GPoM : General introduction

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Generalized Global Polynomial Modelling (GPoM)

GPoM is an R package dedicated to the global modelling technique. It has been developed at the *Centre d'Etudes Spatiales de la Biosphère*[1, 2]. The global modelling technique is a model-building approach: its aim is to obtain difference, delayed of differential equations from time series. Model-building from data takes its background from the fields of Electrical Engineering and Statistics and was originally mostly dedicated to linear problems [3]. The more advanced developments of the model-building approaches also incorporate the Theory of Nonlinear Dynamical Systems in its background. Thanks to it, global modelling technique is well adapted to model deterministic behaviours of various degree of nonlinarity (linear, weakly or strongly nonlinear, chaotic), and it is also well designed to model dynamical behaviors characterized by a high sentivity to the initial conditions.

In its Ordinary Differential Equations (ODEs) formulation, the global modelling technique was initiated in the early 1990s[4]. Its first illustrations were obtained thanks to a formalism developped by G. Gouesbet and his colleagues[5, 6]. The ability to obtain equations for one chosen system may be very different depending on what variable is used to reconstruct the equations. This property plays a very important role when trying to retrieve governing equations for any dynamical system. This question was investigated during the last decades[7, 8].

It is only in the 2000s that a set of ODEs could be directly obtained from real world data set[9]. New algorithms were developed at the begining of the 2010s[10] that have proven to have very high levels of performance to model dynamical behaviors observed under real environmental conditions: cereal crops cycles, snow area cycles, eco-epidemiology, etc.[11, 12], etc.

All these developments were initialy developped to model dynamical behaviors from single time series. Recent developments have shown that the global modelling technique can also be applied to model multivariate

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[3]L. A. Aguirre & C. Letellier, Modeling nonlinear dynamics and chaos: A review, *Mathematical Problems in Engineering*, 2009, 238960.

[4]J. P. Crutchfield & B. S. McNamara, 1987. Equations of motion from a data series, *Complex Systems* **1**, 417-452.

[5]G. Gouesbet & J. Maquet, 1992. Construction of phenomenological models from numerical scalar time series, *Physica D*, **58**, 202-215.

[6]G. Gouesbet & C. Letellier, 1994. Global vector-field reconstruction by using a multivariate polynomial L2 approximation on nets, *Physical Review E*, **49**, 4955-4972.

[7]C. Letellier, L. A. Aguirre, & J. Maquet, 2005. Relation between observability and differential embeddings for nonlinear dynamics, *Physical Review E*, **71**, 066213.

[8]L.A. Aguirre & C. Letellier, 2011. Investigating observability properties from data in non-linear dynamics. *Physical Review E*, **83**, 066209.

[9]J. Maquet, C. Letellier & L. A. Aguirre 2007. Global models from the Canadian Lynx cycles as a first evidence for chaos in real ecosystems, *Journal of Mathematical Biology*, **55**(1), 21-39.

[10]S. Mangiarotti, R. Coudret, L. Drapeau & L. Jarlan, 2012. Polynomial search and global modeling: Two algorithms for modeling chaos," *Physical Review E*, **86**(4), 046205.

[11]S. Mangiarotti, L. Drapeau & C. Letellier, 2014. Two chaotic global models for cereal crops cycles observed from satellite in Northern Morocco, *Chaos*, **24**, 023130.

[12]S. Mangiarotti, Modélisation globale et caractérisation topologique de dynamiques environnementales: de l'analyse des enveloppes fluides et du couvert de surface de la Terre à la caractérisation topolodynamique du chaos, Habilitation to Direct Researches, Université de Toulouse 3, 2014.

couplings[13, 14].

The present package provides global modelling tools for the modelling of linear and nonlinear behaviors directly from time series.

Seven illustrative `vignettes` are provided to introduce the package which can be used as a tutorial and as a demonstrator. These are as follows:

(1) `I_Generate` introduces the conventions used to formulate sets of ODEs of polynomial form with `GPoM` and shows how to integrate them numerically,

(2) `II_PreProcessing` provides some simple examples of time series preprocessing before applying the global modelling technique,

(3) `III_Modelling` is dedicated to the global modelling itself. Several case studies are presented considering single and multiple time series, both for modelling or detecting causal couplings,

(4) `IV_VisuOutput` shows how to get an overview of the output obtained with global modelling functions and explains how these are organised,

(5) `V_Predictability` aims to provide basic examples of validation considering the models performances in term of predictability,

(6) `VI_Robustness` aims to illustrate the robustness of the global modelling technique under degraded conditions of noisy time series, subsampling/resampling, time series length, sensitivity to initial conditions,

(7) `VII_Retromodelling` aims to test the ability of the global modelling technique to retrieve the original equations considering various dynamical systems.

The present GPoM package is made available to whom would like to use it. It includes most of the latest developments presently available for global modelling in ODE form, and we are happy to share it with you. Please refer to the following publications when using the present tools.

For univariate time series modelling: [1] S. Mangiarotti, R. Coudret, L. Drapeau & L. Jarlan, 2012. Polynomial search and global modeling: Two algorithms for modeling chaos," *Physical Review E*, **86**(4), 046205.

For infering causal couplings and for detecting or analysing multivariate couplings: [2] S. Mangiarotti, 2015. Low dimensional chaotic models for the plague epidemic in Bombay, *Chaos, Solitons & Fractals*, **81**(A), 184-196.

For using the generalized formulation of global modelling (that combines multiariate time series and some of their derivatives): [3] S. Mangiarotti, M. Peyre & M. Huc, 2016. A chaotic model for the epidemic of Ebola virus disease in West Africa (2013–2016). *Chaos*, **26**, 113112.

For the time series resampling (before applying the global modelling technique): [4] S. Mangiarotti, 2018. The global modelling classification technique applied to the detection of chaotic attractors. *Supplementary Material A* to "Can the global modelling technique be used for crop classification?" by S. Mangiarotti, A.K. Sharma, S. Corgne, L. Hubert-Moy, L. Ruiz, M. Sekhar, Y. Kerr, 2018. *Chaos, Solitons & Fractals*, **106**, 363-378.

For modelling the dynamics of aggregated (or associated) time series: [5] S. Mangiarotti, F. Le Jean, M. Huc, C. Letellier, 2016. Global modeling of aggregated and associated chaotic dynamics. *Chaos Solitons Fractals*, **83**, 82–96.

Note that various validation methods have been introduced in [3]. A validation of high precision is also presented in the supplementary matials of [1].

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[13]S. Mangiarotti, 2015. Low dimensional chaotic models for the plague epidemic in Bombay, *Chaos, Solitons & Fractals*, **81**(A), 184-196.

[14]S. Mangiarotti, M. Peyre & M. Huc, 2016. A chaotic model for the epidemic of Ebola virus disease in West Africa (2013–2016). *Chaos*, **26**, 113112.

The authors of the package decline any responsability about the results and interpretations obtained and made by other users.