

# Package ‘InvariantCausalPrediction’

October 3, 2017

**Type** Package

**Title** Invariant Causal Prediction

**Version** 0.7-1

**Date** 2017-10-02

**Author** Nicolai Meinshausen [aut, cre], Dominik Rothenhaeusler [aut]

**Depends** glmnet, mboost, linprog, parallel, RColorBrewer

**Maintainer** Nicolai Meinshausen <meinshausen@stat.math.ethz.ch>

**Description** Confidence intervals for causal effects, using data collected in different experimental or environmental conditions. Hidden variables can be included in the model with a more experimental version.

**License** GPL

**NeedsCompilation** no

**Repository** CRAN

**Date/Publication** 2017-10-03 20:50:45 UTC

## R topics documented:

InvariantCausalPrediction-package . . . . .	2
causalDantzig . . . . .	2
hiddenICP . . . . .	4
ICP . . . . .	7
plot.causalDantzig . . . . .	11
plot.InvariantCausalPrediction . . . . .	12
summary.causalDantzig . . . . .	13
summary.InvariantCausalPrediction . . . . .	14

<b>Index</b>	<b>15</b>
--------------	-----------

---

InvariantCausalPrediction-package  
*Invariant causal prediction*

---

## Description

Confidence intervals for causal prediction in a regression setting. An experimental version is also available for classification.

## Details

Package: InvariantCausalPrediction  
Type: Package  
Version: 0.7-1  
Date: 2017-10-02  
License: GPL

Confidence intervals can be computed with function ICP, both for regression and binary classification. For causal inference with hidden variables, use causalDantzig. print, plot and summary methods are available.

## Author(s)

Nicolai Meinshausen <meinshausen@stat.math.ethz.ch>, Dominik Rothenhaeusler <rothenhaeusler@stat.math.ethz.ch>  
Maintainer: Nicolai Meinshausen <meinshausen@stat.math.ethz.ch>

---

causalDantzig      *Causal Dantzig*

---

## Description

Causal inference in linear structural equation models with hidden variables under additive interventions. This function implements both regularized and unregularized causal Dantzig. For unregularized causal Dantzig, the function returns point estimates and asymptotic confidence intervals. For regularized causal Dantzig, point estimates and a ranking via stability selection are given.

## Usage

```
causalDantzig(X, Y, ExpInd, regularization = FALSE, alpha = 0.05,  
             lambda = NULL, scale = TRUE, mc.cores = 1)
```

**Arguments**

X	A nxp matrix with the predictor variables for all experimental settings, where n is the total number of samples and p is the number of predictors.
Y	The numeric response or target variable of interest of length n.
ExpInd	Indicator of the experiment or the intervention type an observation belongs to. A discrete vector of the same length as Y with K unique entries if there are K different experiments (for example entry 1 for all observational data and entry 2 for intervention data).
regularization	Use regularization=FALSE for unregularized causal Dantzig and regularization=TRUE for regularized causal Dantzig.
alpha	The significance level of the procedure. This parameter is ignored for regularized causal Dantzig. It defaults to alpha=0.05 for two-sided 95% confidence intervals.
lambda	User supplied regularization parameter for regularized causal Dantzig. If lambda=NULL, then the regularization parameter is chosen by cross-validation. The option lambda=NULL is experimental.
scale	The default scale=TRUE scales the variables before running regularized causal Dantzig. This parameter is ignored for unregularized causal Dantzig.
mc.cores	Number of CPU cores used for cross-validation. Defaults to mc.cores=1. This parameter is ignored for unregularized causal Dantzig.

**References**

Dominik Rothenhausler, Peter Buhlmann, Nicolai Meinshausen (2017):

Causal Dantzig: fast inference in linear structural equation models with hidden variables under additive interventions

arxiv preprint <https://arxiv.org/abs/1706.06159>

**See Also**

summary and plot methods for "causalDantzig" objects. Furthermore, see ICP in the package InvariantCausalPrediction for causal inference in a setting without hidden variables.

**Examples**

```
set.seed(1)
H <- rnorm(2000)
X2 <- H+c(rnorm(1000),rnorm(1000,sd=2))
Y <- X2 + H + rnorm(2000)
X1 <- Y + X2 + c(rnorm(1000),rnorm(1000,sd=2))
X3 <- X1 + H + c(rnorm(1000),rnorm(1000,sd=2))
ExpInd <- c(rep(0,1000),rep(1,1000))

X <- cbind(X1,X2,X3)

fit <- causalDantzig(X,Y,ExpInd,regularization=FALSE)
summary(fit)
```

```

fit <- causalDantzig(X,Y,ExpInd,regularization=TRUE)
# for parallel processing:
# mc.cores <- 2
# fit <- causalDantzig(X,Y,ExpInd,regularization=TRUE,mc.cores=mc.cores)
summary(fit)
plot(fit)

```

---

hiddenICP

*Invariant Causal Prediction with hidden variables*


---

### Description

Confidence intervals for causal effects in a regression setting with possible confounders.

### Usage

```
hiddenICP(X, Y, ExpInd, alpha = 0.1, mode = "asymptotic", intercept=FALSE)
```

### Arguments

X	A matrix (or data frame) with the predictor variables for all experimental settings
Y	The response or target variable of interest. Can be numeric for regression or a factor with two levels for binary classification.
ExpInd	Indicator of the experiment or the intervention type an observation belongs to. Can be a numeric vector of the same length as Y with K unique entries if there are K different experiments (for example entry 1 for all observational data and entry 2 for intervention data). Can also be a list, where each element of the list contains the indices of all observations that belong to the corresponding grouping in the data (for example two elements: first element is a vector that contains indices of observations that are observational data and second element is a vector that contains indices of all observations that are of interventional type).
alpha	The level of the test procedure. Use the default alpha=0.1 to obtain 90% confidence intervals.
mode	Currently only mode "asymptotic" is implemented; the argument is thus in the current version without effect.
intercept	Boolean variable; if TRUE, an intercept is added to the design matrix (but coefficients are returned without intercept term).

### Value

A list with elements

betahat            The point estimator for the causal effects

maximinCoefficients	The value in the confidence interval for each variable effects that is closest to 0. Is hence non-zero for variables with significant effects.
ConfInt	The matrix with confidence intervals for the causal coefficient of all variables. First row is the upper bound and second row the lower bound.
pvalues	The p-values of all variables.
colnames	The column-names of the predictor variables.
alpha	The chosen level.

**Author(s)**

Nicolai Meinshausen <meinshausen@stat.math.ethz.ch>

**References**

Dominik Rothenhausler, Peter Buhlmann, Nicolai Meinshausen (2017):

Causal Dantzig: fast inference in linear structural equation models with hidden variables under additive interventions

arxiv preprint <https://arxiv.org/abs/1706.06159>

**See Also**

[causalDantzig](#) for a more recent version of this function. [ICP](#) for reconstructing the parents of a variable under arbitrary interventions on all other variables (but no hidden variables). See package "backShift" for constructing point estimates of causal cyclic models in the presence of hidden variables (again under shift interventions) .

**Examples**

```
#####
##### 1st example:
##### Simulate data with interventions
  set.seed(1)
  ## sample size n
  n <- 2000
  ## 4 predictor variables
  p <- 4
  ## simulate as independent Gaussian variables
  X <- matrix(rnorm(n*p),nrow=n)
  ## divide data into observational (ExpInd=1) and interventional (ExpInd=2)
  ExpInd <- c(rep(1,n/2),rep(2,n/2))
  ## for interventional data (ExpInd==2): change distribution
  nI <- sum(ExpInd==2)
  X[ExpInd==2,] <- X[ExpInd==2,] + matrix( 5*rt( nI*p,df=3),ncol=p)
  ## add hidden variables
  W <- rnorm(n) * 5
  X <- X + outer(W, rep(1,4))

  ## first two variables are the causal predictors of Y
```

```

    beta <- c(1,1,0,0)
    ## response variable Y
    Y <- as.numeric(X%*%beta - 2*W + rnorm(n))

##### Compute "hidden Invariant Causal Prediction" Confidence Intervals
    icp <- hiddenICP(X,Y,ExpInd,alpha=0.01)
    print(icp)

##### Print point estimates and points in the confidence interval closest to 0
    print(icp$betahat)
    print(icp$maximinCoefficients)
    cat("true coefficients are:", beta)

#### compare with coefficients from a linear model
    cat("coefficients from linear model:")
    print(summary(lm(Y ~ X-1)))

#####
##### 2nd example:
##### Simulate model X -> Y -> Z with hidden variables, trying to
##### estimate causal effects from (X,Z) on Y
    set.seed(1)
    ## sample size n
    n <- 10000
    ## simulate as independent Gaussian variables
    W <- rnorm(n)
    noiseX <- rnorm(n)
    noiseY <- rnorm(n)
    noiseZ <- rnorm(n)
    ## divide data into observational (ExpInd=1) and interventional (ExpInd=2)
    ExpInd <- c(rep(1,n/2),rep(2,n/2))
    noiseX[ which(ExpInd==2)] <- noiseX[ which(ExpInd==2)] * 5
    noiseZ[ which(ExpInd==2)] <- noiseZ[ which(ExpInd==2)] * 3

    ## simulate equilibrium data
    beta <- -0.5
    alpha <- 0.9
    X <- noiseX + 3*W
    Y <- beta* X + noiseY + 3*W
    Z <- alpha*Y + noiseZ

##### Compute "Invariant Causal Prediction" Confidence Intervals
    icp <- hiddenICP(cbind(X,Z),Y,ExpInd,alpha=0.1)
    print(icp)

##### Print/plot/show summary of output (truth here is (beta,0))
    print(signif(icp$betahat,3))
    print(signif(icp$maximinCoefficients,3))

```

```

cat("true coefficients are:", beta,0)

#### compare with coefficients from a linear model
cat("coefficients from linear model:")
print(summary(lm(Y ~ X + Z -1)))

```

ICP

*Invariant Causal Prediction***Description**

Confidence intervals for causal effects in a regression setting.

**Usage**

```

ICP(X, Y, ExpInd, alpha=0.01, test = "normal",
    selection = c("lasso", "all", "stability", "boosting")
      [if (ncol(X) <=8) 2 else 4],
    maxNoVariables = 8, maxNoVariablesSimult = 8,
    maxNoObs = 200,
    showAcceptedSets = TRUE,
    showCompletion = TRUE,
    stopIfEmpty = FALSE,
    gof = max(0.01,alpha))

```

**Arguments**

X	A matrix (or data frame) with the predictor variables for all experimental settings
Y	The response or target variable of interest. Can be numeric for regression or a factor with two levels for binary classification.
ExpInd	Indicator of the experiment or the intervention type an observation belongs to. Can be a numeric vector of the same length as Y with K unique entries if there are K different experiments (for example entry 1 for all observational data and entry 2 for intervention data). Can also be a list, where each element of the list contains the indices of all observations that belong to the corresponding grouping in the data (for example two elements: first element is a vector that contains indices of observations that are observational data and second element is a vector that contains indices of all observations that are of interventional type).
alpha	Determines coverage of the confidence regions. alpha is the probability of non-coverage, so the default of 0.01 produces 99% confidence intervals. If no model has a p-value of at least alpha, the model assumptions seem violated and modelReject will return TRUE.

<code>test</code>	Use "exact" for an exact test in a regression setting, especially if sample size is small. However, this test is computationally demanding if sample size is high. The default "normal" tests for a shift in mean and variance of the residuals between different populations/environments. Using "correlation" tests additionally for vanishing correlation between predictor variables and residuals in each environment. Other options are "ranks" that uses a rank-based alternative and "ks" for a Kolmogorov-Smirnov test to detect differences in the distributions of the residuals between different environments. It is also possible to supply a function of the form <code>function(x, z)</code> that takes samples <code>x</code> and <code>z</code> of two populations as arguments and whose return value is the p-value for the null hypothesis that the two underlying distribution are identical.
<code>selection</code>	The method for pre-selection of variables to save computational resources. Can use "all" for no pre-selection (which guarantees coverage but might take longer to compute), "boosting" for a boosting-type, "lasso" for Lasso cross-validated or "stability" for a stability-selection-type pre-selection. Default is "all" if <code>p</code> does not exceed 10 and "boosting" otherwise.
<code>maxNoVariables</code>	The maximal number of variables to pre-select (choosing smaller values saves computational resources but increases approximation error).
<code>maxNoVariablesSimult</code>	The maximal size of sets of variables considered in the procedure (same comment as for <code>maxNoVariables</code> ).
<code>maxNoObs</code>	The maximal number of observations used for the "exact" test (same comment as for <code>maxNoVariables</code> ).
<code>showAcceptedSets</code>	If TRUE, print out information about accepted sets of variables.
<code>showCompletion</code>	If TRUE, print out information about progress of computation.
<code>stopIfEmpty</code>	If TRUE, the procedure will stop computing confidence intervals if the empty set has been accepted (and hence no variable can have a significant causal effect). Setting to TRUE will save computational time in these cases, but means that the confidence intervals lose their coverage properties for values different to 0.
<code>gof</code>	If no set of variables (including the empty set) leads to a p-value larger than the goodness-of-fit cutoff <code>gof</code> , the whole model will be rejected. If the model is correct, this will happen with a probability of <code>gof</code> and this option protects again making statements when the model is obviously not suitable for the data.

## Value

	A list with elements
<code>ConfInt</code>	The matrix with confidence intervals for the causal coefficient of all variables. First row is the upper bound and second row the lower bound.
<code>maximinCoefficients</code>	The value in the confidence interval closest to 0. Is hence non-zero for variables with significant effects.
<code>colnames</code>	The names of the variables (replaced with generic "Variable 1" etc. if not available).



factor	Logical indicating whether the response is a factor or not.
dimX	The dimensions of the matrix with predictor variables.
Coeff	A list which contains for all variables the vector with point-estimates among all accepted sets where the variable was part of the set.
CoeffVar	Same as Coeff but with the standard deviation of the point-estimate.
modelReject	Logical indicating if the whole model was rejected (the p-value of the best fitting model is too low).
acceptedSets	A list with one element per accepted set. For each accepted model the list entry is a vector that contains the indices of the variables in the accepted set.
usedvariables	The pre-selected variables if not all variables are used for the analysis; otherwise all variables.
pvalues	The p-values of all variables.
stopIfEmpty	A boolean value indicating whether computations stop as soon as intersection of accepted sets is empty.
noEnv	The number of distinct environments.
gof	The goodness-of-fit cutoff gof used.
bestModel	The largest p-value across all tested sets of variables.

### Author(s)

Nicolai Meinshausen <meinshausen@stat.math.ethz.ch>

### References

Jonas Peters, Peter Buhlmann, Nicolai Meinshausen (2015):  
 Causal inference using invariant prediction: identification and confidence intervals  
 arxiv preprint <http://arxiv.org/abs/1501.01332>

### See Also

[causalDantzig](#) for reconstructing the parents of a variable in the presence of hidden variables (but assuming shift/additive interventions), which also allows construction of confidence intervals for the causal coefficients. See package "backShift" for constructing point estimates of causal cyclic models in the presence of hidden variables (again under shift interventions).

### Examples

```
#####
##### 1st example:
##### Simulate data with interventions
set.seed(1)
  ## sample size n
  n <- 4000
  ## 5 predictor variables
  p <- 5
  ## simulate as independent Gaussian variables
```

```

    X <- matrix(rnorm(n*p),nrow=n)
  ## divide data into observational (ExpInd=1) and interventional (ExpInd=2)
  ExpInd <- c(rep(1,n/2),rep(2,n/2))
  ## for interventional data (ExpInd==2): change distribution
  X[ExpInd==2,] <- sweep(X[ExpInd==2,],2, 5*rnorm(p) ,FUN="*")
  ## first two variables are the causal predictors of Y
  beta <- c(1,1,rep(0,p-2))
  ## response variable Y
  Y <- as.numeric(X%*%beta + rnorm(n))
  ## optional: make last variable a child of Y (so last variable is non-causal for Y)
  X[,p] <- 0.3*Y + rnorm(n)

##### Compute "Invariant Causal Prediction" Confidence Intervals
  icp <- ICP(X,Y,ExpInd)

##### Print/plot/show summary of output
  print(icp)
  plot(icp)

#### compare with linear model
  cat("\n compare with linear model \n")
  print(summary(lm(Y~X)))

#####
##### 2nd example:
##### Simulate a DAG where X1 -> Y, Y -> X2 and Y -> X3
##### noise interventions on second half of data on X1
##### structure of DAG (at Y -> X2) is changing under interventions
  n1 <- 400
  n2 <- 500
  ExpInd <- c(rep(1,n1), rep(2,n2))
  ## index for observational (ExpInd=1) and intervention data (ExpInd=2)
  X1 <- c(rnorm(n1), 2 * rnorm(n2) + 1)
  Y <- 0.5 * X1 + 0.2 * rnorm(n1 + n2)
  X2 <- c(1.5 * Y[1:n1]      + 0.4 * rnorm(n1),
         -0.3 * Y[(n1+1):n2] + 0.4 * rnorm(n2))
  X3 <- -0.4 * Y           + 0.2 * rnorm(n1 + n2)
  X <- cbind(X1, X2, X3)

#### Compute "Invariant Causal Prediction" Confidence Intervals
  ## use a rank-based test to detect shift in distribution of residuals
  icp <- ICP(X, Y, ExpInd,test="ranks")
  ## use a Kolmogorov-Smirnov test to detect shift in distribution of residuals
  icp <- ICP(X, Y, ExpInd,test="ks")
  ## can also supply test as a function
  ## here chosen to be equivalent to option "ks" above
  icp <- ICP(X, Y, ExpInd,test=function(x,z) ks.test(x,z)$p.val)
  ## use a test based on normal approximations
  icp <- ICP(X, Y, ExpInd, test="normal")

#### Print/plot/show summary of output

```

```

print(icp)
plot(icp)

#### compare with linear model
cat("\n compare with linear model \n")
print(summary(lm(Y~X)))

## Not run:
#####
##### 3rd example:
##### College Distance data
library(AER)
data("CollegeDistance")
CD <- CollegeDistance

## define two experimental settings by
## distance to closest 4-year college
ExpInd <- list()
ExpInd[[1]] <- which(CD$distance < quantile(CD$distance,0.5))
ExpInd[[2]] <- which(CD$distance >= quantile(CD$distance,0.5))

## target variable is binary (did education lead at least to BA degree?)
Y <- as.factor(CD$education>=16)
## use these predictors
X <- CD[,c("gender","ethnicity","score","fcollege","mcollege","home",
"urban","unemp","wage","tuition","income","region")]

## searching all subsets (use selection="lasso" or selection="stability"
## to select a subset of subsets to search)
## with selection="all" the function will take several minutes
icp <- ICP(X,Y,ExpInd,selection="all",alpha=0.1)

## Print/plot/show summary of output
print(icp)
summary(icp)
plot(icp)

## End(Not run)

```

**Description**

Gives a plot of the coefficient paths for a fitted "causalDantzig" object with regularization.

**Usage**

```
## S3 method for class 'causalDantzig'
plot(x, ...)
```

**Arguments**

x                    fitted "causalDantzig" model with regularization  
...                    Additional inputs to generic plot function (not used).

**See Also**

causalDantzig and summary method.

---

plot.InvariantCausalPrediction

*Plots of "InvariantCausalPrediction" objects*

---

**Description**

Plots confidence intervals for invariant causal prediction (output of [ICP](#) function)

**Usage**

```
## S3 method for class 'InvariantCausalPrediction'
plot(x,
      maxshow = 50, col1 = NULL, col2 = NULL, col3 = NULL,
      mar = c(10, 4, 3, 1), lwd = 1, ...)
```

**Arguments**

x                    Object of class "InvariantCausalPrediction", as generated by function [ICP](#).  
maxshow            Maximal number of variables to show confidence intervals for  
col1                Colour of confidence intervals generated by accepted sets of variables (defaults to light red).  
col2                Colour of point-estimates generated by accepted sets of variables (defaults to light red).  
col3                Colour of confidence intervals generated by procedure (defaults to blue).  
mar                 Margins for the figure (might need to be adjusted to allow for long variable names).  
lwd                 Scaling of the width of the lines used for individual confidence intervals.  
...                 Additional inputs to generic plot function (not used).

**Value**

Does not return an object.

**Author(s)**

Nicolai Meinshausen <meinshausen@stat.math.ethz.ch>

**References**

Jonas Peters, Peter Buhlmann, Nicolai Meinshausen (2015):  
Causal inference using invariant prediction: identification and confidence intervals  
arxiv preprint <http://arxiv.org/abs/1501.01332>

**See Also**

[ICP](#)

---

summary.causalDantzig *Summarizes "causalDantzig" fits*

---

**Description**

summary method for class "causalDantzig".

**Usage**

```
## S3 method for class 'causalDantzig'  
summary(object, ...)
```

**Arguments**

object	fitted "causalDantzig" model with or without regularization
...	Additional inputs to generic summary function (not used).

**See Also**

causalDantzig and plot method.

---

summary.InvariantCausalPrediction

*Summary of "InvariantCausalPrediction" objects*

---

## Description

Prints confidence intervals for invariant causal prediction (output of [ICP](#) function)

## Usage

```
## S3 method for class 'InvariantCausalPrediction'  
summary(object, maxshow = 50, ...)
```

## Arguments

object	Object of class "InvariantCausalPrediction", as generated by function <a href="#">ICP</a> .
maxshow	Maximal number of variables to show in the summary.
...	Additional inputs to generic summary function (not used).

## Value

Does not return an object.

## Author(s)

Nicolai Meinshausen <meinshausen@stat.math.ethz.ch>

## References

Jonas Peters, Peter Buhlmann, Nicolai Meinshausen (2015):  
Causal inference using invariant prediction: identification and confidence intervals  
arxiv preprint <http://arxiv.org/abs/1501.01332>

## See Also

[ICP](#)

# Index

## \*Topic **Causality**

- hiddenICP, [4](#)
- ICP, [7](#)
- plot.InvariantCausalPrediction, [12](#)
- summary.InvariantCausalPrediction,  
[14](#)

## \*Topic **Classification**

- ICP, [7](#)
- plot.InvariantCausalPrediction, [12](#)
- summary.InvariantCausalPrediction,  
[14](#)

## \*Topic **Hidden Variables**

- hiddenICP, [4](#)

## \*Topic **Regression**

- hiddenICP, [4](#)
- ICP, [7](#)
- plot.InvariantCausalPrediction, [12](#)
- summary.InvariantCausalPrediction,  
[14](#)

causalDantzig, [2](#), [5](#), [9](#)

hiddenICP, [4](#)

ICP, [5](#), [7](#), [12–14](#)

InvariantCausalPrediction  
(InvariantCausalPrediction-package),  
[2](#)

InvariantCausalPrediction-package, [2](#)

plot.causalDantzig, [11](#)

plot.InvariantCausalPrediction, [12](#)

summary.causalDantzig, [13](#)

summary.InvariantCausalPrediction, [14](#)