Package 'matlib'

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Type Package

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Description A collection of matrix functions for teaching and learning matrix linear algebra as used in multivariate statistical methods. These functions are mainly for tutorial purposes in learning matrix algebra ideas using R. In some cases, functions are provided for concepts available elsewhere in R, but where the function call or name is not obvious. In other cases, functions are provided to show or demonstrate an algorithm. In addition, a collection of functions are provided for drawing vector diagrams in 2D and 3D.

License GPL (>= 2)

URL https://github.com/friendly/matlib

BugReports https://github.com/friendly/matlib/issues

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R topics documented:

`arc` *Draw an arc showing the angle between vectors*

Description

A utility function for drawing vector diagrams. Draws a circular arc to show the angle between two vectors in 2D or 3D.

Usage

```
arc(p1, p2, p3, d = 0.1, absolute = TRUE, ...)
```

Arguments

`p1`	Starting point of first vector
`p2`	End point of first vector, and also start of second vector
`p3`	End point of second vector
`d`	The distance from `p2` along each vector for drawing their corner
`absolute`	logical; if `TRUE`, `d` is taken as an absolute distance along the vectors; otherwise it is calculated as a relative distance, i.e., a fraction of the length of the vectors.
`...`	Arguments passed to `link[graphics]{lines}` or to `link[rgl]{lines3d}`

Details

In this implementation, the two vectors are specified by three points, `p1`, `p2`, `p3`, meaning a line from `p1` to `p2`, and another line from `p2` to `p3`.

Value

none

References

http://math.stackexchange.com/questions/1507248/find-arc-between-two-tips-of-vectors-in-3d

See Also

Other vector diagrams: `Proj`, `arrows3d`, `corner`, `plot.regvec3d`, `point_on_line`, `regvec3d`, `vectors3d`, `vectors`

Examples

```
library(rgl)
vec <- rbind(diag(3), c(1,1,1))
rownames(vec) <- c("X", "Y", "Z", "J")
open3d()
aspect3d("iso")
vectors3d(vec, col=c(rep("black",3), "red"), lwd=2)
# draw the XZ plane, whose equation is Y=0
planes3d(0, 0, 1, 0, col="gray", alpha=0.2)
# show projections of the unit vector J
segments3d(rbind( c(1,1,1), c(1, 1, 0)))
segments3d(rbind( c(0,0,0), c(1, 1, 0)))
segments3d(rbind( c(1,0,0), c(1, 1, 0)))
segments3d(rbind( c(0,1,0), c(1, 1, 0)))
segments3d(rbind( c(1,1,1), c(1, 0, 0)))

# show some orthogonal vectors
p1 <- c(0,0,0)
p2 <- c(1,1,0)
p3 <- c(1,1,1)
p4 <- c(1,0,0)
# show some angles
arc(p1, p2, p3, d=.2)
arc(p4, p1, p2, d=.2)
arc(p3, p1, p2, d=.2)
```

`arrows3d` *Draw 3D arrows*

Description

Draws nice 3D arrows with `cone3d`s at their tips.

Usage

```
arrows3d(coords, headlength = 0.035, head = "end", scale = NULL,
  radius = NULL, ref.length = NULL, draw = TRUE, ...)
```

Arguments

`coords`	A 2n x 3 matrix giving the start and end (x,y,z) coordinates of n arrows, in pairs. The first vector in each pair is taken as the starting coordinates of the arrow, the second as the end coordinates.
`headlength`	Length of the arrow heads, in device units
`head`	Position of the arrow head. Only `head="end"` is presently implemented.
`scale`	Scale factor for base and tip of arrow head, a vector of length 3, giving relative scale factors for X, Y, Z
`radius`	radius of the base of the arrow head

`ref.length`	length of vector to be used to scale all of the arrow heads (permits drawing arrow heads of the same size as in a previous call); if `NULL`, arrows are scaled relative to the longest vector
`draw`	if `TRUE` (the default) draw the arrow(s)
`...`	rgl arguments passed down to `segments3d` and `cone3d`, for example, `col` and `lwd`

Details

This function is meant to be analogous to `arrows`, but for 3D plots using `rgl`. `headlength`, `scale` and `radius` set the length, scale factor and base radius of the arrow head, a 3D cone. The units of these are all in terms of the ranges of the current rgl 3D scene.

Value

invisibly returns the length of the vector used to scale the arrow heads

Author(s)

January Weiner, borrowed from the **pca3d** package, slightly modified by John Fox

See Also

`vectors3d`

Other vector diagrams: `Proj`, `arc`, `corner`, `plot.regvec3d`, `point_on_line`, `regvec3d`, `vectors3d`, `vectors`

Examples

```
#none yet
```

`buildTmat` *Build/Get tranformation matricies*

Description

Recover the history of the row operations that have been performed. This function combines the transformation matricies into a single transformation matrix representing all row operations or may optionally print all the individual operations which have been performed.

Usage

```
buildTmat(x, all = FALSE)

## S3 method for class 'trace'
as.matrix(x, ...)

## S3 method for class 'trace'
print(x, ...)
```

Arguments

x	a matrix A, joined with a vector of constants, b, that has been passed to gaussianElimination or the row operator functions
all	logical; print individual tranformation matricies?
...	additional arguments

Value

the tranformation matrix or a list of individual transformation matricies

See Also

echelon, gaussianElimination

Examples

```
A <- matrix(c(2, 1, -1,
             -3, -1, 2,
             -2,  1, 2), 3, 3, byrow=TRUE)
b <- c(8, -11, -3)

# using row operations to reduce below diagonal to 0
Abt <- Ab <- cbind(A, b)
Abt <- rowadd(Abt, 1, 2, 3/2)
Abt <- rowadd(Abt, 1, 3, 1)
Abt <- rowadd(Abt, 2, 3, -4)
Abt

# build T matrix and multiply by original form
(T <- buildTmat(Abt))
T %*% Ab    # same as Abt

# print all transformation matricies
buildTmat(Abt, TRUE)

# invert transformation matrix to reverse operations
inv(T) %*% Abt

# gaussian elimination
(soln <- gaussianElimination(A, b))
T <- buildTmat(soln)
inv(T) %*% soln
```

cholesky *Cholesky Square Root of a Matrix*

Description

Returns the Cholesky square root of the non-singular, symmetric matrix `X`. The purpose is mainly to demonstrate the algorithm used by Kennedy & Gentle (1980).

Usage

```
cholesky(X, tol = sqrt(.Machine$double.eps))
```

Arguments

`X`	a square symmetric matrix
`tol`	tolerance for checking for 0 pivot

Value

the Cholesky square root of `X`

Author(s)

John Fox

References

Kennedy W.J. Jr, Gentle J.E. (1980). *Statistical Computing*. Marcel Dekker.

See Also

`chol` for the base R function

`gsorth` for Gram-Schmidt orthogonalization of a data matrix

Examples

```
C <- matrix(c(1,2,3,2,5,6,3,6,10), 3, 3) # nonsingular, symmetric
C
cholesky(C)
cholesky(C) %*% t(cholesky(C))  # check
```

circle3d *Draw a horizontal circle*

Description

A utility function for drawing a horizontal circle in a 3D graph

Usage

```
circle3d(center, radius, segments = 100, fill = FALSE, ...)
```

Arguments

`center`	A vector of length 3.
`radius`	A positive number.
`segments`	An integer specifying the number of line segments to use to draw the circle (default, 100).
`fill`	logical; if `TRUE`, the circle is filled (the default is `FALSE`).
`...`	**rgl** material properties for the circle.

class *Class Data Set*

Description

A small artificial data set used to illustrate statistical concepts.

Usage

```
data("class")
```

Format

A data frame with 15 observations on the following 4 variables.

`sex` a factor with levels `F` `M`

`age` a numeric vector

`height` a numeric vector

`weight` a numeric vector

Examples

```
data(class)
plot(class)
```

cofactor *Cofactor of A[i,j]*

Description

Returns the cofactor of element (i,j) of the square matrix A, i.e., the signed minor of the sub-matrix that results when row i and column j are deleted.

Usage

```
cofactor(A, i, j)
```

Arguments

`A`	a square matrix
`i`	row index
`j`	column index

Value

the cofactor of A[i,j]

Author(s)

Michael Friendly

See Also

`row_cofactors` for all cofactors of a given row

Other determinants: `minor`, `row_cofactors`, `row_minors`

Examples

```
M <- matrix(c(4, -12, -4,
              2,   1,  3,
             -1,  -3,  2), 3, 3, byrow=TRUE)
cofactor(M, 1, 1)
cofactor(M, 1, 2)
cofactor(M, 1, 3)
```

cone3d — *Draw a 3D cone*

Description

Draws a cone in 3D from a `base` point to a `tip` point, with a given `radius` at the base. This is used to draw nice arrow heads in `arrows3d`.

Usage

```
cone3d(base, tip, radius = 10, col = "grey", scale = NULL, ...)
```

Arguments

`base`	coordinates of base of the cone
`tip`	coordinates of tip of the cone
`radius`	radius of the base
`col`	color
`scale`	scale factor for base and tip
`...`	rgl arguments passed down; see `rgl.material`

Value

returns the integer object ID of the shape that was added to the scene

Author(s)

January Weiner, borrowed from from the **pca3d** package

See Also

`arrows3d`

Examples

```
# none yet
```

corner — *Draw a corner showing the angle between two vectors*

Description

A utility function for drawing vector diagrams. Draws two line segments to indicate the angle between two vectors, typically used for indicating orthogonal vectors are at right angles in 2D and 3D diagrams.

Usage

```
corner(p1, p2, p3, d = 0.1, absolute = TRUE, ...)
```

Arguments

p1	Starting point of first vector
p2	End point of first vector, and also start of second vector
p3	End point of second vector
d	The distance from p2 along each vector for drawing their corner
absolute	logical; if TRUE, d is taken as an absolute distance along the vectors; otherwise it is calculated as a relative distance, i.e., a fraction of the length of the vectors. See point_on_line for the precise definition.
...	Arguments passed to link[graphics]{lines} or to link[rgl]{lines3d}

Details

In this implementation, the two vectors are specified by three points, p1, p2, p3, meaning a line from p1 to p2, and another line from p2 to p3.

Value

none

See Also

Other vector diagrams: Proj, arc, arrows3d, plot.regvec3d, point_on_line, regvec3d, vectors3d, vectors

Examples

```
# none yet
```

echelon *Echelon Form of a Matrix*

Description

Returns the reduced row-echelon form of the matrix `X`, using `gaussianElimination`. It is nothing more than a synonym for Gaussian elimination, but offers the possibility to show the steps using `verbose=TRUE`.

Usage

```
echelon(X, ...)
```

Arguments

`X`	a matrix
`...`	other arguments passed to `gaussianElimination`

Details

When the matrix `x` is square and non-singular, the result will be the identity matrix. Otherwise, the result will have some all-zero rows, and the rank of the matrix is the number of not all-zero rows.

Value

the reduced echelon form of `X`.

Author(s)

John Fox

Examples

```
A <- matrix(c(2, 1, -1,
             -3, -1, 2,
             -2,  1, 2), 3, 3, byrow=TRUE)
b <- c(8, -11, -3)
echelon(A, b, verbose=TRUE, fractions=TRUE)

A <- matrix(c(1,2,3,4,5,6,7,8,10), 3, 3) # a nonsingular matrix
A
echelon(A) # the reduced row-echelon form of A

b <- 1:3
echelon(A, b)  # solving the matrix equation Ax = b
echelon(A, diag(3)) # inverting A

B <- matrix(1:9, 3, 3) # a singular matrix
B
```

```
echelon(B)
echelon(B, b)
echelon(B, diag(3))
```

---

eig — *Eigen Decomposition of a Square Symmetric Matrix*

Description

eig calculates the eigenvalues and eigenvectors of a square, symmetric matrix using the iterated QR decomposition

Usage

```
eig(X, tol = sqrt(.Machine$double.eps), max.iter = 100,
  retain.zeroes = TRUE)
```

Arguments

X	a square symmetric matrix
tol	tolerance passed to QR
max.iter	maximum number of QR iterations
retain.zeroes	logical; retain 0 eigenvalues?

Value

a list of two elements: values– eigenvalues, vectors– eigenvectors

Author(s)

John Fox and Georges Monette

See Also

eigen

SVD

Examples

```
C <- matrix(c(1,2,3,2,5,6,3,6,10), 3, 3) # nonsingular, symmetric
C
EC <- eig(C) # eigenanalysis of C
EC$vectors %*% diag(EC$values) %*% t(EC$vectors) # check
```

gaussianElimination *Gaussian Elimination*

Description

gaussianElimination demonstrates the algorithm of row reduction used for solving systems of linear equations of the form $Ax = B$. Optional arguments verbose and fractions may be used to see how the algorithm works.

Usage

```
gaussianElimination(A, B, tol = sqrt(.Machine$double.eps), verbose = FALSE,
  latex = FALSE, fractions = FALSE)
```

Arguments

A	coefficient matrix
B	right-hand side vector or matrix. If B is a matrix, the result gives solutions for each column as the right-hand side of the equations with coefficients in A.
tol	tolerance for checking for 0 pivot
verbose	logical; if TRUE, print intermediate steps
latex	logical; if TRUE, and verbose is TRUE, print intermediate steps using LaTeX equation outputs rather than R output
fractions	logical; if TRUE, try to express non-integers as rational numbers

Value

If B is absent, returns the reduced row-echelon form of A. If B is present, returns the reduced row-echelon form of A, with the same operations applied to B.

Author(s)

John Fox

Examples

```
A <- matrix(c(2, 1, -1,
             -3, -1, 2,
             -2,  1, 2), 3, 3, byrow=TRUE)
b <- c(8, -11, -3)
gaussianElimination(A, b)
gaussianElimination(A, b, verbose=TRUE, fractions=TRUE)
gaussianElimination(A, b, verbose=TRUE, fractions=TRUE, latex=TRUE)

# determine whether matrix is solvable
gaussianElimination(A, numeric(3))
```

```
# find inverse matrix by elimination: A = I -> A^-1 A = A^-1 I -> I = A^-1
gaussianElimination(A, diag(3))
inv(A)
```

Ginv *Generalized Inverse of a Matrix*

Description

`Ginv` returns an arbitrary generalized inverse of the matrix `A`, using `gaussianElimination`.

Usage

```
Ginv(A, tol = sqrt(.Machine$double.eps), verbose = FALSE,
  fractions = FALSE)
```

Arguments

`A`	numerical matrix
`tol`	tolerance for checking for 0 pivot
`verbose`	logical; if `TRUE`, print intermediate steps
`fractions`	logical; if `TRUE`, try to express non-integers as rational numbers

Details

A generalized inverse is a matrix $\mathbf{A}^-$ satisfying $\mathbf{A}\mathbf{A}^-\mathbf{A} = \mathbf{A}$.

The purpose of this function is mainly to show how the generalized inverse can be computed using Gaussian elimination.

Value

the generalized inverse of `A`, expressed as fractions if `fractions=TRUE`, or rounded

Author(s)

John Fox

See Also

`ginv` for a more generally usable function

Examples

```
A <- matrix(c(1,2,3,4,5,6,7,8,10), 3, 3) # a nonsingular matrix
A
Ginv(A, fractions=TRUE)  # a generalized inverse of A = inverse of A
round(Ginv(A) %*% A, 6)  # check

B <- matrix(1:9, 3, 3) # a singular matrix
B
Ginv(B, fractions=TRUE)  # a generalized inverse of B
B %*% Ginv(B) %*% B   # check
```

---

gsorth *Gram-Schmidt Orthogonalization of a Matrix*

Description

Calculates a matrix with uncorrelated columns using the Gram-Schmidt process

Usage

```
gsorth(y, order, recenter = TRUE, rescale = TRUE, adjnames = TRUE)
```

Arguments

`y`	a numeric matrix or data frame
`order`	if specified, a permutation of the column indices of `y`
`recenter`	logical; if `TRUE`, the result has same means as the original `y`, else means = 0 for cols 2:p
`rescale`	logical; if `TRUE`, the result has same sd as original, else, sd = residual sd
`adjnames`	logical; if `TRUE`, colnames are adjusted to Y1, Y2.1, Y3.12, ...

Value

a matrix/data frame with uncorrelated columns

Examples

```
set.seed(1234)
A <- matrix(c(1:60 + rnorm(60)), 20, 3)
cor(A)
G <- gsorth(A)
zapsmall(cor(G))
```

Inverse *Inverse of a Matrix*

Description

Uses gaussianElimination to find the inverse of a square, non-singular matrix, X.

Usage

```
Inverse(X, tol = sqrt(.Machine$double.eps), ...)
```

Arguments

X	a square numeric matrix
tol	tolerance for checking for 0 pivot
...	other arguments passed on

Details

The method is purely didactic: The identity matrix, I, is appended to X, giving $X|I$. Applying Gaussian elimination gives $I|X^{-1}$, and the portion corresponding to X^{-1} is returned.

Value

the inverse of X

Author(s)

John Fox

Examples

```
A <- matrix(c(2, 1, -1,
             -3, -1, 2,
             -2,  1, 2), 3, 3, byrow=TRUE)
Inverse(A)
Inverse(A, verbose=TRUE, fractions=TRUE)
```

J *Create a vector, matrix or array of constants*

Description

This function creates a vector, matrix or array of constants, typically used for the unit vector or unit matrix in matrix expressions.

Usage

```
J(..., constant = 1, dimnames = NULL)
```

Arguments

`...`	One or more arguments supplying the dimensions of the array, all non-negative integers
`constant`	The value of the constant used in the array
`dimnames`	Either `NULL` or the names for the dimensions.

Details

The `"dimnames"` attribute is optional: if present it is a list with one component for each dimension, either `NULL` or a character vector of the length given by the element of the `"dim"` attribute for that dimension. The list can be named, and the list names will be used as names for the dimensions.

Examples

```
J(3)
J(2,3)
J(2,3,2)
J(2,3, constant=2, dimnames=list(letters[1:2], LETTERS[1:3]))

X <- matrix(1:6, nrow=2, ncol=3)
dimnames(X) <- list(sex=c("M", "F"), day=c("Mon", "Wed", "Fri"))
J(2) %*% X      # column sums
X %*% J(3)      # row sums
```

len *Length of a Vector or Column Lengths of a Matrix*

Description

`len` calculates the Euclidean length (also called Euclidean norm) of a vector or the length of each column of a numeric matrix.

Usage

```
len(X)
```

Arguments

`X`	a numeric vector or matrix

Value

a scalar or vector containing the length(s)

See Also

`norm` for more general matrix norms

Examples

```
len(1:3)
len(matrix(1:9, 3, 3))
```

`LU` *LU Decomposition*

Description

`LU` computes the LU decomposition of a matrix, A, such that $PA = LU$, where L is a lower triangle matrix, U is an upper triangle, and P is a permutation matrix.

Usage

```
LU(A, b, tol = sqrt(.Machine$double.eps), fractions = FALSE)
```

Arguments

`A`	coefficient matrix
`b`	right-hand side vector. When supplied the returned object will also contain the solved d and `x` elements
`tol`	tolerance for checking for 0 pivot
`fractions`	logical; if `TRUE`, try to express non-integers as rational numbers

Details

The LU decomposition is used to solve the equation $Ax = b$ by calculating $L(Ux - d) = 0$, where $Ld = b$. If row exchanges are necessary for A then the permutation matrix P will be requred to exchange the rows in A; otherwise, P will be an identity matrix and the LU equation will be simplified to $A = LU$.

Value

A list of matrix components of the solution, P, L and U. If b is supplied, the vectors d and x are also returned.

Author(s)

Phil Chalmers

Examples

```
A <- matrix(c(2, 1, -1,
             -3, -1, 2,
             -2,  1, 2), 3, 3, byrow=TRUE)
b <- c(8, -11, -3)
(ret <- LU(A)) # P is an identity; no row swapping
with(ret, L %*% U) # check that A = L * U
LU(A, b)

# permutations required in this example
A <- matrix(c(1,  1, -1,
              2,  2,  4,
              1, -1,  1), 3, 3, byrow=TRUE)
b <- c(1, 2, 9)
(ret <- LU(A, b))
with(ret, P %*% A)
with(ret, L %*% U)
```

---

matlib — *matlib: Matrix Functions for Teaching and Learning Linear Algebra and Multivariate Statistics.*

Description

These functions are mainly for tutorial purposes in learning matrix algebra ideas using R. In some cases, functions are provided for concepts available elsewhere in R, but where the function call or name is not obvious. In other cases, functions are provided to show or demonstrate an algorithm. In addition, a collection of functions are provided for drawing vector diagrams in 2D and 3D.

Details

These are not meant for production uses. Other methods are more efficient for larger problems.

Topics

The functions in this package are grouped under the following topics

- Convenience functions
- Determinants: functions for calculating determinants by cofactor expansion
- Elementary row operations: functions for solving linear equations "manually" by the steps used in row echelon form and Gaussian elimination
- Linear equations: functions to illustrate linear equations of the form $A x = b$
- Gaussian elimination: functions for illustrating Gaussian elimination for solving systems of linear equations of the form $A x = b$.
- Eigenvalues: functions to illustrate the algorithms for calculating eigenvalues and eigenvectors
- Vector diagrams: functions for drawing vector diagrams in 2D and 3D

---

matrix2latex — *Convert matrix to LaTeX equation*

Description

This function provides a soft-wrapper to `xtable::xtableMatharray()` with support for `fractions` output and square `brackets`.

Usage

```
matrix2latex(x, fractions = FALSE, brackets = TRUE, ...)
```

Arguments

`x`	a matrix
`fractions`	logical; if `TRUE`, try to express non-integers as rational numbers
`brackets`	logical; include square brackets around the matricies?
`...`	additional arguments passed to `xtable::xtableMatharray()`

Author(s)

Phil Chalmers

Examples

```
A <- matrix(c(2, 1, -1,
             -3, -1, 2,
             -2,  1, 2), 3, 3, byrow=TRUE)
b <- c(8, -11, -3)

matrix2latex(cbind(A,b))
matrix2latex(cbind(A,b), digits = 0)
matrix2latex(cbind(A/2,b), fractions = TRUE)
```

minor *Minor of A[i,j]*

Description

Returns the minor of element (i,j) of the square matrix A, i.e., the determinant of the sub-matrix that results when row i and column j are deleted.

Usage

```
minor(A, i, j)
```

Arguments

A	a square matrix
i	row index
j	column index

Value

the minor of A[i,j]

Author(s)

Michael Friendly

See Also

row_minors for all minors of a given row

Other determinants: cofactor, row_cofactors, row_minors

Examples

```
M <- matrix(c(4, -12, -4,
              2,   1,  3,
             -1,  -3,  2), 3, 3, byrow=TRUE)
minor(M, 1, 1)
minor(M, 1, 2)
minor(M, 1, 3)
```

mpower *Matrix Power*

Description

A simple function to demonstrate the power of a square symmetric matrix in terms of its eigenvalues and eigenvectors.

Usage

```
mpower(A, p, tol = sqrt(.Machine$double.eps))
```

Arguments

`A`	a square symmetric matrix
`p`	matrix power, not necessarily a positive integer
`tol`	tolerance for determining if the matrix is symmetric

Details

The matrix power `p` can be a fraction or other non-integer. For example, `p=1/2` and `p=1/3` give a square-root and cube-root of the matrix.

Negative powers are also allowed. For example, `p=-1` gives the inverse and `p=-1/2` gives the inverse square-root.

Value

`A` raised to the power `p`: `A^p`

See Also

The `{%^%}` operator in the **expm** package is far more efficient

Examples

```
C <- matrix(c(1,2,3,2,5,6,3,6,10), 3, 3) # nonsingular, symmetric
C
mpower(C, 2)
zapsmall(mpower(C, -1))
solve(C)    # check
```

`plot.regvec3d` *Plot method for regvec3d objects*

Description

The plot method for `regvec3d` objects uses the low-level graphics tools in this package to draw 3D and 3D vector diagrams reflecting the partial and marginal relations of `y` to `x1` and `x2` in a bivariate multiple linear regression model, `lm(y ~ x1 + x2)`.

The `summary` method prints the vectors and their vector lengths, followed by the `summary` for the model.

Print method for regvec3d objects

Usage

```
## S3 method for class 'regvec3d'
plot(x, y, dimension = 3, col = c("black", "red", "blue",
  "brown", "lightgray"), col.plane = "gray", cex.lab = 1.2, show.base = 2,
  show.marginal = FALSE, show.hplane = TRUE, show.angles = TRUE,
  error.sphere = c("none", "e", "y.hat"), scale.error.sphere = x$scale,
  level.error.sphere = 0.95, grid = FALSE, add = FALSE, ...)

## S3 method for class 'regvec3d'
summary(object, ...)

## S3 method for class 'regvec3d'
print(x, ...)
```

Arguments

`x`	A “regvec3d” object
`y`	Ignored; only included for compatibility with the S3 generic
`dimension`	Number of dimensions to plot: `3` (default) or `2`
`col`	A vector of 5 colors
`col.plane`	Color of the base plane in a 3D plot or axes in a 2D plot
`cex.lab`	character expansion applied to vector labels. May be a number or numeric vector corresponding to the the rows of `X`, recycled as necessary.
`show.base`	If `show.base > 0`, draws the base plane in a 3D plot; if `show.base > 1`, the plane is drawn thicker
`show.marginal`	If `TRUE` also draws lines showing the marginal relations of `y` on `x1` and on `x2`
`show.hplane`	If `TRUE`, draws the plane defined by `y`, `yhat` and the origin in the 3D
`show.angles`	If `TRUE`, draw and label the angle between the `x1` and `x2` and between `y` and `yhat`, corresponding respectively to the correlation between the xs and the multiple correlation

`error.sphere`	Plot a sphere (or in 2D, a circle) of radius proportional to the length of the residual vector, centered either at the origin ("e") or at the fitted-values vector ("y.hat"; the default is "none".)
`scale.error.sphere`	Whether to scale the error sphere if `error.sphere="y.hat"`; defaults to `TRUE` if the vectors representing the variables are scaled, in which case the oblique projections of the error spheres can represent confidence intervals for the coefficients; otherwise defaults to `FALSE`.
`level.error.sphere`	The confidence level for the error sphere, applied if `scale.error.sphere=TRUE`.
`grid`	If `TRUE`, draws a light grid on the base plane
`add`	If `TRUE`, add to the current plot; otherwise start a new rgl or plot window
`...`	Parameters passed down to functions [unused now]
`object`	A `regvec3d` object for the `summary` method

Details

A 3D diagram shows the vector `y` and the plane formed by the predictors, `x1` and `x2`, where all variables are represented in deviation form, so that the intercept need not be included.

A 2D diagram, using the first two columns of the result, can be used to show the projection of the space in the `x1`, `x2` plane.

The drawing functions `vectors` and `link{vectors3d}` used by the `plot.regvec3d` method only work reasonably well if the variables are shown on commensurate scales, i.e., with either `scale=TRUE` or `normalize=TRUE`.

Value

None

References

Fox, J. (2016). *Applied Regression Analysis and Generalized Linear Models*, 3rd ed., Sage, Chapter 10.

See Also

`regvec3d`, `vectors3d`, `vectors`

Other vector diagrams: `Proj`, `arc`, `arrows3d`, `corner`, `point_on_line`, `regvec3d`, `vectors3d`, `vectors`

Examples

```
if (require(car)) {
   data("Duncan", package="car")
   dunc.reg <- regvec3d(prestige ~ income + education, data=Duncan)
   plot(dunc.reg)
   plot(dunc.reg, dimension=2)
   plot(dunc.reg, error.sphere="e")
```

```
    summary(dunc.reg)

    # Example showing Simpson's paradox
    states.vec <- regvec3d(SATM ~ pay + percent, data=car::States, scale=TRUE)
    plot(states.vec, show.marginal=TRUE)
    plot(states.vec, show.marginal=TRUE, dimension=2)
    summary(states.vec)
}
```

---

plotEqn *Plot Linear Equations*

Description

Shows what matrices A, b look like as the system of linear equations, $Ax = b$ with two unknowns, x1, x2, by plotting a line for each equation.

Usage

```
plotEqn(A, b, vars, xlim = c(-4, 4), ylim, col = 1:nrow(A), lwd = 2,
  lty = 1, axes = TRUE, labels = TRUE, solution = TRUE)
```

Arguments

A	either the matrix of coefficients of a system of linear equations, or the matrix `cbind(A,b)`. The `A` matrix must have two columns.
b	if supplied, the vector of constants on the right hand side of the equations, of length matching the number of rows of `A`.
vars	a numeric or character vector of names of the variables. If supplied, the length must be equal to the number of unknowns in the equations. The default is `paste0("x", 1:ncol(A)`.
xlim	horizontal axis limits for the first variable
ylim	vertical axis limits for the second variable; if missing, `ylim` is calculated from the range of the set of equations over the `xlim`.
col	scalar or vector of colors for the lines, recycled as necessary
lwd	scalar or vector of line widths for the lines, recycled as necessary
lty	scalar or vector of line types for the lines, recycled as necessary
axes	logical; draw horizontal and vertical axes through (0,0)?
labels	logical, or a vector of character labels for the equations; if `TRUE`, each equation is labeled using the character string resulting from `showEqn`
solution	logical; should the solution points for pairs of equations be marked?

Value

nothing; used for the side effect of making a plot

Author(s)

Michael Friendly

See Also

showEqn

Examples

```
# consistent equations
A<- matrix(c(1,2,3, -1, 2, 1),3,2)
b <- c(2,1,3)
showEqn(A, b)
plotEqn(A,b)

# inconsistent equations
b <- c(2,1,6)
showEqn(A, b)
plotEqn(A,b)
```

`plotEqn3d` *Plot Linear Equations in 3D*

Description

Shows what matrices A, b look like as the system of linear equations, $Ax = b$ with three unknowns, x1, x2, and x3, by plotting a plane for each equation.

Usage

```
plotEqn3d(A, b, vars, xlim = c(-2, 2), ylim = c(-2, 2), zlim,
  col = 2:(nrow(A) + 1), alpha = 1, labels = FALSE, solution = TRUE)
```

Arguments

`A`	either the matrix of coefficients of a system of linear equations, or the matrix `cbind(A,b)` The `A` matrix must have three columns.
`b`	if supplied, the vector of constants on the right hand side of the equations, of length matching the number of rows of `A`.
`vars`	a numeric or character vector of names of the variables. If supplied, the length must be equal to the number of unknowns in the equations. The default is `paste0("x", 1:ncol(A))`.
`xlim`	axis limits for the first variable
`ylim`	axis limits for the second variable
`zlim`	horizontal axis limits for the second variable; if missing, `zlim` is calculated from the range of the set of equations over the `xlim` and `ylim`

col	scalar or vector of colors for the lines, recycled as necessary
alpha	transparency applied to each plane
labels	logical, or a vector of character labels for the equations; not yet implemented.
solution	logical; should the solution point for all equations be marked (if possible)

Value

nothing; used for the side effect of making a plot

Examples

```
# three consistent equations in three unknowns
A <- matrix(c(13, -4, 2, -4, 11, -2, 2, -2, 8), 3,3)
b <- c(1,2,4)
plotEqn3d(A,b)
```

---

point_on_line — *Position of a point along a line*

Description

A utility function for drawing vector diagrams. Find position of an interpolated point along a line from x1 to x2.

Usage

```
point_on_line(x1, x2, d, absolute = TRUE)
```

Arguments

x1	A vector of length 2 or 3, representing the starting point of a line in 2D or 3D space
x2	A vector of length 2 or 3, representing the ending point of a line in 2D or 3D space
d	The distance along the line from x1 to x2 of the point to be found.
absolute	logical; if TRUE, d is taken as an absolute distance along the line; otherwise it is calculated as a relative distance, i.e., a fraction of the length of the line.

Details

The function takes a step of length d along the line defined by the difference between the two points, x2 - x1. When absolute=FALSE, this step is proportional to the difference, while when absolute=TRUE, the difference is first scaled to unit length so that the step is always of length d. Note that the physical length of a line in different directions in a graph depends on the aspect ratio of the plot axes, and lines of the same length will only appear equal if the aspect ratio is one (asp=1 in 2D, or aspect3d("iso") in 3D).

Value

The interpolated point, a vector of the same length as x1

See Also

Other vector diagrams: Proj, arc, arrows3d, corner, plot.regvec3d, regvec3d, vectors3d, vectors

Examples

```
x1 <- c(0, 0)
x2 <- c(1, 4)
point_on_line(x1, x2, 0.5)
point_on_line(x1, x2, 0.5, absolute=FALSE)
point_on_line(x1, x2, 1.1)

y1 <- c(1, 2, 3)
y2 <- c(3, 2, 1)
point_on_line(y1, y2, 0.5)
point_on_line(y1, y2, 0.5, absolute=FALSE)
```

power_method *Power Method for Eigenvectors*

Description

Finds a dominant eigenvalue and its corresponding eigenvector of a square matrix by applying the Power Method with scaling

Usage

```
power_method(X, v = NULL, eps = 1e-06, maxiter = 100, verbose = FALSE)
```

Arguments

X	a square numeric matrix
v	optional starting vector
eps	convergence threshold
maxiter	maximum number of iterations
verbose	logical; if TRUE, show the approximation to the eigenvector at each iteration

Value

a list containing the eigenvector, eigenvalue, and iterations

Author(s)

Gaston Sanchez (from matrixkit)

Examples

```
A = cbind(c(2, 1), c(12, 5))
power_method(A)
eigen(A)$vectors[,1]  # check

B = cbind(c(1, 2, 0), c(2, 1, 3), c(0, 3, 1))
power_method(B)
```

Proj *Projection of Vector y on columns of X*

Description

Fitting a linear model, `lm(y ~ X)`, by least squares can be thought of geometrically as the orthogonal projection of `y` on the column space of `X`. This function is designed to allow exploration of projections and orthogonality.

Usage

```
Proj(y, X, list = FALSE)
```

Arguments

`y`	a vector, treated as a one-column matrix
`X`	a vector or matrix. Number of rows of `y` and `X` must match
`list`	logical; if FALSE, return just the projected vector; otherwise returns a list

Details

The projection is defined as Py where $P = X(X'X)^{-}X'$ and X^{-} is a generalized inverse.

Value

the projection of `y` on `X` (if `list=FALSE`) or a list with elements `y` and `P`

Author(s)

Michael Friendly

See Also

Other vector diagrams: `arc`, `arrows3d`, `corner`, `plot.regvec3d`, `point_on_line`, `regvec3d`, `vectors3d`, `vectors`

Examples

```
X <- matrix( c(1, 1, 1, 1, 1, -1, 1, -1), 4,2, byrow=TRUE)
y <- 1:4
Proj(y, X[,1])  # project y on unit vector
Proj(y, X[,2])
Proj(y, X)

# orthogonal complements
yp <-Proj(y, X, list=TRUE)
yp$y
P <- yp$P
IP <- diag(4) - P
yc <- c(IP %*% y)
crossprod(yp$y, yc)

# P is idempotent:  P P = P
P %*% P
all.equal(P, P %*% P)
```

QR — *QR Decomposition by Graham-Schmidt Orthonormalization*

Description

QR computes the QR decomposition of a matrix, X, that is an orthonormal matrix, Q and an upper triangular matrix, R, such that $X = QR$.

Usage

```
QR(X, tol = sqrt(.Machine$double.eps))
```

Arguments

X	a numeric matrix
tol	tolerance for detecting linear dependencies in the columns of X

Details

The QR decomposition plays an important role in many statistical techniques. In particular it can be used to solve the equation $Ax = b$ for given matrix A and vector b. The function is included here simply to show the algorithm of Gram-Schmidt orthogonalization. The standard qr function is faster and more accurate.

Value

a list of three elements, consisting of an orthonormal matrix Q, an upper triangular matrix R, and the rank of the matrix X

Author(s)

John Fox and Georges Monette

See Also

qr

Examples

```
A <- matrix(c(1,2,3,4,5,6,7,8,10), 3, 3) # a square nonsingular matrix
res <- QR(A)
res
q <- res$Q
zapsmall( t(q) %*% q)   # check that q' q = I
r <- res$R
q %*% r                 # check that q r = A

# relation to determinant: det(A) = prod(diag(R))
det(A)
prod(diag(r))

B <- matrix(1:9, 3, 3) # a singular matrix
QR(B)
```

R *Rank of a Matrix*

Description

Returns the rank of a matrix X, using the QR decomposition, QR. Included here as a simple function, because rank does something different and it is not obvious what to use for matrix rank.

Usage

```
R(X)
```

Arguments

X	a matrix

Value

rank of X

See Also

qr

Examples

```
M <- outer(1:3, 3:1)
M
R(M)

M <- matrix(1:9, 3, 3)
M
R(M)
# why rank=2?
echelon(M)

set.seed(1234)
M <- matrix(sample(1:9), 3, 3)
M
R(M)
```

`regvec3d` *Vector space representation of a two-variable regression model*

Description

`regvec3d` calculates the 3D vectors that represent the projection of a two-variable multiple regression model from n-D *observation* space into the 3D mean-deviation *variable* space that they span, thus showing the regression of `y` on `x1` and `x2` in the model `lm(y ~ x1 + x2)`. The result can be used to draw 2D and 3D vector diagrams accurately reflecting the partial and marginal relations of `y` to `x1` and `x2` as vectors in this representation.

Usage

```
regvec3d(x1, ...)

## S3 method for class 'formula'
regvec3d(formula, data = NULL, which = 1:2, name.x1,
  name.x2, name.y, name.e, name.y.hat, name.b1.x1, name.b2.x2, abbreviate = 0,
  ...)

## Default S3 method:
regvec3d(x1, x2, y, scale = FALSE, normalize = TRUE,
  name.x1 = deparse(substitute(x1)), name.x2 = deparse(substitute(x2)),
  name.y = deparse(substitute(y)), name.e = "residuals",
  name.y.hat = paste0(name.y, "hat"), name.b1.x1 = paste0("b1", name.x1),
  name.b2.x2 = paste0("b2", name.x2), name.y1.hat = paste0(name.y, "hat 1"),
  name.y2.hat = paste0(name.y, "hat 2"), ...)
```

Arguments

`x1`	The generic argument or the first predictor passed to the default method
`...`	Arguments passed to methods

formula	A two-sided formula for the linear regression model. It must contain two quantitative predictors (x1 and x2) on the right-hand-side. If further predictors are included, y, x1 and x2 are taken as residuals from the their linear fits on these variables.
data	A data frame in which the variables in the model are found
which	Indices of predictors variables in the model taken as x1 and x2
name.x1	Name for x1 to be used in the result and plots. By default, this is taken as the name of the x1 variable in the formula, possibly abbreviated according to abbreviate.
name.x2	Ditto for the name of x2
name.y	Ditto for the name of y
name.e	Name for the residual vector. Default: "residuals"
name.y.hat	Name for the fitted vector
name.b1.x1	Name for the vector corresponding to the partial coefficient of x1
name.b2.x2	Name for the vector corresponding to the partial coefficient of x2
abbreviate	An integer. If abbreviate >0, the names of x1, x2 and y are abbreviated to this length before being combined with the other name.* arguments
x2	second predictor variable in the model
y	response variable in the model
scale	logical; if TRUE, standardize each of y, x1, x2 to standard scores
normalize	logical; if TRUE, normalize each vector relative to the maximum length of all
name.y1.hat	Name for the vector corresponding to the marginal coefficient of x1
name.y2.hat	Name for the vector corresponding to the marginal coefficient of x2

Details

If additional variables are included in the model, e.g., `lm(y ~ x1 + x2 + x3 + ...)`, then y, x1 and x2 are all taken as *residuals* from their separate linear fits on `x3 + ...`, thus showing their partial relations net of (or adjusting for) these additional predictors.

A 3D diagram shows the vector y and the plane formed by the predictors, x1 and x2, where all variables are represented in deviation form, so that the intercept need not be included.

A 2D diagram, using the first two columns of the result, can be used to show the projection of the space in the x1, x2 plane.

In these views, the ANOVA representation of the various sums of squares for the regression predictors appears as the lengths of the various vectors. For example, the error sum of squares is the squared length of the e vector, and the regression sum of squares is the squared length of the yhat vector.

The drawing functions vectors and link{vectors3d} used by the plot.regvec3d method only work reasonably well if the variables are shown on commensurate scales, i.e., with either scale=TRUE or normalize=TRUE.

Value

An object of class "regvec3d", containing the following components

model — The "lm" object corresponding to `lm(y ~ x1 + x2)`.

vectors — A 9 x 3 matrix, whose rows correspond to the variables in the model, the residual vector, the fitted vector, the partial fits for `x1`, `x2`, and the marginal fits of `y` on `x1` and `x2`. The columns effectively represent `x1`, `x2`, and `y`, but are named `"x"`, `"y"` and `"z"`.

Methods (by class)

- `formula`: Formula method for regvec3d
- `default`: Default method for regvec3d

References

Fox, J. (2016). *Applied Regression Analysis and Generalized Linear Models*, 3rd ed., Sage, Chapter 10.

See Also

`plot.regvec3d`

Other vector diagrams: `Proj`, `arc`, `arrows3d`, `corner`, `plot.regvec3d`, `point_on_line`, `vectors3d`, `vectors`

Examples

```
library(rgl)
therapy.vec <- regvec3d(therapy ~ perstest + IE, data=therapy)
therapy.vec
plot(therapy.vec)
plot(therapy.vec, dimension="2")
```

---

rowadd — *Add multiples of rows to other rows*

Description

The elementary row operation `rowadd` adds multiples of one or more rows to other rows of a matrix. This is usually used as a means to solve systems of linear equations, of the form $Ax = b$, and `rowadd` corresponds to adding equals to equals.

Usage

```
rowadd(x, from, to, mult)
```

Arguments

x	a numeric matrix, possibly consisting of the coefficient matrix, A, joined with a vector of constants, b.
from	the index of one or more source rows. If from is a vector, it must have the same length as to.
to	the index of one or more destination rows
mult	the multiplier(s)

Details

The functions rowmult and rowswap complete the basic operations used in reduction to row echelon form and Gaussian elimination. These functions are used for demonstration purposes.

Value

the matrix x, as modified

See Also

echelon, gaussianElimination

Other elementary row operations: rowmult, rowswap

Examples

```
A <- matrix(c(2, 1, -1,
             -3, -1, 2,
             -2,  1, 2), 3, 3, byrow=TRUE)
b <- c(8, -11, -3)

# using row operations to reduce below diagonal to 0
Ab <- cbind(A, b)
(Ab <- rowadd(Ab, 1, 2, 3/2))  # row 2 <- row 2 + 3/2 row 1
(Ab <- rowadd(Ab, 1, 3, 1))    # row 3 <- row 3 + 1 row 1
(Ab <- rowadd(Ab, 2, 3, -4))   # row 3 <- row 3 - 4 row 2
# multiply to make diagonals = 1
(Ab <- rowmult(Ab, 1:3, c(1/2, 2, -1)))
# The matrix is now in triangular form

# Could continue to reduce above diagonal to zero
echelon(A, b, verbose=TRUE, fractions=TRUE)
```

rowmult — *Multiply Rows by Constants*

Description

Multiplies one or more rows of a matrix by constants. This corresponds to multiplying or dividing equations by constants.

Usage

```
rowmult(x, row, mult)
```

Arguments

x	a matrix, possibly consisting of the coefficient matrix, A, joined with a vector of constants, b.
row	index of one or more rows.
mult	row multiplier(s)

Value

the matrix x, modified

See Also

echelon, gaussianElimination

Other elementary row operations: rowadd, rowswap

Examples

```
A <- matrix(c(2, 1, -1,
             -3, -1, 2,
             -2,  1, 2), 3, 3, byrow=TRUE)
b <- c(8, -11, -3)

# using row operations to reduce below diagonal to 0
Ab <- cbind(A, b)
(Ab <- rowadd(Ab, 1, 2, 3/2))  # row 2 <- row 2 + 3/2 row 1
(Ab <- rowadd(Ab, 1, 3, 1))    # row 3 <- row 3 + 1 row 1
(Ab <- rowadd(Ab, 2, 3, -4))
# multiply to make diagonals = 1
(Ab <- rowmult(Ab, 1:3, c(1/2, 2, -1)))
# The matrix is now in triangular form
```

rowswap — *Interchange two rows of a matrix*

Description

This elementary row operation corresponds to interchanging two equations.

Usage

```
rowswap(x, from, to)
```

Arguments

`x`	a matrix, possibly consisting of the coefficient matrix, A, joined with a vector of constants, b.
`from`	source row.
`to`	destination row

Value

the matrix `x`, with rows `from` and `to` interchanged

See Also

`echelon`, `gaussianElimination`

Other elementary row operations: `rowadd`, `rowmult`

row_cofactors — *Row Cofactors of A[i,]*

Description

Returns the vector of cofactors of row i of the square matrix A. The determinant, `det(A)`, can then be found as `M[i,] %*% row_cofactors(M,i)` for any row, i.

Usage

```
row_cofactors(A, i)
```

Arguments

`A`	a square matrix
`i`	row index

Value

a vector of the cofactors of A[i,]

Author(s)

Michael Friendly

See Also

det for the determinant

Other determinants: cofactor, minor, row_minors

Examples

```
M <- matrix(c(4, -12, -4,
              2,   1,  3,
             -1,  -3,  2), 3, 3, byrow=TRUE)
minor(M, 1, 1)
minor(M, 1, 2)
minor(M, 1, 3)
row_cofactors(M, 1)
det(M)
# expansion by cofactors of row 1
M[1,] %*% row_cofactors(M,1)
```

---

row_minors *Row Minors of A[i,]*

Description

Returns the vector of minors of row i of the square matrix A

Usage

```
row_minors(A, i)
```

Arguments

A	a square matrix
i	row index

Value

a vector of the minors of A[i,]

Author(s)

Michael Friendly

See Also

Other determinants: cofactor, minor, row_cofactors

Examples

```
M <- matrix(c(4, -12, -4,
              2,   1,  3,
             -1,  -3,  2), 3, 3, byrow=TRUE)
minor(M, 1, 1)
minor(M, 1, 2)
minor(M, 1, 3)
row_minors(M, 1)
```

---

showEig   *Show the eigenvectors associated with a covariance matrix*

Description

This function is designed for illustrating the eigenvectors associated with the covariance matrix for a given bivariate data set. It draws a data ellipse of the data and adds vectors showing the eigenvectors of the covariance matrix.

Usage

```
showEig(X, col.vec = "blue", lwd.vec = 3, mult = sqrt(qchisq(levels, 2)),
  asp = 1, levels = c(0.5, 0.95), plot.points = TRUE,
  add = !plot.points, ...)
```

Arguments

`X`	A two-column matrix or data frame
`col.vec`	color for eigenvectors
`lwd.vec`	line width for eigenvectors
`mult`	length multiplier(s) for eigenvectors
`asp`	aspect ratio of plot, set to `asp=1` by default, and passed to dataEllipse
`levels`	passed to dataEllipse determining the coverage of the data ellipse(s)
`plot.points`	logical; should the points be plotted?
`add`	logical; should this call add to an existing plot?
`...`	other arguments passed to `link[car]{dataEllipse}`

Author(s)

Michael Friendly

See Also

`link[car]{dataEllipse}`

Examples

```
x <- rnorm(200)
y <- .5 * x + .5 * rnorm(200)
X <- cbind(x,y)
showEig(X)

# Duncan data
data(Duncan, package="car")
showEig(Duncan[, 2:3], levels=0.68)
showEig(Duncan[,2:3], levels=0.68, robust=TRUE, add=TRUE, fill=TRUE)
```

`showEqn` *Show Matrices (A, b) as Linear Equations*

Description

Shows what matrices A, b look like as the system of linear equations, $Ax = b$, but written out as a set of equations.

Usage

```
showEqn(A, b, vars, simplify = FALSE, fractions = FALSE, latex = FALSE)
```

Arguments

`A`	either the matrix of coefficients of a system of linear equations, or the matrix `cbind(A,b)`
`b`	if supplied, the vector of constants on the right hand side of the equations
`vars`	a numeric or character vector of names of the variables. If supplied, the length must be equal to the number of unknowns in the equations. The default is `paste0("x", 1:ncol(A)`.
`simplify`	logical; try to simplify the equations?
`fractions`	logical; express numbers as rational fractions?
`latex`	logical; print equations in a form suitable for LaTeX output?

Value

a one-column character matrix, one row for each equation

Author(s)

Michael Friendly and John Fox

See Also

plotEqn, plotEqn3d

Examples

```
A <- matrix(c(2, 1, -1,
             -3, -1, 2,
             -2,  1, 2), 3, 3, byrow=TRUE)
b <- c(8, -11, -3)
showEqn(A, b)
# show numerically
x <- solve(A, b)
showEqn(A, b, vars=x)

showEqn(A, b, simplify=TRUE)
showEqn(A, b, latex=TRUE)
```

Solve	*Solve and Display Solutions for Systems of Linear Simultaneous Equations*

Description

Solve the equation system $Ax = b$, given the coefficient matrix A and right-hand side vector b, using `link{gaussianElimination}`. Display the solutions using showEqn.

Usage

```
Solve(A, b, verbose = FALSE, simplify = TRUE, fractions = FALSE, ...)
```

Arguments

`A,`	the matrix of coefficients of a system of linear equations
`b,`	the vector of constants on the right hand side of the equations
`verbose,`	logical; show the steps of the Gaussian elimination algorithm?
`simplify`	logical; try to simplify the equations?
`fractions`	logical; express numbers as rational fractions?
`...,`	arguments to be passed to `link{gaussianElimination}` and showEqn

Value

the function is used primarily for its side effect of printing the solution in a readable form, but it invisibly returns the solution as a character vector

Author(s)

John Fox

See Also

gaussianElimination, showEqn

Examples

```
A1 <- matrix(c(2, 1, -1,
             -3, -1, 2,
             -2,  1, 2), 3, 3, byrow=TRUE)
b1 <- c(8, -11, -3)
Solve(A1, b1) # unique solution

A2 <- matrix(1:9, 3, 3)
b2 <- 1:3
Solve(A2,  b2, fractions=TRUE) # underdetermined

b3 <- c(1, 2, 4)
Solve(A2, b3, fractions=TRUE) # overdetermined
```

SVD *Singular Value Decomposition of a Matrix*

Description

Compute the singular-value decomposition of a matrix X from the eigenstructure of $X'X$. The result consists of two orthonormal matrices, U, and V and the vector d of singular values, such that $X = Udiag(d)V'$. Singular values of zero are not retained in the solution.

Usage

```
SVD(X, tol = sqrt(.Machine$double.eps))
```

Arguments

X	a square symmetric matrix
tol	tolerance passed to QR

Value

a list of three elements: d– singular values, U– left singular vectors, V– right singular vectors

Author(s)

John Fox and Georges Monette

See Also

svd, the standard svd function

eig

Examples

```
C <- matrix(c(1,2,3,2,5,6,3,6,10), 3, 3) # nonsingular, symmetric
C
SVD(C)

# least squares by the SVD
data("workers")
X <- cbind(1, as.matrix(workers[, c("Experience", "Skill")]))
head(X)
y <- workers$Income
head(y)
(svd <- SVD(X))
VdU <- svd$V %*% diag(1/svd$d) %*%t(svd$U)
(b <- VdU %*% y)
coef(lm(Income ~ Experience + Skill, data=workers))
```

`swp` *The Matrix Sweep Operator*

Description

The `swp` function "sweeps" a matrix on the rows and columns given in `index` to produce a new matrix with those rows and columns "partialled out" by orthogonalization. This was defined as a fundamental statistical operation in multivariate methods by Beaton (1964) and expanded by Dempster (1969). It is closely related to orthogonal projection, but applied to a cross-products or covariance matrix, rather than to data.

Usage

```
swp(M, index)
```

Arguments

`M`	a numeric matrix
`index`	a numeric vector indicating the rows/columns to be swept. The entries must be less than or equal to the number or rows or columns in `M`. If missing, the function sweeps on all rows/columns `1:min(dim(M))`.

Details

If M is the partitioned matrix

$$\begin{bmatrix} \mathbf{R} & \mathbf{S} \\ \mathbf{T} & \mathbf{U} \end{bmatrix}$$

where R is $q \times q$ then swp(M, 1:q) gives

$$\begin{bmatrix} \mathbf{R}^{-1} & \mathbf{R}^{-1}\mathbf{S} \\ -\mathbf{T}\mathbf{R}^{-1} & \mathbf{U} - \mathbf{T}\mathbf{R}^{-1}\mathbf{S} \end{bmatrix}$$

Value

the matrix M with rows and columns in indices swept.

References

Beaton, A. E. (1964), *The Use of Special Matrix Operations in Statistical Calculus*, Princeton, NJ: Educational Testing Service.

Dempster, A. P. (1969) *Elements of Continuous Multivariate Analysis*. Addison-Wesley Publ. Co., Reading, Mass.

See Also

Proj, QR

Examples

```
data(therapy)
mod3 <- lm(therapy ~ perstest + IE + sex, data=therapy)
X <- model.matrix(mod3)
XY <- cbind(X, therapy=therapy$therapy)
XY
M <- crossprod(XY)
swp(M, 1)
swp(M, 1:2)
```

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therapy *Therapy Data*

Description

A toy data set on outcome in therapy in relation to a personality test (perstest) and a scale of internal-external locus of control (IE) used to illustrate linear and multiple regression.

Usage

```
data("therapy")
```

Format

A data frame with 10 observations on the following 4 variables.

`sex` a factor with levels `F` `M`

`perstest` score on a personality test, a numeric vector

`therapy` outcome in psychotherapy, a numeric vector

`IE` score on a scale of internal-external locus of control, a numeric vector

Examples

```
data(therapy)
plot(therapy ~ perstest, data=therapy, pch=16)
abline(lm(therapy ~ perstest, data=therapy), col="red")

plot(therapy ~ perstest, data=therapy, cex=1.5, pch=16,
col=ifelse(sex=="M", "red","blue"))
```

`tr` *Trace of a Matrix*

Description

Calculates the trace of a square numeric matrix, i.e., the sum of its diagonal elements

Usage

```
tr(X)
```

Arguments

`X` a numeric matrix

Value

a numeric value, the sum of `diag(X)`

Examples

```
X <- matrix(1:9, 3, 3)
tr(X)
```

vandermode — *Vandermode Matrix*

Description

The function returns the Vandermode matrix of a numeric vector, `x`, whose columns are the vector raised to the powers `0:n`.

Usage

```
vandermode(x, n)
```

Arguments

`x`	a numeric vector
`n`	a numeric scalar

Value

a matrix of size `length(x)` x `n`

Examples

```
vandermode(1:5, 4)
```

vec — *Vectorize a Matrix*

Description

Returns a 1-column matrix, stacking the columns of `x`, a matrix or vector

Usage

```
vec(x)
```

Arguments

`x`	A matrix or vector

Value

A one-column matrix containing the elements of `x` in column order

Examples

```
vec(1:3)
vec(matrix(1:6, 2, 3))
vec(c("hello", "world"))
```

vectors — *Draw geometric vectors in 2D*

Description

This function draws vectors in a 2D plot, in a way that facilitates constructing vector diagrams. It allows vectors to be specified as rows of a matrix, and can draw labels on the vectors.

Usage

```
vectors(X, origin = c(0, 0), lwd = 2, angle = 13, length = 0.15,
  labels = TRUE, cex.lab = 1.5, pos.lab = 4, frac.lab = 1, ...)
```

Arguments

`X`	a vector or two-column matrix representing a set of geometric vectors; if a matrix, one vector is drawn for each row
`origin`	the origin from which they are drawn, a vector of length 2.
`lwd`	line width(s) for the vectors, a constant or vector of length equal to the number of rows of `X`.
`angle`	the `angle` argument passed to `arrows` determining the angle of arrow heads.
`length`	the `length` argument passed to `arrows` determining the length of arrow heads.
`labels`	a logical or a character vector of labels for the vectors. If `TRUE` and `X` is a matrix, labels are taken from `rownames(X)`. If `NULL`, no labels are drawn.
`cex.lab`	character expansion applied to vector labels. May be a number or numeric vector corresponding to the the rows of `X`, recycled as necessary.
`pos.lab`	label position relative to the label point as in `text`, recycled as necessary.
`frac.lab`	location of label point, as a fraction of the distance between `origin` and `X`, recycled as necessary. Values `frac.lab > 1` locate the label beyond the end of the vector.
`...`	other arguments passed on to graphics functions.

Value

none

See Also

`arrows`, `text`

Other vector diagrams: `Proj`, `arc`, `arrows3d`, `corner`, `plot.regvec3d`, `point_on_line`, `regvec3d`, `vectors3d`

Examples

```
# shows addition of vectors
u <- c(3,1)
v <- c(1,3)
sum <- u+v

xlim <- c(0,5)
ylim <- c(0,5)
# proper geometry requires asp=1
plot( xlim, ylim, type="n", xlab="X", ylab="Y", asp=1)
abline(v=0, h=0, col="gray")

vectors(rbind(u,v,`u+v`=sum), col=c("red", "blue", "purple"), cex.lab=c(2, 2, 2.2))
# show the opposing sides of the parallelogram
vectors(sum, origin=u, col="red", lty=2)
vectors(sum, origin=v, col="blue", lty=2)

# projection of vectors
vectors(Proj(v,u), labels="P(v,u)", lwd=3)
vectors(v, origin=Proj(v,u))
corner(c(0,0), Proj(v,u), v, col="grey")
```

`vectors3d` *Draw 3D vectors*

Description

This function draws vectors in a 3D plot, in a way that facilitates constructing vector diagrams. It allows vectors to be specified as rows of a matrix, and can draw labels on the vectors.

Usage

```
vectors3d(X, origin = c(0, 0, 0), headlength = 0.035, ref.length = NULL,
  radius = 1/60, labels = TRUE, cex.lab = 1.2, adj.lab = 0.5,
  frac.lab = 1.1, draw = TRUE, ...)
```

Arguments

`X`	a vector or three-column matrix representing a set of geometric vectors; if a matrix, one vector is drawn for each row
`origin`	the origin from which they are drawn, a vector of length 3.
`headlength`	the `headlength` argument passed to `arrows3d` determining the length of arrow heads
`ref.length`	vector length to be used in scaling arrow heads so that they are all the same size; if `NULL` the longest vector is used to scale the arrow heads
`radius`	radius of the base of the arrow heads

labels	a logical or a character vector of labels for the vectors. If TRUE and X is a matrix, labels are taken from rownames(X). If NULL, no labels are drawn.
cex.lab	character expansion applied to vector labels. May be a number or numeric vector corresponding to the the rows of X, recycled as necessary.
adj.lab	label position relative to the label point as in text3d, recycled as necessary.
frac.lab	location of label point, as a fraction of the distance between origin and X, recycled as necessary. Values frac.lab > 1 locate the label beyond the end of the vector.
draw	if TRUE (the default), draw the vector(s).
...	other arguments passed on to graphics functions.

Value

invisibly returns the vector ref.length used to scale arrow heads

Bugs

At present, the color (color=) argument is not handled as expected when more than one vector is to be drawn.

Author(s)

Michael Friendly

See Also

arrows3d, codetexts3d, codergl.material

Other vector diagrams: Proj, arc, arrows3d, corner, plot.regvec3d, point_on_line, regvec3d, vectors

Examples

```
vec <- rbind(diag(3), c(1,1,1))
rownames(vec) <- c("X", "Y", "Z", "J")
library(rgl)
open3d()
vectors3d(vec, color=c(rep("black",3), "red"), lwd=2)
# draw the XZ plane, whose equation is Y=0
planes3d(0, 0, 1, 0, col="gray", alpha=0.2)
vectors3d(c(1,1,0), col="green", lwd=2)
# show projections of the unit vector J
segments3d(rbind(c(1,1,1), c(1, 1, 0)))
segments3d(rbind(c(0,0,0), c(1, 1, 0)))
segments3d(rbind(c(1,0,0), c(1, 1, 0)))
segments3d(rbind(c(0,1,0), c(1, 1, 0)))
# show some orthogonal vectors
p1 <- c(0,0,0)
p2 <- c(1,1,0)
p3 <- c(1,1,1)
```

```
p4 <- c(1,0,0)
corner(p1, p2, p3, col="red")
corner(p1, p4, p2, col="red")
corner(p1, p4, p3, col="blue")

rgl.bringtotop()
```

---

workers *Workers Data*

Description

A toy data set comprised of information on workers `Income` in relation to other variables, used for illustrating linear and multiple regression.

Usage

```
data("workers")
```

Format

A data frame with 10 observations on the following 4 variables.

`Income` income from the job, a numeric vector

`Experience` number of years of experience, a numeric vector

`Skill` skill level in the job, a numeric vector

`Gender` a factor with levels `Female` `Male`

Examples

```
data(workers)
plot(Income ~ Experience, data=workers, main="Income ~ Experience", pch=20, cex=2)

# simple linear regression
reg1 <- lm(Income ~ Experience, data=workers)
abline(reg1, col="red", lwd=3)

# quadratic fit?
plot(Income ~ Experience, data=workers, main="Income ~ poly(Experience,2)", pch=20, cex=2)
reg2 <- lm(Income ~ poly(Experience,2), data=workers)
fit2 <-predict(reg2)
abline(reg1, col="red", lwd=1, lty=1)
lines(workers$Experience, fit2, col="blue", lwd=3)

# How does Income depend on a factor?
plot(Income ~ Gender, data=workers, main="Income ~ Gender")
points(workers$Gender, jitter(workers$Income), cex=2, pch=20)
means<-aggregate(workers$Income,list(workers$Gender),mean)
points(means,col="red", pch="+", cex=2)
lines(means,col="red", lwd=2)
```

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