forecast coefficients are then multiplied by the principal components to obtain a forecast curve.

Usage

```
## S3 method for class 'ftsm'
forecast(object, h = 10, method = c("ets", "arima", "ar", "ets.na",
 "rwdrift", "rw", "struct", "arfima"), level = 80, jumpchoice = c("fit",
  "actual"), pimethod = c("parametric", "nonparametric"), B = 100,
   usedata = nrow(object$coeff), adjust = TRUE, model = NULL,
    damped = NULL, stationary = FALSE, ...)
```

Arguments

`object`	Output from `ftsm`.
`h`	Forecast horizon.
`method`	Univariate time series forecasting methods. Current possibilities are "ets", "arima", "ets.na", "rwdrift" and "rw".
`level`	Coverage probability of prediction intervals.
`jumpchoice`	Jump-off point for forecasts. Possibilities are "actual" and "fit". If "actual", the forecasts are bias-adjusted by the difference between the fit and the last year of observed data. Otherwise, no adjustment is used. See Booth et al. (2006) for the detail on jump-off point.
`pimethod`	Indicates if parametric method is used to construct prediction intervals.
`B`	Number of bootstrap samples.
`usedata`	Number of time periods to use in forecasts. Default is to use all.
`adjust`	If `adjust = TRUE`, adjusts the variance so that the one-step forecast variance matches the empirical one-step forecast variance.
`model`	If the `ets` method is used, `model` allows a model specification to be passed to `ets()`.
`damped`	If the `ets` method is used, `damped` allows the damping specification to be passed to `ets()`.
`stationary`	If `stationary = TRUE`, `method` is set to `method = "ar"` and only stationary AR models are used.
`...`	Other arguments passed to forecast routine.

Details

1. Obtain a smooth curve $f_t(x)$ for each t using a nonparametric smoothing technique.

2. Decompose the smooth curves via a functional principal component analysis.

3. Fit a univariate time series model to each of the principal component scores.

4. Forecast the principal component scores using the fitted time series models.

5. Multiply the forecast principal component scores by fixed principal components to obtain forecasts of $f_{n+h}(x)$.

6. The estimated variances of the error terms (smoothing error and model residual error) are used to compute prediction intervals for the forecasts.

Value

List with the following components:

`mean`	An object of class `fts` containing point forecasts.
`lower`	An object of class `fts` containing lower bound for prediction intervals.
`upper`	An object of class `fts` containing upper bound for prediction intervals.
`fitted`	An object of class `fts` of one-step-ahead forecasts for historical data.
`error`	An object of class `fts` of one-step-ahead errors for historical data.
`coeff`	List of objects of type `forecast` containing the coefficients and their forecasts.
`coeff.error`	One-step-ahead forecast errors for each of the coefficients.
`var`	List containing the various components of variance: model, error, mean, total and coeff.
`model`	Fitted `ftsm` model.
`bootsamp`	An array of $dimension = c(p, B, h)$ containing the bootstrapped point forecasts. p is the number of variables. B is the number of bootstrap samples. h is the forecast horizon.

Author(s)

Rob J Hyndman

References

H. Booth and R. J. Hyndman and L. Tickle and P. D. Jong (2006) "Lee-Carter mortality forecasting: A multi-country comparison of variants and extensions", *Demographic Research*, **15**, 289-310.

B. Erbas and R. J. Hyndman and D. M. Gertig (2007) "Forecasting age-specific breast cancer mortality using functional data model", *Statistics in Medicine*, **26**(2), 458-470.

R. J. Hyndman and M. S. Ullah (2007) "Robust forecasting of mortality and fertility rates: A functional data approach", *Computational Statistics and Data Analysis*, **51**(10), 4942-4956.

R. J. Hyndman and H. Booth (2008) "Stochastic population forecasts using functional data models for mortality, fertility and migration", *International Journal of Forecasting*, **24**(3), 323-342.

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H. L. Shang, P. W. F. Smith, J. Bijak, A. Wisniowski (2015) "A multilevel functional data method for forecasting population, with an application to the United Kingdom, *International Journal of Forecasting*, forthcoming.

See Also

`ftsm`, `forecastfplsr`, `plot.ftsf`, `plot.fm`, `residuals.fm`, `summary.fm`

Examples

```
# ElNino is an object of class sliced functional time series.
# Via functional principal component decomposition, the dynamic was captured
# by a few principal components and principal component scores.
# By using an exponential smoothing method,
# the principal component scores are forecasted.
# The forecasted curves are constructed by forecasted principal components
# times fixed principal components plus the mean function.
forecast(object = ftsm(ElNino), h = 10, method = "ets")
forecast(object = ftsm(ElNino, weight = TRUE))
```

`forecastfplsr` *Forecast functional time series*

Description

The decentralized response is forecasted by multiplying the estimated regression coefficient with the new decentralized predictor

Usage

```
forecastfplsr(object, components = 2, h = 20)
```

Arguments

object	An object of class fts.
components	Number of optimal components.
h	Forecast horizon.

Value

A fts class object, containing forecasts of responses.

Author(s)

Han Lin Shang

References

R. J. Hyndman and H. L. Shang (2009) "Forecasting functional time series" (with discussion), *Journal of the Korean Statistical Society*, **38**(3), 199-221.

See Also

forecast.ftsm, ftsm, plot.fm, plot.ftsf, residuals.fm, summary.fm

Examples

```
# A set of functions are decomposed by functional partial least squares decomposition.
# By forecasting univariate partial least squares scores, the forecasted curves are
# obtained by multiplying the forecasted scores by fixed functional partial least
# squares function plus fixed mean function.
forecastfplsr(object = Australiasmoothfertility, h = 5)
```

---

fplsr *Functional partial least squares regression*

Description

Fits a functional partial least squares (PLSR) model using nonlinear partial least squares (NIPALS) algorithm or simple partial least squares (SIMPLS) algorithm.

Usage

```
fplsr(data, order = 6, type = c("simpls", "nipals"), unit.weights =
 TRUE, weight = FALSE, beta = 0.1, interval = FALSE, method =
  c("delta", "boota"), alpha = 0.05, B = 100, adjust = FALSE,
   backh = 10)
```

Arguments

data	An object of class fts.
order	Number of principal components to fit.
type	When type = "nipals", uses the NIPALS algorithm; when type = "simpls", uses the SIMPLS algorithm.
unit.weights	Constrains predictor loading weights to have unit norm.
weight	When weight = TRUE, a set of geometrically decaying weights is applied to the decentralized data.
beta	When weight = TRUE, the speed of geometric decay is governed by a weight parameter.
interval	When interval = TRUE, produces distributional forecasts.
method	Method used for computing prediction intervals.
alpha	1-alpha gives the nominal coverage probability.
B	Number of replications.
adjust	When adjust = TRUE, an adjustment is performed.
backh	When adjust = TRUE, an adjustment is performed by evaluating the difference between predicted and actual values in a testing set. backh specifies the testing set.

Details

Point forecasts:

The NIPALS function implements the orthogonal scores algorithm, as described in Martens and Naes (1989). This is one of the two classical PLSR algorthms, the other is the simple partial least squares regression in DeJong (1993). The difference between these two approaches is that the NIPALS deflates the original predictors and responses, while the SIMPLS deflates the covariance matrix of original predictors and responses. Thus, SIMPLS is more computationally efficient than NIPALS.

In a functional data set, the functional PLSR can be performed by setting the functional responses to be 1 lag ahead of the functional predictors. This idea has been adopted from the Autoregressive Hilbertian processes of order 1 (ARH(1)) of Bosq (2000).

Distributional forecasts:

Parametric method:

Influenced by the works of Denham (1997) and Phatak et al. (1993), one way of constructing prediction intervals in the PLSR is via a local linearization method (also known as the Delta method). It can be easily understood as the first two terms in a Taylor series expansion. The variance of coefficient estimators can be approximated, from which an analytic-formula based prediction intervals are constructed.

Nonparametric method:

After discretizing and decentralizing functional data $f_t(x)$ and $g_s(y)$, a PLSR model with K latent components is built. Then, the fit residuals $o_s(y_i)$ between $g_s(y_i)$ and $\hat{g}_s(y_i)$ are calculated as

$$o_s(y_i) = g_s(y_i) - \hat{g}_s(y_i), i = 1, ..., p.$$

The next step is to generate B bootstrap samples $o_s^b(y_i)$ by randomly sampling with replacement from $[o_1(y_i), ..., o_n(y_i)]$. Adding bootstrapped residuals to the original response variables in order to generate new bootstrap responses,

$$g_s^b(y_i) = g_s(y_i) + o_s^b(y_i).$$

Then, the PLSR models are constructed using the centered and discretized predictors and bootstrapped responses to obtain the boostrapped regression coefficients and point forecasts, from which the empirical prediction intervals and kernel density plots are constructed.

Value

A list containing the following components is returned.

`B`	$(p \times m)$ matrix containing the regression coefficients. p is the number of variables in the predictors and m is the number of variables in the responses.
`P`	$(p \times order)$ matrix containing the predictor loadings.
`Q`	$(m \times order)$ matrix containing the response loadings.
`T`	`(ncol(data$y)-1) x order` matrix containing the predictor scores.
`R`	$(p \times order)$ matrix containing the weights used to construct the latent components of predictors.
`Yscores`	`(ncol(data$y)-1) x order` matrix containing the response scores.
`projection`	$(p \times order)$ projection matrix used to convert predictors to predictor scores.
`meanX`	An object of class `fts` containing the column means of predictors.
`meanY`	An object of class `fts` containing the column means of responses.
`Ypred`	An object of class `fts` containing the 1-step-ahead predicted values of the responses.
`fitted`	An object of class `fts` containing the fitted values.
`residuals`	An object of class `fts` containing the regression residuals.
`Xvar`	A vector with the amount of predictor variance explained by each number of component.
`Xtotvar`	Total variance in predictors.
`weight`	When `weight = TRUE`, a set of geometrically decaying weights is given. When `weight = FALSE`, weights are all equal 1.
`x1`	Time period of a `fts` object, which can be obtained from `colnames(data$y)`.
`y1`	Variables of a `fts` object, which can be obtained from `data$x`.
`ypred`	Returns the original functional predictors.
`y`	Returns the original functional responses.

bootsamp	Bootstrapped point forecasts.
lb	Lower bound of prediction intervals.
ub	Upper bound of prediction intervals.
lbadj	Adjusted lower bound of prediction intervals.
ubadj	Adjusted upper bound of prediction intervals.
lbadjfactor	Adjusted lower bound factor, which lies generally between 0.9 and 1.1.
ubadjfactor	Adjusted upper bound factor, which lies generally between 0.9 and 1.1.

Author(s)

Han Lin Shang

References

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P. T. Reiss and R. T. Ogden (2007), "Functional principal component regression and functional partial least squares", *Journal of the American Statistical Association*, **102**(479), 984-996.

A. Delaigle and P. Hall (2012), "Methodology and theory for partial least squares applied to functional data", *Annals of Statistics*, **40**(1), 322-352.

C. Preda, G. Saporta (2005) "PLS regression on a stochastic process", *Computational Statistics and Data Analysis*, **48**(1), 149-158.

C. Preda, G. Saporta, C. Leveder (2007) "PLS classification of functional data", *Computational Statistics*, **22**, 223-235.

See Also

ftsm, forecast.ftsm, plot.fm, summary.fm, residuals.fm, plot.fmres

Examples

```
# When weight = FALSE, all observations are assigned equally.
# When weight = TRUE, all observations are assigned geometrically decaying weights.
fplsr(ElNino, order = 6, type = "nipals")
fplsr(data = ElNino, order = 6)
fplsr(data = ElNino, weight = TRUE)
fplsr(data = ElNino, unit.weights = FALSE)
fplsr(data = ElNino, unit.weights = FALSE, weight = TRUE)

# The prediction intervals are calculated numerically.
fplsr(data = ElNino, interval = TRUE, method = "delta")

# The prediction intervals are calculated by bootstrap method.
fplsr(data = ElNino, interval = TRUE, method = "boota")
```

---

ftsm *Fit functional time series model*

Description

Fits a principal component model to a fts object. The function uses optimal orthonormal principal components obtained from a principal components decomposition.

Usage

```
ftsm(y, order = 6, ngrid = max(500, ncol(y$y)), method = c("classical",
 "M", "rapca"), mean = TRUE, level = FALSE, lambda = 3,
  weight = FALSE, beta = 0.1, ...)
```

Arguments

y	An object of class fts.
order	Number of principal components to fit.
ngrid	Number of grid points to use in calculations. Set to maximum of 500 and ncol(y$y).
method	Method to use for principal components decomposition. Possibilities are "M", "rapca" and "classical".
mean	If mean = TRUE, it will estimate mean term in the model before computing basis terms. If mean = FALSE, the mean term is assumed to be zero.

level	If mean = TRUE, it will include an additional (intercept) term that depends on t but not on x.
lambda	Tuning parameter for robustness when method = "M".
weight	When weight = TRUE, a set of geometrically decaying weights is applied to the decentralized data.
beta	When weight = TRUE, the speed of geometric decay is governed by a weight parameter.
...	Additional arguments controlling the fitting procedure.

Details

If method = "classical", then standard functional principal component decomposition is used, as described by Ramsay and Dalzell (1991).

If method = "rapca", then the robust principal component algorithm of Hubert, Rousseeuw and Verboven (2002) is used.

If method = "M", then the hybrid algorithm of Hyndman and Ullah (2005) is used.

Value

Object of class "ftsm" with the following components:

x1	Time period of a fts object, which can be obtained from colnames(y$y).
y1	Variables of a fts object, which can be obtained from y$x.
y	Original functional time series or sliced functional time series.
basis	Matrix of principal components evaluated at value of y$x (one column for each principal component). The first column is the fitted mean or median.
basis2	Matrix of principal components excluded from the selected model.
coeffs	Matrix of coefficients (one column for each coefficient series). The first column is all ones.
coeff2	Matrix of coefficients associated with the principal components excluded from the selected model.
fitted	An object of class fts containing the fitted values.
residuals	An object of class fts containing the regression residuals (difference between observed and fitted).
varprop	Proportion of variation explained by each principal component.
wt	Weight associated with each time period.
v	Measure of variation for each time period.
mean.se	Measure of standar error associated with the mean.

Author(s)

Rob J Hyndman

References

J. O. Ramsay and C. J. Dalzell (1991) "Some tools for functional data analysis (with discussion)", *Journal of the Royal Statistical Society: Series B*, **53**(3), 539-572.

M. Hubert and P. J. Rousseeuw and S. Verboven (2002) "A fast robust method for principal components with applications to chemometrics", *Chemometrics and Intelligent Laboratory Systems*, **60**(1-2), 101-111.

B. Erbas and R. J. Hyndman and D. M. Gertig (2007) "Forecasting age-specific breast cancer mortality using functional data model", *Statistics in Medicine*, **26**(2), 458-470.

R. J. Hyndman and M. S. Ullah (2007) "Robust forecasting of mortality and fertility rates: A functional data approach", *Computational Statistics and Data Analysis*, **51**(10), 4942-4956.

R. J. Hyndman and H. Booth (2008) "Stochastic population forecasts using functional data models for mortality, fertility and migration", *International Journal of Forecasting*, **24**(3), 323-342.

R. J. Hyndman and H. L. Shang (2009) "Forecasting functional time series (with discussion)", *Journal of the Korean Statistical Society*, **38**(3), 199-221.

See Also

`ftsmweightselect`, `forecast.ftsm`, `plot.fm`, `plot.ftsf`, `residuals.fm`, `summary.fm`

Examples

```
# ElNino is an object of class sliced functional time series, constructed
# from a univariate time series.
# By default, all observations are assigned with equal weighting.
ftsm(y = ElNino, order = 6, method = "classical", weight = FALSE)
# When weight = TRUE, geometrically decaying weights are used.
ftsm(y = ElNino, order = 6, method = "classical", weight = TRUE)
```

`ftsmiterativeforecasts` *Forecast functional time series*

Description

The coefficients from the fitted object are forecasted using either an ARIMA model (`method = "arima"`), an AR model (`method = "ar"`), an exponential smoothing method (`method = "ets"`), a linear exponential smoothing method allowing missing values (`method = "ets.na"`), or a random walk with drift model (`method = "rwdrift"`). The forecast coefficients are then multiplied by the principal components to obtain a forecast curve.

Usage

```
ftsmiterativeforecasts(object, components, iteration = 20)
```

Arguments

object	An object of class fts.
components	Number of principal components.
iteration	Number of iterative one-step-ahead forecasts.

Details

1. Obtain a smooth curve $f_t(x)$ for each t using a nonparametric smoothing technique.

2. Decompose the smooth curves via a functional principal component analysis.

3. Fit a univariate time series model to each of the principal component scores.

4. Forecast the principal component scores using the fitted time series models.

5. Multiply the forecast principal component scores by fixed principal components to obtain forecasts of $f_{n+h}(x)$.

6. The estimated variances of the error terms (smoothing error and model residual error) are used to compute prediction intervals for the forecasts.

Value

List with the following components:

mean	An object of class fts containing point forecasts.
lower	An object of class fts containing lower bound for prediction intervals.
upper	An object of class fts containing upper bound for prediction intervals.
fitted	An object of class fts of one-step-ahead forecasts for historical data.
error	An object of class fts of one-step-ahead errors for historical data.
coeff	List of objects of type forecast containing the coefficients and their forecasts.
coeff.error	One-step-ahead forecast errors for each of the coefficients.
var	List containing the various components of variance: model, error, mean, total and coeff.
model	Fitted ftsm model.
bootsamp	An array of $dim = c(p, B, h)$ containing the bootstrapped point forecasts. p is the number of variables. B is the number of bootstrap samples. h is the forecast horizon.

Author(s)

Han Lin Shang

References

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R. J. Hyndman and M. S. Ullah (2007) "Robust forecasting of mortality and fertility rates: A functional data approach", *Computational Statistics and Data Analysis*, **51**(10), 4942-4956.

R. J. Hyndman and H. Booth (2008) "Stochastic population forecasts using functional data models for mortality, fertility and migration", *International Journal of Forecasting*, **24**(3), 323-342.

R. J. Hyndman and H. L. Shang (2009) "Forecasting functional time series" (with discussion), *Journal of the Korean Statistical Society*, **38**(3), 199-221.

See Also

`ftsm`, `plot.ftsf`, `plot.fm`, `residuals.fm`, `summary.fm`

Examples

```
# Iterative one-step-ahead forecasts via functional principal component analysis.
ftsmiterativeforecasts(object = Australiasmoothfertility, components = 2, iteration = 5)
```

`ftsmweightselect` *Selection of the weight parameter used in the weighted functional time series model.*

Description

The geometrically decaying weights are used to estimate the mean curve and functional principal components, where more weights are assigned to the more recent data than the data from the distant past.

Usage

```
ftsmweightselect(data, ncomp = 6, ntrainyear, errorcriterion = c("mae", "mse", "mape"))
```

Arguments

`data`	An object of class `fts`.
`ncomp`	Number of components.
`ntrainyear`	Number of holdout observations used to assess the forecast accuracy.
`errorcriterion`	Error measure.

Details

The data set is split into a fitting period and forecasting period. Using the data in the fitting period, we compute the one-step-ahead forecasts and calculate the forecast error. Then, we increase the fitting period by one, and carry out the same forecasting procedure until the fitting period covers entire data set. The forecast accuracy is determined by the averaged forecast error across the years in the forecasting period. By using an optimization algorithm, we select the optimal weight parameter that would result in the minimum forecast error.

Value

Optimal weight parameter.

Note

Can be computational intensive, as it takes about half-minute to compute. For example, ftsmweightselect(ElNinosmooth, ntrainyear = 1).

Author(s)

Han Lin Shang

References

R. J. Hyndman and H. L. Shang (2009) "Forecasting functional time series (with discussion)", *Journal of the Korean Statistical Society*, **38**(3), 199-221.

See Also

`ftsm`, `forecast.ftsm`

`is.fts` *Test for functional time series*

Description

Tests whether an object is of class `fts`.

Usage

```
is.fts(x)
```

Arguments

`x`	Arbitrary R object.

Author(s)

Rob J Hyndman

Examples

```
# check if ElNino is the class of the functional time series.
is.fts(x = ElNino)
```

`isfe.fts` *Integrated Squared Forecast Error for models of various orders*

Description

Computes integrated squared forecast error (ISFE) values for functional time series models of various orders.

Usage

```
isfe.fts(data, max.order = N - 3, N = 10, h = 5:10, method =
 c("classical", "M", "rapca"), mean = TRUE, level = FALSE,
  fmethod = c("arima", "ar", "ets", "ets.na", "struct", "rwdrift",
   "rw", "arfima"), lambda = 3, ...)
```

Arguments

`data`	An object of class `fts`.
`max.order`	Maximum number of principal components to fit.
`N`	Minimum number of functional observations to be used in fitting a model.
`h`	Forecast horizons over which to average.
`method`	Method to use for principal components decomposition. Possibilities are "M", "rapca" and "classical".
`mean`	Indicates if mean term should be included.
`level`	Indicates if level term should be included.
`fmethod`	Method used for forecasting. Current possibilities are "ets", "arima", "ets.na", "struct", "rwdrift" and "rw".
`lambda`	Tuning parameter for robustness when `method = "M"`.
`...`	Additional arguments controlling the fitting procedure.

Value

Numeric matrix with `(max.order+1)` rows and `length(h)` columns containing ISFE values for models of orders `0:(max.order)`.

Note

This function can be very time consuming for data with large dimensionality or large sample size. By setting `max.order` small, computational speed can be dramatically increased.

Author(s)

Rob J Hyndman

References

R. J. Hyndman and M. S. Ullah (2007) "Robust forecasting of mortality and fertility rates: A functional data approach", *Computational Statistics and Data Analysis*, **51**(10), 4942-4956.

See Also

`ftsm`, `forecast.ftsm`, `plot.fm`, `plot.fmres`, `summary.fm`, `residuals.fm`

---

long_run_covariance_estimation

Estimating long-run covariance function for a functional time series

---

Description

Bandwidth estimation in the long-run covariance function for a functional time series, using different types of kernel function

Usage

```
long_run_covariance_estimation(dat, C0 = 3, H = 3)
```

Arguments

`dat`	A matrix of p by n, where p denotes the number of grid points and n denotes sample size
`C0`	A tuning parameter used in the adaptive bandwidth selection algorithm of Rice
`H`	A tuning parameter used in the adaptive bandwidth selection algorithm of Rice

Value

An estimated covariance function of size (p by p)

Author(s)

Han Lin Shang

References

L. Horvath, G. Rice and S. Whipple (2016) Adaptive bandwidth selection in the long run covariance estimation of functional time series, Computational Statistics and Data Analysis, 100, 676-693.

G. Rice and H. L. Shang (2016) A plug-in bandwidth selection procedure for long run covariance estimation with stationary functional time series, Journal of Time Series Analysis, in press.

See Also

fts

Examples

```
dum = long_run_covariance_estimation(dat = ElNino$y)
```

---

mean.fts *Mean functions for functional time series*

Description

Computes mean of functional time series at each variable.

Usage

```
## S3 method for class 'fts'
mean(x, method = c("coordinate", "FM", "mode", "RP", "RPD", "radius"),
 na.rm = TRUE, alpha, beta, weight, ...)
```

Arguments

x	An object of class fts.
method	Method for computing the mean function.
na.rm	A logical value indicating whether NA values should be stripped before the computation proceeds.
alpha	Tuning parameter when method="radius".
beta	Trimming percentage, by default it is 0.25, when method="radius".
weight	Hard thresholding or soft thresholding.
...	Other arguments.

Details

If method = "coordinate", it computes the coordinate-wise functional mean.

If method = "FM", it computes the mean of trimmed functional data ordered by the functional depth of Fraiman and Muniz (2001).

If method = "mode", it computes the mean of trimmed functional data ordered by h-modal functional depth.

If method = "RP", it computes the mean of trimmed functional data ordered by random projection depth.

If method = "RPD", it computes the mean of trimmed functional data ordered by random projection derivative depth.

If method = "radius", it computes the mean of trimmed functional data ordered by the notion of alpha-radius.

Value

A list containing x = variables and y = mean rates.

Author(s)

Rob J Hyndman, Han Lin Shang

References

O. Hossjer and C. Croux (1995) "Generalized univariate signed rank statistics for testing and estimating a multivariate location parameter", *Journal of Nonparametric Statistics*, **4**(3), 293-308.

A. Cuevas and M. Febrero and R. Fraiman (2006) "On the use of bootstrap for estimating functions with functional data", *Computational Statistics \& Data Analysis*, **51**(2), 1063-1074.

A. Cuevas and M. Febrero and R. Fraiman (2007), "Robust estimation and classification for functional data via projection-based depth notions", *Computational Statistics*, **22**(3), 481-496.

M. Febrero and P. Galeano and W. Gonzalez-Manteiga (2007) "A functional analysis of NOx levels: location and scale estimation and outlier detection", *Computational Statistics*, **22**(3), 411-427.

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M. Febrero and P. Galeano and W. Gonzalez-Manteiga (2010) "Measures of influence for the functional linear model with scalar response", *Journal of Multivariate Analysis*, **101**(2), 327-339.

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D. Gervini (2012) "Outlier detection and trimmed estimation in general functional spaces", *Statistica Sinica*, **22**(4), 1639-1660.

See Also

median.fts, var.fts, sd.fts, quantile.fts

Examples

```
# Calculate the mean function by the different depth measures.
mean(x = ElNino, method = "coordinate")
mean(x = ElNino, method = "FM")
mean(x = ElNino, method = "mode")
mean(x = ElNino, method = "RP")
mean(x = ElNino, method = "RPD")
mean(x = ElNino, method = "radius", alpha = 0.5, beta = 0.25, weight = "hard")
mean(x = ElNino, method = "radius", alpha = 0.5, beta = 0.25, weight = "soft")
```

median.fts *Median functions for functional time series*

Description

Computes median of functional time series at each variable.

Usage

```
## S3 method for class 'fts'
median(x, na.rm, method = c("hossjercroux", "coordinate", "FM", "mode",
 "RP", "RPD", "radius"), alpha, beta, weight, ...)
```

Arguments

x	An object of class fts.
na.rm	Remove any missing value.
method	Method for computing median.
alpha	Tuning parameter when method="radius".
beta	Trimming percentage, by default it is 0.25, when method="radius".
weight	Hard thresholding or soft thresholding.
...	Other arguments.

Details

If method = "coordinate", it computes a coordinate-wise median.

If method = "hossjercroux", it computes the L1-median using the Hossjer-Croux algorithm.

If method = "FM", it computes the median of trimmed functional data ordered by the functional depth of Fraiman and Muniz (2001).

If method = "mode", it computes the median of trimmed functional data ordered by h-modal functional depth.

If method = "RP", it computes the median of trimmed functional data ordered by random projection depth.

If method = "RPD", it computes the median of trimmed functional data ordered by random projection derivative depth.

If method = "radius", it computes the mean of trimmed functional data ordered by the notion of alpha-radius.

Value

A list containing x = variables and y = median rates.

Author(s)

Rob J Hyndman, Han Lin Shang

References

O. Hossjer and C. Croux (1995) "Generalized univariate signed rank statistics for testing and estimating a multivariate location parameter", *Journal of Nonparametric Statistics*, **4**(3), 293-308.

A. Cuevas and M. Febrero and R. Fraiman (2006) "On the use of bootstrap for estimating functions with functional data", *Computational Statistics \& Data Analysis*, **51**(2), 1063-1074.

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M. Febrero and P. Galeano and W. Gonzalez-Manteiga (2007) "A functional analysis of NOx levels: location and scale estimation and outlier detection", *Computational Statistics*, **22**(3), 411-427.

M. Febrero and P. Galeano and W. Gonzalez-Manteiga (2008) "Outlier detection in functional data by depth measures, with application to identify abnormal NOx levels", *Environmetrics*, **19**(4), 331-345.

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J. A. Cuesta-Albertos and A. Nieto-Reyes (2010) "Functional classification and the random Tukey depth. Practical issues", Combining Soft Computing and Statistical Methods in Data Analysis, Advances in Intelligent and Soft Computing, Volume 77, 123-130.

D. Gervini (2012) "Outlier detection and trimmed estimation in general functional spaces", *Statistica Sinica*, **22**(4), 1639-1660.

See Also

`mean.fts`, `var.fts`, `sd.fts`, `quantile.fts`

Examples

```
# Calculate the median function by the different depth measures.
median(x = ElNino, method = "hossjercroux")
median(x = ElNino, method = "coordinate")
median(x = ElNino, method = "FM")
median(x = ElNino, method = "mode")
median(x = ElNino, method = "RP")
median(x = ElNino, method = "RPD")
median(x = ElNino, method = "radius", alpha = 0.5, beta = 0.25, weight = "hard")
median(x = ElNino, method = "radius", alpha = 0.5, beta = 0.25, weight = "soft")
```

MFDM *Multilevel functional data method*

Description

Fit a multilevel functional principal component model. The function uses two-step functional principal component decompositions.

Usage

```
MFDM(mort_female, mort_male, mort_ave, percent_1 = 0.95, percent_2 = 0.95, fh,
     level = 80, alpha = 0.2, MCMCiter = 100, fmethod = c("auto_arima", "ets"),
   BC = c(FALSE, TRUE), lambda)
```

Arguments

mort_female	Female mortality.
mort_male	Male mortality.
mort_ave	Total mortality.
percent_1	Cumulative percentage used for determining the number of common functional principal components.
percent_2	Cumulative percentage used for determining the number of sex-specific functional principal components.
fh	Forecast horizon.
level	Nominal coverage probability of a prediction interval.
alpha	1 - Nominal coverage probability.
MCMCiter	Number of MCMC iterations.
fmethod	Univariate time-series forecasting method.
BC	If Box-Cox transformation is performed.
lambda	If BC = TRUE, specify a Box-Cox transformation parameter.

Details

The basic idea of multilevel functional data method is to decompose functions from different subpopulations into an aggregated average, a sex-specific deviation from the aggregated average, a common trend, a sex-specific trend and measurement error. The common and sex-specific trends are modelled by projecting them onto the eigenvectors of covariance operators of the aggregated and sex-specific centred stochastic process, respectively.

Value

`first_percent`	Percentage of total variation explained by the first common functional principal component.
`female_percent`	Percentage of total variation explained by the first female functional principal component in the residual.
`male_percent`	Percentage of total variation explained by the first male functional principal component in the residual.
`mort_female_fore`	Forecast female mortality in the original scale.
`mort_male_fore`	Forecast male mortality in the original scale.

Note

It can be quite time consuming, especially when MCMCiter is large.

Author(s)

Han Lin Shang

References

C. M. Crainiceanu and J. Goldsmith (2010) "Bayesian functional data analysis using WinBUGS", *Journal of Statistical Software*, **32**(11).

C-Z. Di and C. M. Crainiceanu and B. S. Caffo and N. M. Punjabi (2009) "Multilevel functional principal component analysis", *The Annals of Applied Statistics*, **3**(1), 458-488.

V. Zipunnikov and B. Caffo and D. M. Yousem and C. Davatzikos and B. S. Schwartz and C. Crainiceanu (2015) "Multilevel functional principal component analysis for high-dimensional data", *Journal of Computational and Graphical Statistics*, **20**, 852-873.

H. L. Shang, P. W. F. Smith, J. Bijak, A. Wisniowski (2016) "A multilevel functional data method for forecasting population, with an application to the United Kingdom", *International Journal of Forecasting*, **forthcoming**.

See Also

`ftsm`, `forecast.ftsm`, `fplsr`, `forecastfplsr`

---

pcscorebootstrapdata *Bootstrap independent and identically distributed functional data or functional time series*

Description

Computes bootstrap or smoothed bootstrap samples based on either independent and identically distributed functional data or functional time series.

Usage

```
pcscorebootstrapdata(dat, bootrep, statistic, bootmethod = c("st", "sm",
"mvn", "stiefel", "meboot"), smo)
```

Arguments

dat	An object of class matrix.
bootrep	Number of bootstrap samples.
statistic	Summary statistics.
bootmethod	Bootstrap method. When bootmethod = "st", the sampling with replacement is implemented. To avoid the repeated bootstrap samples, the smoothed boostrap method can be implemented by adding multivariate Gaussian random noise. When bootmethod = "mvn", the bootstrapped principal component scores are drawn from a multivariate Gaussian distribution with the mean and covariance matrices of the original principal component scores. When bootmethod = "stiefel", the bootstrapped principal component scores are drawn from a Stiefel manifold with the mean and covariance matrices of the original principal component scores. When bootmethod = "meboot", the bootstrapped principal component scores are drawn from a maximum entropy algorithm of Vinod (2004).
smo	Smoothing parameter.

Details

We will presume that each curve is observed on a grid of T points with $0 \leq t_1 < t_2 \ldots < t_T \leq \tau$. Thus, the raw data set $(X_1, X_2, \ldots, X_n)$ of n observations will consist of an n by T data matrix. By applying the singular value decomposition, $X_1, X_2, \ldots, X_n$ can be decomposed into $X = ULR^\top$, where the crossproduct of U and R is identity matrix.

Holding the mean and L and R fixed at their realized values, there are four re-sampling methods that differ mainly by the ways of re-sampling U.

(a) Obtain the re-sampled singular column matrix by randomly sampling with replacement from the original principal component scores.

(b) To avoid the appearance of repeated values in bootstrapped principal component scores, we adapt a smooth bootstrap procedure by adding a white noise component to the bootstrap.

(c) Because principal component scores follow a standard multivariate normal distribution asymptotically, we can randomly draw principal component scores from a multivariate normal distribution with mean vector and covariance matrix of original principal component scores.

(d) Because the crossproduct of U is identitiy matrix, U is considered as a point on the Stiefel manifold, that is the space of n orthogonal vectors, thus we can randomly draw principal component scores from the Stiefel manifold.

Value

bootdata	Bootstrap samples. If the original data matrix is p by n, then the bootstrapped data are p by n by $bootrep$.
meanfunction	Bootstrap summary statistics. If the original data matrix is p by n, then the bootstrapped summary statistics is p by $bootrep$.

Author(s)

Han Lin Shang

References

H. D. Vinod (2004), "Ranking mutual funds using unconventional utility theory and stochastic dominance", *Journal of Empirical Finance*, **11**(3), 353-377.

A. Cuevas, M. Febrero, R. Fraiman (2006), "On the use of the bootstrap for estimating functions with functional data", *Computational Statistics and Data Analysis*, **51**(2), 1063-1074.

D. S. Poskitt and A. Sengarapillai (2013), "Description length and dimensionality reduction in functional data analysis", *Computational Statistics and Data Analysis*, **58**, 98-113.

H. L. Shang (2015), "Re-sampling techniques for estimating the distribution of descriptive statistics of functional data", *Communications in Statistics–Simulation and Computation*, **44**(3), 614-635.

See Also

fbootstrap

Examples

```
# Bootstrapping the distribution of a summary statistics of functional data.
boot1 = pcscorebootstrapdata(ElNino$y, 400, "mean", bootmethod = "st")
boot2 = pcscorebootstrapdata(ElNino$y, 400, "mean", bootmethod = "sm", smo = 0.05)
boot3 = pcscorebootstrapdata(ElNino$y, 400, "mean", bootmethod = "mvn")
boot4 = pcscorebootstrapdata(ElNino$y, 400, "mean", bootmethod = "stiefel")
boot5 = pcscorebootstrapdata(ElNino$y, 400, "mean", bootmethod = "meboot")
```

---

plot.fm *Plot fitted model components for a functional model*

Description

When `class(x)[1] = ftsm`, plot showing the principal components in the top row of plots and the coefficients in the bottom row of plots.

When `class(x)[1] = fm`, plot showing the predictor scores in the top row of plots and the response loadings in the bottom row of plots.

Usage

```
## S3 method for class 'fm'
plot(x, order, xlab1 = x$y$xname, ylab1 = "Principal component",
 xlab2 = "Time", ylab2 = "Coefficient", mean.lab = "Mean",
  level.lab = "Level", main.title = "Main effects", interaction.title
   = "Interaction", basiscol = 1, coeffcol = 1, outlier.col = 2,
    outlier.pch = 19, outlier.cex = 0.5, ...)
```

Arguments

x	Output from ftsm or fplsr.
order	Number of principal components to plot. Default is all principal components in a model.
xlab1	x-axis label for principal components.
xlab2	x-axis label for coefficient time series.
ylab1	y-axis label for principal components.
ylab2	y-axis label for coefficient time series.
mean.lab	Label for mean component.
level.lab	Label for level component.
main.title	Title for main effects.
interaction.title	Title for interaction terms.
basiscol	Colors for principal components if plot.type = "components".
coeffcol	Colors for time series coefficients if plot.type = "components".
outlier.col	Colors for outlying years.
outlier.pch	Plotting character for outlying years.
outlier.cex	Size of plotting character for outlying years.
...	Plotting parameters.

Value

Function produces a plot.

Author(s)

Rob J Hyndman

References

R. J. Hyndman and M. S. Ullah (2007) "Robust forecasting of mortality and fertility rates: A functional data approach", *Computational Statistics \& Data Analysis*, **51**(10), 4942-4956.

R. J. Hyndman and H. Booth (2008) "Stochastic population forecasts using functional data models for mortality, fertility and migration", *International Journal of Forecasting*, **24**(3), 323-342.

R. J. Hyndman and H. L. Shang (2009) "Forecasting functional time series (with discussion)", *Journal of the Korean Statistical Society*, **38**(3), 199-221.

See Also

ftsm, forecast.ftsm, residuals.fm, summary.fm, plot.fmres, plot.ftsf

Examples

```
plot(x = ftsm(y = ElNino))
```

plot.fmres *Plot residuals from a fitted functional model.*

Description

Functions to produce a plot of residuals from a fitted functional model.

Usage

```
## S3 method for class 'fmres'
plot(x, type = c("image", "fts", "contour", "filled.contour",
 "persp"), xlab = "Year", ylab = "Age", zlab = "Residual", ...)
```

Arguments

x	Generated by `residuals(fit)`, where fit is the output from `ftsm` or `fplsr`.
type	Type of plot to use. Possibilities are `image`, `fts`, `contour`, `filled.contour` and `persp`.
xlab	Label for x-axis.
ylab	Label for y-axis.
zlab	Label for z-axis.
...	Plotting parameters.

Value

Produces a plot.

Author(s)

Rob J Hyndman

See Also

`ftsm`, `forecast.ftsm`, `plot.fm`, `plot.fmres`, `residuals.fm`, `summary.fm`

Examples

```
# colorspace package was used to provide a more coherent color option.
plot(residuals(ftsm(y = ElNino)), type = "filled.contour", xlab = "Month",
     ylab = "Residual sea surface temperature")
```

plot.ftsf *Plot fitted model components for a functional time series model*

Description

Plot fitted model components for a fts object.

Usage

```
## S3 method for class 'ftsf'
plot(x, plot.type = c("function", "components", "variance"),
 components, xlab1 = fit$y$xname, ylab1 = "Basis function",
  xlab2 = "Time", ylab2 = "Coefficient", mean.lab = "Mean",
   level.lab = "Level", main.title = "Main effects",
    interaction.title = "Interaction", vcol = 1:3, shadecols = 7,
     fcol = 4, basiscol = 1, coeffcol = 1, outlier.col = 2,
      outlier.pch = 19, outlier.cex = 0.5,...)
```

Arguments

x	Output from forecast.ftsm.
plot.type	Type of plot.
components	Number of principal components.
xlab1	x-axis label for principal components.
xlab2	x-axis label for coefficient time series.
ylab1	y-axis label for principal components.
ylab2	y-axis label for coefficient time series.
mean.lab	Label for mean component.
level.lab	Label for level component.
main.title	Title for main effects.
interaction.title	Title for interaction terms.
vcol	Colors to use if plot.type = "variance".
shadecols	Color for shading of prediction intervals when plot.type = "components".
fcol	Color of point forecasts when plot.type = "components".
basiscol	Colors for principal components if plot.type = "components".
coeffcol	Colors for time series coefficients if plot.type = "components".
outlier.col	Colors for outlying years.
outlier.pch	Plotting character for outlying years.
outlier.cex	Size of plotting character for outlying years.
...	Plotting parameters.

Details

When `plot.type = "function"`, it produces a plot of the forecast functions;

When `plot.type = "components"`, it produces a plot of the principla components and coefficients with forecasts and prediction intervals for each coefficient;

When `plot.type = "variance"`, it produces a plot of the variance components.

Value

Function produces a plot.

Author(s)

Rob J Hyndman

References

R. J. Hyndman and M. S. Ullah (2007) "Robust forecasting of mortality and fertility rates: A functional data approach", *Computational Statistics and Data Analysis*, **51**(10), 4942-4956.

R. J. Hyndman and H. Booth (2008) "Stochastic population forecasts using functional data models for mortality, fertility and migration", *International Journal of Forecasting*, **24**(3), 323-342.

R. J. Hyndman and H. L. Shang (2009) "Forecasting functional time series (with discussion)", *Journal of the Korean Statistical Society*, **38**(3), 199-221.

See Also

`ftsm`, `plot.fm`, `plot.fmres`, `residuals.fm`, `summary.fm`

Examples

```
plot(x = forecast(object = ftsm(y = ElNino)))
```

---

plot.ftsm *Plot fitted model components for a functional time series model*

Description

Plot showing the basis functions in the top row of plots and the coefficients in the bottom row of plots.

Usage

```
## S3 method for class 'ftsm'
plot(x, components, xlab1 = x$y$xname, ylab1 = "Basis function",
 xlab2 = "Time", ylab2 = "Coefficient", mean.lab = "Mean",
  level.lab = "Level", main.title = "Main effects",
   interaction.title = "Interaction", basiscol = 1, coeffcol = 1,
    outlier.col = 2, outlier.pch = 19, outlier.cex = 0.5, ...)
```

Arguments

x	Output from ftsm.
components	Number of principal components to plot.
xlab1	x-axis label for basis functions.
xlab2	x-axis label for coefficient time series.
ylab1	y-axis label for basis functions.
ylab2	y-axis label for coefficient time series.
mean.lab	Label for mean component.
level.lab	Label for level component.
main.title	Title for main effects.
interaction.title	Title for interaction terms.
basiscol	Colors for basis functions if plot.type="components".
coeffcol	Colors for time series coefficients if plot.type="components".
outlier.col	Colour for outlying years.
outlier.pch	Plotting character for outlying years.
outlier.cex	Size of plotting character for outlying years.
...	Plotting parameters.

Value

None. Function produces a plot.

Author(s)

Rob J Hyndman

References

R. J. Hyndman and M. S. Ullah (2007) "Robust forecasting of mortality and fertility rates: A functional data approach", *Computational Statistics and Data Analysis*, **51**(10), 4942-4956.

R. J. Hyndman and H. L. Shang (2009) "Forecasting functional time series" (with discussion), *Journal of the Korean Statistical Society*, **38**(3), 199-221.

See Also

forecast.ftsm, ftsm, plot.fm, plot.ftsf, residuals.fm, summary.fm

Examples

```
# plot different principal components.
plot.ftsm(ftsm(y = ElNino, order = 2), components = 2)
```

plotfplsr *Plot fitted model components for a functional time series model*

Description

Plot showing the basis functions of the predictors in the top row, followed by the basis functions of the responses in the second row, then the coefficients in the bottom row of plots.

Usage

```
plotfplsr(x, xlab1 = x$ypred$xname, ylab1 = "Basis function", xlab2 = "Time",
 ylab2 = "Coefficient", mean.lab = "Mean", interaction.title = "Interaction")
```

Arguments

x	Output from fplsr.
xlab1	x-axis label for basis functions.
ylab1	y-axis label for basis functions.
xlab2	x-axis label for coefficient time series.
ylab2	y-axis label for coefficient time series.
mean.lab	Label for mean component.
interaction.title	Title for interaction terms.

Value

None. Function produces a plot.

Author(s)

Han Lin Shang

References

R. J. Hyndman and M. S. Ullah (2007) "Robust forecasting of mortality and fertility rates: A functional data approach", *Computational Statistics and Data Analysis*, **51**(10), 4942-4956.

R. J. Hyndman and H. L. Shang (2009) "Forecasting functional time series" (with discussion), *Journal of the Korean Statistical Society*, **38**(3), 199-221.

See Also

forecast.ftsm, ftsm, plot.fm, plot.ftsf, residuals.fm, summary.fm

Examples

```
# Fit the data by the functional partial least squares.
ausfplsr = fplsr(data = Australiasmoothfertility, order = 2)
plotfplsr(x = ausfplsr)
```

pm_10_GR *Particulate Matter Concentrations (pm10)*

Description

This data set consists of half-hourly measurement of the concentrations (measured in ug/m3) of particular matter with an aerodynamic diameter of less than 10um, abbreviated PM10, in ambient air taken in Graz-Mitte, Austria from October 1, 2010 until March 31, 2011. To stabilise the variance, a square-root transformation can be applied to the data.

Usage

```
data(pm_10_GR)
```

Details

As epidemiological and toxicological studies have pointed to negative health effects, European Union (EU) regulation sets pollution standards for the level of the concentration. Policy makers have to ensure compliance with these EU rules and need reliable statistical tools to determine, and justify the public, appropriate measures such as partial traffic regulation (see Stadlober, Hormann and Pfeiler, 2008).

Source

Thanks Professor Siegfried. Hormann for providing this data set. The original data source is http://www.umwelt.steiermark.at/cms/

References

A. Aue, D. D. Norinho, S. Hormann (2015) "On the prediction of stationary functional time series", *Journal of the American Statistical Association*, **110**(509), 378-392.

E. Stadlober, S. Hormann, B. Pfeiler (2008) "Quality and performance of a PM10 daily forecasting model", *Atmospheric Environment*, **42**, 1098-1109.

S. Hormann, B. Pfeiler, E. Stadlober (2005) "Analysis and prediction of particulate matter PM10 for the winter season in Graz", *Austrian Journal of Statistics*, **34**(4), 307-326.

Examples

```
plot(pm_10_GR)
```

quantile *Quantile*

Description

Generic functions for quantile.

Usage

```
quantile(x, ...)
```

Arguments

x	Numeric vector whose sample quantiles are wanted, or an object of a class for which a method has been defined.
...	Arguments passed to specific methods.

Value

Refer to specific methods. For numeric vectors, see the quantile functions in the stats package.

Author(s)

Han Lin Shang

See Also

quantile.fts

quantile.fts *Quantile functions for functional time series*

Description

Computes quantiles of functional time series at each variable.

Usage

```
## S3 method for class 'fts'
quantile(x, probs, ...)
```

Arguments

x	An object of class fts.
probs	Quantile percentages.
...	Other arguments.

Value

Return quantiles for each variable.

Author(s)

Han Lin Shang

See Also

mean.fts, median.fts, var.fts, sd.fts

Examples

```
quantile(x = ElNino)
```

---

residuals.fm *Compute residuals from a functional model*

Description

After fitting a functional model, it is useful to inspect the residuals. This function extracts the relevant information from the fit object and puts it in a form suitable for plotting with image, persp, contour, filled.contour, etc.

Usage

```
## S3 method for class 'fm'
residuals(object, ...)
```

Arguments

object	Output from ftsm or fplsr.
...	Other arguments.

Value

Produces an object of class "fmres" containing the residuals from the model.

Author(s)

Rob J Hyndman

References

B. Erbas and R. J. Hyndman and D. M. Gertig (2007) "Forecasting age-specific breast cancer mortality using functional data model", *Statistics in Medicine*, **26**(2), 458-470.

R. J. Hyndman and M. S. Ullah (2007) "Robust forecasting of mortality and fertility rates: A functional data approach", *Computational Statistics \& Data Analysis*, **51**(10), 4942-4956.

R. J. Hyndman and H. Booth (2008) "Stochastic population forecasts using functional data models for mortality, fertility and migration", *International Journal of Forecasting*, **24**(3), 323-342.

H. L. Shang (2012) "Point and interval forecasts of age-specific fertility rates: a comparison of functional principal component methods", *Journal of Population Research*, **29**(3), 249-267.

H. L. Shang (2012) "Point and interval forecasts of age-specific life expectancies: a model averaging", *Demographic Research*, **27**, 593-644.

See Also

`ftsm`, `forecast.ftsm`, `summary.fm`, `plot.fm`, `plot.fmres`

Examples

```
plot(residuals(object = ftsm(y = ElNino)), xlab = "Year", ylab = "Month")
```

`sd` *Standard deviation*

Description

Generic functions for standard deviation.

Usage

```
sd(...)
```

Arguments

`...`	Arguments passed to specific methods.

Details

The `sd` functions in the `stats` package are replaced by `sd.default`.

Value

Refer to specific methods. For numeric vectors, see the `sd` functions in the `stats` package.

Author(s)

Han Lin Shang

See Also

sd.fts

---

sd.fts *Standard deviation functions for functional time series*

Description

Computes standard deviation of functional time series at each variable.

Usage

```
## S3 method for class 'fts'
sd(x, method = c("coordinate", "FM", "mode", "RP", "RPD", "radius"),
 trim = 0.25, alpha, weight,...)
```

Arguments

x	An object of class fts.
method	Method for computing median.
trim	Percentage of trimming.
alpha	Tuning parameter when method="radius".
weight	Hard thresholding or soft thresholding.
...	Other arguments.

Details

If method = "coordinate", it computes coordinate-wise standard deviation functions.

If method = "FM", it computes the standard deviation functions of trimmed functional data ordered by the functional depth of Fraiman and Muniz (2001).

If method = "mode", it computes the standard deviation functions of trimmed functional data ordered by h-modal functional depth.

If method = "RP", it computes the standard deviation functions of trimmed functional data ordered by random projection depth.

If method = "RPD", it computes the standard deviation functions of trimmed functional data ordered by random projection with derivative depth.

If method = "radius", it computes the standard deviation function of trimmed functional data ordered by the notion of alpha-radius.

Value

A list containing x = variables and y = standard deviation rates.

Author(s)

Han Lin Shang

References

O. Hossjer and C. Croux (1995) "Generalized univariate signed rank statistics for testing and estimating a multivariate location parameter", *Nonparametric Statistics*, **4**(3), 293-308.

A. Cuevas and M. Febrero and R. Fraiman (2006) "On the use of bootstrap for estimating functions with functional data", *Computational Statistics \& Data Analysis*, **51**(2), 1063-1074.

A. Cuevas and M. Febrero and R. Fraiman (2007), "Robust estimation and classification for functional data via projection-based depth notions", *Computational Statistics*, **22**(3), 481-496.

M. Febrero and P. Galeano and W. Gonzalez-Manteiga (2007) "A functional analysis of NOx levels: location and scale estimation and outlier detection", *Computational Statistics*, **22**(3), 411-427.

M. Febrero and P. Galeano and W. Gonzalez-Manteiga (2008) "Outlier detection in functional data by depth measures, with application to identify abnormal NOx levels", *Environmetrics*, **19**(4), 331-345.

M. Febrero and P. Galeano and W. Gonzalez-Manteiga (2010) "Measures of influence for the functional linear model with scalar response", *Journal of Multivariate Analysis*, **101**(2), 327-339.

J. A. Cuesta-Albertos and A. Nieto-Reyes (2010) "Functional classification and the random Tukey depth. Practical issues", Combining Soft Computing and Statistical Methods in Data Analysis, Advances in Intelligent and Soft Computing, Volume 77, 123-130.

D. Gervini (2012) "Outlier detection and trimmed estimation in general functional spaces", *Statistica Sinica*, **22**(4), 1639-1660.

See Also

`mean.fts`, `median.fts`, `var.fts`, `quantile.fts`

Examples

```
# Fraiman-Muniz depth was arguably the oldest functional depth.
sd(x = ElNino, method = "FM")
sd(x = ElNino, method = "coordinate")
sd(x = ElNino, method = "mode")
sd(x = ElNino, method = "RP")
sd(x = ElNino, method = "RPD")
sd(x = ElNino, method = "radius", alpha = 0.5, weight = "hard")
sd(x = ElNino, method = "radius", alpha = 0.5, weight = "soft")
```

summary.fm *Summary for functional time series model*

Description

Summarizes a basis function model fitted to a functional time series. It returns various measures of goodness-of-fit.

Usage

```
## S3 method for class 'fm'
summary(object, ...)
```

Arguments

object	Output from ftsm or fplsr.
...	Other arguments.

Value

None.

Author(s)

Rob J Hyndman

See Also

ftsm, forecast.ftsm, residuals.fm, plot.fm, plot.fmres

Examples

```
summary(object = ftsm(y = ElNino))
```

T_stationary *Testing stationarity of functional time series*

Description

A hypothesis test for stationarity of functional time series.

Usage

```
T_stationary(sample, L = 49, J = 500, MC_rep = 1000, cumulative_var = .90,
    Ker1 = FALSE, Ker2 = TRUE, h = ncol(sample)^.5, pivotal = FALSE)
```

Arguments

sample	A matrix of discretised curves of dimension (p by n), where p represents the dimensionality and n represents sample size.
L	Number of Fourier basis functions.
J	Truncation level used to approximate the distribution of the squared integrals of Brownian bridges that appear in the limit distribution.
MC_rep	Number of replications.
cumulative_var	Amount of variance explained.
Ker1	Flat top kernel in (4.1) of Horvath et al. (2014).
Ker2	Flat top kernel in (7) of Politis (2003).
h	Kernel bandwidth.
pivotal	If `pivotal = TRUE`, a pivotal statistic is used.

Details

As in traditional (scalar and vector) time series analysis, many inferential procedures for functional time series assume stationarity. Stationarity is required for functional dynamic regression models, for bootstrap and resampling methods for functional time series and for the functional analysis of volatility.

Value

p-value	When `p-value` is less than any level of significance, we reject the null hypothesis and conclude that the tested functional time series is not stationary.

Author(s)

Greg. Rice and Han Lin Shang

References

L. Horvath, P. Kokoszka, G. Rice (2014) "Testing stationarity of functional time series", *Journal of Econometrics*, **179**(1), 66-82.

D. N. Politis (2003) "Adaptive bandwidth choice", *Journal of Nonparametric Statistics*, **15**(4-5), 517-533.

See Also

farforecast

Examples

```
result = T_stationary(pm_10_GR_sqrt$y)
result_pivotal = T_stationary(pm_10_GR_sqrt$y, J = 100, MC_rep = 5000, h = 20, pivotal = TRUE)
```

var *Variance*

Description

Generic functions for variance.

Usage

```
var(...)
```

Arguments

...	Arguments passed to specific methods.

Details

The cor functions in the stats package are replaced by var.default.

Value

Refer to specific methods. For numeric vectors, see the cor functions in the stats package.

Author(s)

Rob J Hyndman and Han Lin Shang

See Also

var.fts

var.fts *Variance functions for functional time series*

Description

Computes variance functions of functional time series at each variable.

Usage

```
## S3 method for class 'fts'
var(x, method = c("coordinate", "FM", "mode", "RP", "RPD", "radius"),
 trim = 0.25, alpha, weight, ...)
```

Arguments

x	An object of class fts.
method	Method for computing median.
trim	Percentage of trimming.
alpha	Tuning parameter when method="radius".
weight	Hard thresholding or soft thresholding.
...	Other arguments.

Details

If method = "coordinate", it computes coordinate-wise variance.

If method = "FM", it computes the variance of trimmed functional data ordered by the functional depth of Fraiman and Muniz (2001).

If method = "mode", it computes the variance of trimmed functional data ordered by h-modal functional depth.

If method = "RP", it computes the variance of trimmed functional data ordered by random projection depth.

If method = "RPD", it computes the variance of trimmed functional data ordered by random projection derivative depth.

If method = "radius", it computes the standard deviation function of trimmed functional data ordered by the notion of alpha-radius.

Value

A list containing x = variables and y = variance rates.

Author(s)

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References

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See Also

`mean.fts`, `median.fts`, `sd.fts`, `quantile.fts`

Examples

```
# Fraiman-Muniz depth was arguably the oldest functional depth.
var(x = ElNino, method = "FM")
var(x = ElNino, method = "coordinate")
var(x = ElNino, method = "mode")
var(x = ElNino, method = "RP")
var(x = ElNino, method = "RPD")
var(x = ElNino, method = "radius", alpha = 0.5, weight = "hard")
var(x = ElNino, method = "radius", alpha = 0.5, weight = "soft")
```

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