

Package ‘fAsianOptions’

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and valuating Asian Options together with tools for analyzing
and modeling Exponential Brownian Motion (EBM).
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fAsianOptions-package *EBM and Asian Option Valuation*

Description

The Rmetrics "fAsianOptions" package is a collection of functions to analyze and model Exponential Brownian Motion and to value Asian options.

Details

```
Package:  \tab fAsianOptions\cr
Type:    \tab Package\cr
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```

1 Introduction

The fAsianOptions package provides functions for pricing and valuating Asian Options together with tools for analyzing and modeling Exponential Brownian Motion (EBM).

2 Asian Option Pricing

This is a collection of functions used in the theory of exponential Brownian Motion, EBM, and in the valuation of Asian options.

MomentMatchedAsianOption	value moment matched option prices
- "LN"	Log-Normal Approx of Levy, Turnbull and Wakeman
- "RG"	Reciprocal-Gamma Approx of Milevski and Posner
- "JI"	Johnson Type I Approx of Posner and Milevsky
MomentMatchedAsianDensity	value moment matched option densities
- "LN"	Log-Normal Approximation
- "RG"	Reciprocal-Gamma Approximation
- "JI"	Johnson Type I Approximation
GramCharlierAsianOption	calculates Gram-Charlier option prices
AsianOptionMoments	methods to calculate Asian Moments
- "A"	using moments from Abrahamson's Formula
- "D"	using moments from Dufresne's Formula
- "TW"	using first 2 Moments from Turnbull-Wakeman
- "T"	including asymptotic Behavior after Tolmatz

ZhangAsianOption	Asian option price by Zhang's 1D PDE
VecerAsianOption	Asian option price by Vecer's 1D PDE
gGemanYor	Function to be Laplace inverted
GemanYorAsianOption	Asian option price by Laplace Inversion
gLinetzky	Function to be integrated
LinetzkyAsianOption	Asian option price by Spectral Expansion
BoundsOnAsianOption	using lower and upper bonds on Asian calls
CurranThompsonAsianOption	using Thompson's continuous limit
RogerShiThompsonAsianOption	using Thompson's single integral formula
ThompsonAsianOption	using Thompson's upper bound
TolmatzAsianOption	using lower bound from Tolmatz' asymptotics
CallPutParityAsianOption	using Call-Put parity relation
WithDividendsAsianOption	adding dividends to Asian option formula
FuMadanWangTable	returns table from Fu, Madan and Wang's paper
FusaiTaglianiTable	returns table from Fusai und tagliani's paper
GemanTable	returns table from Geman's paper
LinetzkyTable	returns table from Linetzky's paper
ZhangTable	returns table from Zhang's paper
ZhangLongTable	returns long table from Zhang's paper
ZhangShortTable	returns short table from Zhang's paper

2 EBM Distribution

In this section we summarize distributions and related functions which are useful in the theory of exponential Brownian motion and Asian option valuation. The functions compute densities and probabilities for the log-Normal distribution, the Gamma distribution, the Reciprocal-Gamma distribution, and the Johnson Type-I distribution. Functions are made available for the computation of moments including the Normal, the log-Normal, the Reciprocal-Gamma, and the Asian-Option Density. In addition a function is given to compute numerically first and second derivatives of a given function.

dlognorm	returns the log-Normal density and derivatives
plognorm	returns the log-Normal, a synonyme for R's plnorm
dgam	returns the Gamma density, a synonyme for R's dgamma
pgam	returns the Gamma probability, a synonyme for R's pgamma

drgam	returns the Reciprocal-Gamma density
prgam	returns the Reciprocal-Gamma probability
djohnson	returns the Johnson Type I density
pjohnson	returns the Johnson Type I probability
mnorm	returns the moments of Normal density
mlognorm	returns the moments of log-Normal density
mrgam	returns the moments of reciprocal-Gamma density
masian	returns the moments of Asian Option density
derivative	returns the first and second numerical derivative

3 Bessel Functions

This section offers special mathematical functions which compute the modified Bessel functions of integer order of the first and second kind as well as their derivatives.

BesselI	computes modified Bessel function of the 1st kind
BesselDI	computes its derivative
BesselK	computes the modified Bessel function of the 3rd kind
BesselDK	computes its derivative

4 Gamma Function

This section offers special mathematical functions which compute the Gamma function and related functions. The functions include the error function, the Psi function, the incomplete Gamma function, the Gamma function for complex argument, and the Pochhammer symbol. The Gamma function the logarithm of the Gamma function, their first four derivatives, and the Beta function and the logarithm of the Beta function are part of R's base package (marked by an asterisk). For example, these functions are required to value Asian Options based on the theory of exponential Brownian motion.

erf	computes the Error function
gamma*	computes the Gamma function
lgamma*	the logarithm of the Gamma function
digamma*	the first derivative of the Log Gamma function
trigamma*	the second derivative of the Log Gamma function
tetragamma*	the third derivative of the Log Gamma function
pentagamma*	the fourth derivative of the Log Gammafunction
beta*	the Beta function
lbeta*	the logarithm of the Beta function
Psi	computes the Psi or Digamma function
igamma	computes the incomplete Gamma function
cgamma	computes the Gamma function for complex argument
Pochhammer	returns the Pochhammer symbol

5 Hypergeometric Function

This section offers special mathematical functions which compute the confluent hypergeometric and related functions. These functions are required to value Asian Options based on the theory of exponential Brownian motion.

kummerM	the Confluent Hypergeometric Function of the 1st kind
kummerU	the Confluent Hypergeometric Function of the 2nd kind
whittakerM	the Whittaker M Function
whittakerW	the Whittaker W Function
hermiteH	the Hermite Polynomials

About Rmetrics

The fOptions Rmetrics package is written for educational support in teaching "Computational Finance and Financial Engineering" and licensed under the GPL.

BesselFunctions *Modified Bessel Functions*

Description

A collection and description of special mathematical functions which compute the modified Bessel functions of integer order of the first and second kind as well as their derivatives.

The functions are:

BesselI	modified Bessel function of the 1st Kind,
BesselDI	its derivative,
BesselK	the modified Bessel function of the 3rd Kind,
BesselDK	its derivative.

Usage

```
BesselI(x, nu, expon.scaled = FALSE)
BesselK(x, nu, expon.scaled = FALSE)
BesselDI(x, nu)
BesselDK(x, nu)
```

Arguments

expon.scaled	a logical; if TRUE, the results are exponentially scaled.
nu	an integer value greater or equal to zero, the integer order of the modified Bessel function.
x	a positive numeric value or a vector of positive numerical values.

Value

The functions return the values of the selected special mathematical function.

Author(s)

Diethelm Wuertz for the Rmetrics R-port.

References

Abramowitz M., Stegun I.A. (1972); *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th printing, New York, Dover Publishing.

Weisstein E.W. (2004); *MathWorld – A Wolfram Web Resource*, <http://mathworld.wolfram.com>

Examples

```
## Bessel I0 and K0 -
# Abramowitz-Stegun: Table 9.8, p. 416-422
x = c(0.0, 0.01, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50)
data.frame(x, I = exp(-x)*BesselI(x, 0), K = exp(x)*BesselK(x, 0))
# Compare with R's internal function:
# data.frame(x, ratio = BesselI(x, 0) / besselI(x, 0))
# data.frame(x, ratio = BesselK(x, 0) / besselK(x, 0))

## x = 0:
c(BesselI(0, 0), BesselI(0, 1), BesselI(0, 2), BesselI(0, 5))
# Compare with R's internal function:
# c(besselI(0, 0), besselI(0, 1), besselI(0, 2), besselI(0, 5))
c(BesselK(0, 0), BesselK(0, 1), BesselK(0, 2), BesselK(0, 5))
# Compare with R's internal function:
# c(besselK(0, 0), besselK(0, 1), besselK(0, 2), besselK(0, 5))

## Bessel I2 and K2 -
# Abramowitz-Stegun: Table 9.8, p. 416-422
x = c(0.0, 0.01, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50)
data.frame(x, I = BesselI(x, 2)/x^2, K = BesselK(x, 2)*x^2)
# Compare with R's internal function:
# data.frame(x, ratio = BesselI(x, 0) / besselI(x, 0))
# data.frame(x, ratio = BesselK(x, 0) / besselK(x, 0))
# data.frame(x, ratio = BesselI(x, 1) / besselI(x, 1))
# data.frame(x, ratio = BesselK(x, 1) / besselK(x, 1))
# data.frame(x, ratio = BesselI(x, 5) / besselI(x, 5))
# data.frame(x, ratio = BesselK(x, 5) / besselK(x, 5))
# data.frame(x, ratio = BesselI(x,50) / besselI(x,50))
# data.frame(x, ratio = BesselK(x,50) / besselK(x,50))
```

 EBMASianOptions *Exponential Brownian Motion Distributions*

Description

A collection and description of functions used in the theory of exponential Brownian Motion and in the valuation of Asian options.

The functions for Moment matching and Series Expansions are:

MomentMatchedAsianOption	Valuate moment matched option prices,
... method="LN"	Log-Normal Approximation of Levy, Turnbull and Wakeman,
... method="RG"	Reciprocal-Gamma Approximation of Milevski and Posner,
... method="JI"	Johnson Type I Approximation of Posner and Milevsky,
MomentMatchedAsianDensity	Valuate moment matched option densities,
... method="LN"	Log-Normal Approximation,
... method="RG"	Reciprocal-Gamma Approximation,
... method="JI"	Johnson Type I Approximation,
GramCharlierAsianOption	Calculate Gram-Charlier option prices.
AsianOptionMoments	Methods to calculate Asian Moments,
... method="A"	Moments from Abrahamson's Formula,
... method="D"	Moments from Dufresne's Formula,
... method="TW"	First 2 Moments from Turnbull-Wakeman,
... method="T"	Asymptotic Behavior after Tolmatz.
ZhangAsianOption	Asian option price by Zhang's 1D PDE,
VecerAsianOption	Asian option price by Vecer's 1D PDE.
gGemanYor	Function to be Laplace inverted,
GemanYorAsianOption	Asian option price by Laplace Inversion,
gLinetzky	Function to be integrated,
LinetzkyAsianOption	Asian option price by Spectral Expansion.
BoundsOnAsianOption	Lower and upper bounds on Asian calls,
CurrantThompsonAsianOption	From Thompson's continuous limit,
RogerShiThompsonAsianOption	From Thompson's single integral formula,
ThompsonAsianOption	Thompson's upper bound,
TolmatzAsianOption	Lower Bound from Tolmatz' symptotics.
CallPutParityAsianOption	Call-Put parity Relation,
WithDividendsAsianOption	Adds dividends to Asian option formula.

FuMadanWangTable	Table from Fu, Madan and Wang's paper,
FusaiTaglianiTable	Table from Fusai und tagliani's paper,
GemanTable	Table from Geman's paper,
LinetzkyTable	Table from Linetzky's paper,
ZhangTable	Table from Zhang's paper,
ZhangLongTable	Long Table from Zhang's paper,
ZhangShortTable	Short Table from Zhang's paper.

Usage

```

MomentMatchedAsianOption(TypeFlag = c("c", "p"), S = 100, X = 100, Time = 1,
  r = 0.09, sigma = 0.30, table = NA, method = c("LN", "RG", "JI"))
MomentMatchedAsianDensity(x, Time = 1, r = 0.09, sigma = 0.30,
  method = c("LN", "RG", "JI"))
GramCharlierAsianOption(TypeFlag = c("c", "p"), S = 100, X = 100, Time = 1,
  r = 0.09, sigma = 0.30, table = NA, method = c("LN", "RG", "JI"))

AsianOptionMoments(M = 4, Time = 1, r = 0.045, sigma = 0.30, log = FALSE,
  method = c("A", "D", "TW", "T"))

ZhangAsianOption(TypeFlag = c("c", "p"), S = 100, X = 100, Time = 1,
  r = 0.09, sigma = 0.30, table = NA, correction = TRUE, nint = 800,
  eps = 1.0e-8, dt = 1.0e-10)
VecerAsianOption(TypeFlag = c("c", "p"), S = 100, X = 100, Time = 1,
  r = 0.09, sigma = 0.30, table = NA, nint = 800, eps = 1.0e-8,
  dt = 1.0e-10)

gGemanYor(lambda, S = 100, X = 100, Time = 1, r = 0.05, sigma = 0.30,
  log = FALSE, doplot = FALSE)
GemanYorAsianOption(TypeFlag = c("c", "p"), S = 100, X = 100, Time = 1,
  r = 0.09, sigma = 0.30, doprint = FALSE)
gLinetzky(x, y, tau, nu, ip = 0)
LinetzkyAsianOption(TypeFlag = c("c", "p"), S = 2, X = 2, Time = 1,
  r = 0.02, sigma = 0.1, table = NA, lower = 0, upper = 100,
  method = "adaptive", subdivisions = 100, ip = 0, doprint = TRUE,
  doplot = TRUE, ...)

BoundsOnAsianOption(TypeFlag = c("c", "p"), S = 100, X = 100, Time = 1,
  r = 0.09, sigma = 0.30, table = NA, method = c("CT", "RST", "T"))
CurranThompsonAsianOption(TypeFlag = c("c", "p"), S = 100, X = 100,
  Time = 1, r = 0.09, sigma = 0.30)
RogerShiThompsonAsianOption(TypeFlag = c("c", "p"), S = 100, X = 100,

```



```

    Time = 1, r = 0.09, sigma = 0.30)
ThompsonAsianOption(TypeFlag = c("c", "p"), S = 100, X = 100, Time = 1,
  r = 0.09, sigma = 0.30)
TolmatzAsianOption(TypeFlag = c("c", "p"), S = 100, X = 100, Time = 1,
  r = 0.09, sigma = 0.30)

CallPutParityAsianOption(TypeFlag = "p", Price = 8.828759, S = 100,
  X = 100, Time = 1, r = 0.09, sigma = 0.3, table = NA)
WithDividendsAsianOption(TypeFlag = "c", Dividends = 0.45, S = 100,
  X = 100, Time = 1, r = 0.09, sigma = 0.3,
  calculator = MomentMatchedAsianOption, method = "LN")

FuMadanWangTable()
FusaiTaglianiTable()
GemanTable()
LinetzkyTable()
ZhangTable()
ZhangLongTable()
ZhangShortTable()

```

Arguments

calculator	[WithDividendsAsianOption] - the name of the function selecting the option calculator to be used.
correction	[ZhangAsianOption] - xxx.
Dividends	[WithDividendsAsianOption] - xxx.
doplot	[gGemanYor][LinetzkyAsianOption] - xxx.
doprint	[GemanYorAsianOption][LinetzkyAsianOption] - xxx.
dt	[VecerAsianOption][ZhangAsianOption] - xxx.
eps	[VecerAsianOption][ZhangAsianOption] - xxx.
ip	[gLinetzky] - xxx.
lambda	[gGemanYor] - xxx.
log	[AsianOptionMoments][gGemanYor] - xxx.
lower, upper	[LinetzkyAsianOption] - xxx.
M	[*] - xxx.

method	[*] - xxx.
nint	[*] - xxx.
nu	[*] - xxx.
Price	[*] - xxx.
r	a numeric value, the annualized rate of interest; e.g. 0.25 means 25% pa.
S	the asset price, a numeric value.
sigma	a numeric value, the annualized volatility of the underlying security; e.g. 0.3 means 30% volatility pa.
subdivisions	[*] - xxx.
table	[*] - xxx.
tau	[*] - xxx.
Time	a numeric value, the time to maturity measured in years; e.g. 0.5 means 6 months.
TypeFlag	a character string either "c" for a call option or a "p" for a put option.
x	[*] - xxx.
X	a numeric value, the exercise price.
y	[*] - xxx.
...	[*] - xxx.

Author(s)

Diethelm Wuertz for the Rmetrics R-port.

Examples

```
## Examples:
# none ...
```

Description

A collection and description of distributions and related functions which are useful in the theory of exponential Brownian motion and Asian option valuation. The functions compute densities and probabilities for the log-Normal distribution, the Gamma distribution, the Reciprocal-Gamma distribution, and the Johnson Type-I distribution. Functions are made available for the computation of moments including the Normal, the log-Normal, the Reciprocal-Gamma, and the Asian-Option Density. In addition a function is given to compute numerically first and second derivatives of a given function.

The functions are:

dlognorm	the log-Normal density and derivatives,
plognorm	the log-Normal, a synonyme for R's plnorm,
dgam	the Gamma density, a synonyme for R's dgamma,
pgam	the Gamma probability, a synonyme for R's pgamma,
drgam	the Reciprocal-Gamma density,
prgam	the Reciprocal-Gamma probability,
djohnson	the Johnson Type I density,
pjohnson	the Johnson Type I probability,
mnorm	the Moments of Normal density,
mlognorm	the Moments of log-Normal density,
mrgam	the Moments of reciprocal-Gamma density,
masian	the Moments of Asian Option density,
derivative	the First and second numerical derivative.

Usage

```
dlognorm(x, meanlog = 0, sdlog = 1, deriv = c(0, 1, 2))
plognorm(q, meanlog = 0, sdlog = 1)
dgam(x, alpha, beta)
pgam(q, alpha, beta, lower.tail = TRUE)
drgam(x, alpha, beta, deriv = c(0, 1, 2))
prgam(q, alpha, beta, lower.tail = TRUE)
djohnson(x, a = 0, b = 1, c = 0, d = 1, deriv = c(0, 1, 2))
pjohnson(q, a = 0, b = 1, c = 0, d = 1)

mnorm(mean = 0, sd = 1)
mlognorm(meanlog = 0, sdlog = 1)
mrgam(alpha = 1/2, beta = 1)
mjohnson(a, b, c, d)
masian(Time = 1, r = 0.045, sigma = 0.30)
```

```
derivative(x, y, deriv = c(1, 2))
```

```
dEBM(u, t = 1)
```

```
pEBM(u, t = 1)
```

```
d2EBM(u, t = 1)
```

```
dasymEBM(u, t = 1)
```

Arguments

a, b, c, d	[*johnson] - the parameters of the Johnson Type I distribution. The default values are a=1, b=1, c=0, and d=1.
alpha, beta	[*gam] - the parameters of the Gamma distribution.
deriv	an integer value, the degree of differentiation, either 0, 1 or 2.
lower.tail	a logical, if TRUE, the default, then the probabilities are $P[X \leq x]$, otherwise, $P[X > x]$.
mean, sd	[*lognorm] - the parameters of the Normal distribution, the mean and the standard deviation respectively. The default values are mean=0 and sd=1.
meanlog, sdlog	[*lognorm] - the parameters of the Log Normal distribution, the mean and the standard deviation respectively. The default values are mean=0 and sd=1.
q	a real numeric value or vector.
t	...
Time, r, sigma	the parameters of the Asian Option distribution.
u	...
x	a real numeric value or vector.
y	[derivative] - a real numeric value or vector, the function values from which to compute the first and second derivative.

Value

The functions d* and p* return the values or numeric vectors of the density and probability of the the corresponding distribution.

The functions m* return a list with three elements, the values of the first four moments rawMoments, the values of the first four central moments centralMoments, and the skewness and kurtosis fisher, also called Fisher parameters.

The function derivative returns a list of two elemtes, \$x and \$y, where \$y(\$x) is either the first or second derivative of $y(x)$ as selected by the argument deriv.

Author(s)

Diethelm Wuertz for the Rmetrics R-port.

Examples

```
## dlognorm -
# Calculate Log-Normal Density and its Derivatives:
x = exp(seq(-2.8, 1.2, length = 100))
y0 = dlognorm(x, deriv = 0)
y1 = dlognorm(x, deriv = 1)
y2 = dlognorm(x, deriv = 2)

## derivative -
# Compare with Numerical Differentiation:
par(mfrow = c(2, 2))
xa = exp(seq(-2.5, 1.5, length = 20))
plot(x, y0, type = "l", main = "Log-Normal Density")
plot(x, y1, type = "l", main = "1st Derivative")
z = derivative(xa, dlognorm(xa, deriv = 0), deriv = 1)
points(z$x, z$y, col = "steelblue")
plot(x, y2, type = "l", main = "2nd Derivative")
z = derivative(xa, dlognorm(xa, deriv = 0), deriv = 2)
points(z$x, z$y, col = "steelblue")
```

GammaFunctions

*Gamma and Related Functions***Description**

A collection and description of special mathematical functions. The functions include the error function, the Psi function, the incomplete Gamma function, the Gamma function for complex argument, and the Pochhammer symbol. The Gamma function the logarithm of the Gamma function, their first four derivatives, and the Beta function and the logarithm of the Beta function are part of R's base package. For example, these functions are required to valuate Asian Options based on the theory of exponential Brownian motion.

The functions are:

erf	the Error function,
gamma*	the Gamma function,
lgamma*	the logarithm of the Gamma function,
digamma*	the first derivative of the Log Gamma function,
trigamma*	the second derivative of the Log Gamma function,
tetragamma*	the third derivative of the Log Gamma function,
pentagamma*	the fourth derivative of the Log Gammafunction,
beta*	the Beta function,
lbeta*	the logarithm of the Beta function,
Psi	Psi(x) the Psi or Digamma function,
igamma	P(a,x) the incomplete Gamma function,
cgamma	Gamma function for complex argument,
Pochhammer	the Pochhammer symbol.

The functions marked by an asterisk are part of R's base package.

Usage

```
erf(x)
Psi(x)
igamma(x, a)
cgamma(x, log = FALSE)
Pochhammer(x, n)
```

Arguments

x	[erf] - a real numeric value or vector. [Psi][*gamma][Pochhammer] - a complex numeric value or vector.
a	a complex numeric value or vector.
n	an integer value $n \geq 0$. A notation used in the theory of special functions for the rising factorial, also known as the rising factorial power, Graham et al. 1994.
log	a logical, if TRUE the logarithm of the complex Gamma function is calculated otherwise if FALSE, the complex Gamma function itself will be calculated.

Value

The functions return the values of the selected special mathematical function.

Author(s)

Diethelm Wuertz for the Rmetrics R-port.

References

Abramowitz M., Stegun I.A. (1972); *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th printing, New York, Dover Publishing.

Artin, E. (1964); *The Gamma Function*, New York, Holt, Rinehart, and Winston Publishing.

Weisstein E.W. (2004); *MathWorld—A Wolfram Web Resource*, <http://mathworld.wolfram.com>

Examples

```
## Calculate Error, Gamma and Related Functions

## gamma -
# Abramowitz-Stegun: Figure 6.1
x = seq(-4.01, 4.01, by = 0.011)
plot(x, gamma(x), ylim = c(-5,5), type = "l", main = "Gamma Function")
lines(x = c(-4, 4), y = c(0, 0))

## Psi -
# Abramowitz-Stegun: Figure 6.1
```

```

x = seq(-4.01, 4.01, by = 0.011)
plot(x, Psi(x), ylim = c(-5, 5), type = "l", main = "Psi Function")
lines(x = c(-4, 4), y = c(0, 0))
# Note: Is digamma defined for positive values only ?

## igamma -
# Abramowitz-Stegun: Figure 6.3.
gammaStar = function(x, a) { igamma(x,a)/x^a }
# ... create Figure as an exercise.

## igamma -
# Abramowitz-Stegun: Formula 6.5.12
# Relation to Confluent Hypergeometric Functions
a = sqrt(2)
x = pi
Re ( (x^a/a) * kummerM(-x, a, 1+a) )
Re ( (x^a*exp(-x)/a) * kummerM(x, 1, 1+a) )
pgamma(x,a) * gamma(a)
igamma(x, a)

## cgamma -
# Abramowitz-Stegun: Tables 6.7
x = 1
y = seq(0, 5, by = 0.1); x = rep(x, length = length(y))
z = complex(real = x, imag = y)
c = cgamma(z, log = TRUE)
cbind(y, Re(c), Im(c))

## cgamma -
# Abramowitz-Stegun: Examples 4-8:
options(digits = 10)
gamma(6.38); lgamma(56.38) # 1/2
Psi(6.38); Psi(56.38) # 3/4
cgamma(complex(real = 1, imag = -1), log = TRUE ) # 5
cgamma(complex(real = 1/2, imag = 1/2), log = TRUE ) # 6
cgamma(complex(real = 3, imag = 7), log = TRUE ) # 7/8

```

HypergeometricFunctions

Confluent Hypergeometric Functions

Description

A collection and description of special mathematical functions which compute the confluent hypergeometric and related functions. For example, these functions are required to value Asian Options based on the theory of exponential Brownian motion.

The functions are:

kummerM the Confluent Hypergeometric Function of the 1st Kind,

kummerU	the Confluent Hypergeometric Function of the 2nd Kind,
whittakerM	the Whittaker M Function,
whittakerW	the Whittaker W Function,
hermiteH	the Hermite Polynomials.

Usage

```
kummerM(x, a, b, lncf = 0, ip = 0)
kummerU(x, a, b, ip = 0)
whittakerM(x, kappa, mu, ip = 0)
whittakerW(x, kappa, mu, ip = 0)
hermiteH(x, n, ip = 0)
```

Arguments

x	[kummer*] - a complex numeric value or vector.
a, b	[kummer*] - complex numeric indexes of the Kummer functions.
ip	an integer value that specifies how many array positions are desired, usually 10 is sufficient. Setting ip=0 causes the function to estimate the number of array positions.
kappa, mu	complex numeric indexes of the Whittaker functions.
lncf	an integer value which selects how the result should be represented. A 0 will return the value in standard exponential form, a 1 will return the LOG of the result.
n	[hermiteH] - the index of the Hermite polynomial, a positive integer value.

Details

The functions use the TOMS707 Algorithm by M. Nardin, W.F. Perger and A. Bhalla (1989). A numerical evaluator for the confluent hypergeometric function for complex arguments with large magnitudes using a direct summation of the Kummer series. The method used allows an accuracy of up to thirteen decimal places through the use of large real arrays and a single final division.

The confluent hypergeometric function is the solution to the differential equation:

$$zf''(z) + (a-z)f'(z) - bf(z) = 0$$

The Whittaker functions and the Hermite Polynomials are derived from Kummer's functions.

Value

The functions return the values of the selected special mathematical function.

Author(s)

Diethelm Wuertz for the Rmetrics R-port.

References

Abramowitz M., Stegun I.A. (1972); *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th printing, New York, Dover Publishing.

Weisstein E.W. (2004); *MathWorld – A Wolfram Web Resource*, <http://mathworld.wolfram.com>

Examples

```
## kummerM -
# Abramowitz-Stegun: Formula 13.6.3/13.6.21
x = c(0.001, 0.01, 0.1, 1, 10, 100, 1000)
nu = 1; a = nu+1/2; b = 2*nu+1
M = Re ( kummerM(x = 2*x, a = a, b = b) )
Bessel = gamma(1+nu) * exp(x)*(x/2)^(-nu) * BesselI(x, nu)
cbind(x, M, Bessel)

## kummerM -
# Abramowitz-Stegun: Formula 13.6.14
x = c(0.001, 0.01, 0.1, 1, 10, 100, 1000)
M = Re ( kummerM(2*x, a = 1, b = 2) )
Sinh = exp(x)*sinh(x)/(x)
cbind(x, M, Sinh)
# Now the same for complex x:
y = rep(1, length = length(x))
x = complex(real = x, imag = y)
M = kummerM(2*x, a = 1, b = 2)
Sinh = exp(x)*sinh(x)/(x)
cbind(x, M, Sinh)

## kummerU -
# Abramowitz-Stegun: Formula 13.1.3
x = c(0.001, 0.01, 0.1, 1, 10, 100, 1000)
a = 1/3; b = 2/3
U = Re ( kummerU(x, a = a, b = b) )
cbind(x, U)

## whittakerM -
# Abramowitz-Stegun: Example 13
AS = c(1.10622, 0.57469)
W = c(
  whittakerM(x = 1, kappa = 0, mu = -0.4),
  whittakerW(x = 1, kappa = 0, mu = -0.4) )
data.frame(AS, W)

## kummerM
# Abramowitz-Stegun: Example 17
x = seq(0, 16, length = 200)
plot(x = x, y = kummerM(x, -4.5, 1), type = "l", ylim = c(-25,125),
     main = "Figure 13.2: M(-4.5, 1, x)")
lines(x = c(0, 16), y = c(0, 0), col = 2)
```

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