

# Package ‘fda.usc’

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**Imports** methods, grDevices, graphics, utils, stats, nlme

**Description** Routines for exploratory and descriptive analysis of functional data such as depth measurements, atypical curves detection, regression models, supervised classification, unsupervised classification and functional analysis of variance.

**License** GPL-2

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**LazyLoad** yes

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fda.usc-package	<i>Functional Data Analysis and Utilities for Statistical Computing (fda.usc)</i>
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## Description

This package carries out exploratory and descriptive analysis of functional data exploring its most important features: such as depth measurements or functional outliers detection, among others. It also helps to explain and model the relationship between a dependent variable and independent (regression models) and make predictions. Methods for supervised or unsupervised classification of a set of functional data regarding a feature of the data are also included. Finally, it can perform analysis of variance model (ANOVA) for functional data.

Sections of fda.usc-package:

- A.- Functional Data Representation
- B.- Functional Outlier Detection
- C.- Functional Regression Model
- D.- Functional Supervised Classification
- E.- Functional Non-Supervised Classification
- F.- Functional ANOVA
- G.- Auxiliary functions:

### A.- Functional Data Representation

The functions included in this section allow to define, transform, manipulate and represent a functional dataset in many ways including derivatives, non-parametric kernel methods or basis representation.

[fdata](#)  
[plot.fdata](#)  
[fdata.deriv](#)  
[CV.S](#)  
[GCV.S](#)

min.np  
min.basis  
S.NW  
S.LLR  
S.basis  
Var.e  
Var.y

## B.- Functional Depth and Functional Outlier Detection

The functional data depth calculated by the different depth functions implemented that could be use as a measure of centrality or outlyingness.

### B.1-Depth methods [Depth](#):

depth.FM  
depth.mode  
depth.RP  
depth.RT  
depth.RPD  
[Descriptive](#)

### B.2-Functional Outliers detection methods:

outliers.depth.trim  
outliers.depth.pond  
outliers.thres.lrt  
outliers.lrt

## C.- Functional Regression Models

### C.1. Functional explanatory covariate and scalar response

The functions included in this section allow the estimation of different functional regression models with a scalar response and a single functional explicative covariate.

fregre.pc  
fregre.pc.cv  
fregre.pls  
fregre.pls.cv  
fregre.basis  
fregre.basis.cv

fregre.np  
fregre.np.cv

C.2. Test for the functional linear model (FLM) with scalar response.

f`lm.Ftest`, F-test for the FLM with scalar response  
f`lm.test`, Goodness-of-fit test for the FLM with scalar response  
f`PCvM.statistic`, PCvM statistic for the FLM with scalar response

C.3. Functional and non functional explanatory covariates.

The functions in this section extends those regression models in previous section in several ways. Semifunctional partial linear regression `fregre.plm` is an extension of functional nonparametric regression `fregre.np` allowing include non-functional variables.

Functional linear regression `fregre.lm`, functional generalized linear regression `fregre.glm` and functional generalized spectral additive model `fregre.gsam` are an extensions of `fregre.basis` and `fregre.pc` allowing include more than one functional variable and other non-functional variables, as `lm` or `glm` functions.

fregre.plm  
fregre.lm  
fregre.glm  
fregre.gsam  
fregre.gkam

C.4. Functional response model (`fregre.basis.fr`) allows the estimation of functional regression models with a functional response and a single functional explicative covariate.

C.5. `fregre.gls` fits functional linear model using generalized least squares. `fregre.igls` fits iteratively a functional linear model using generalized least squares.

D.- Functional Supervised Classification

This section allows the estimation of the groups in a training set of functional data `fdata` class by different nonparametric methods of supervised classification. Once these classifiers have been trained, they can be used to predict on new functional data.

Package allows the estimation of the groups in a training set of functional data by different methods of supervised classification.

classif.knn  
classif.kernel

`classif.glm`  
`classif.gsam`  
`classif.gkam`  
`classif.DD`  
`classif.depth`

#### E.- Functional Non-Supervised Classification

This section allows the estimation of the groups in a functional data set `fdata` class by `kmeans` method.

`kmeans.fd`

#### F.- Functional ANOVA

`anova.onefactor`  
`anova.RPm`  
`anova.hetero`

#### G.- Utilities and auxiliary functions:

`fdata.bootstrap`  
`fdata2fd`  
`fdata2pc`  
`fdata2pls`  
`summary.fdata.comp`  
`cond.F`  
`cond.quantile`  
`cond.mode`  
`FDR`  
`Kernel`  
`Kernel.asymmetric`  
`Kernel.integrate`  
`metric.lp`  
`metric.kl`  
`metric.hausdorff`  
`metric.dist`  
`semimetric.NPFDA`  
`semimetric.basis`

### **Details**

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### References

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

---

aemet

*aemet data*

---

### Description

Series of daily summaries of 73 spanish weather stations selected for the period 1980-2009. The dataset contains geographic information of each station and the average for the period 1980-2009 of daily temperature, daily precipitation and daily wind speed.

Meteorological State Agency of Spain (AEMET), <http://www.aemet.es/>.  
Government of Spain.

### Usage

data(aemet)

### Format

Elements of aemet:

..\$df: Dataframe with information of each wheather station:  
 ind: Indicated weather station.  
 name: Station Name. 36 marked UTF-8 strings  
 province: Province (region) of Spain. 36 marked UTF-8 strings



altitude: Altitude of the station (in meters).  
year.ini: Start year.  
year.end: End year.  
longitude: x geographic coordinate of the station (in decimal degrees) .  
latitude: y geographic coordinate of the station (in decimal degrees) .

The functional variables:

...\$temp: mean curve of the average daily temperature for the period 1980-2009 (in degrees Celsius, marked with UTF-8 string).  
...\$wind.speed: mean curve of the average daily wind speed for the period 1980-2009 (in m/s).  
...\$logprec: mean curve of the log precipitation for the period 1980-2009 (in log mm).

### Details

It marks 36 UTF-8 string of names of stations and 3 UTF-8 string names of provinces through the function `iconv`.

In leap years temperatures for February 28 and 29 were averaged.  
Negligible precipitation (less than 1 tenth of mm) is replaced by 0.05 and no precipitation (0.0 mm) is replaced by 0.01. Then the logarithm is applied.

### Author(s)

Manuel Febrero Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

### Source

The data were obtained from the FTP of AEMET in 2009.

### Examples

```
data(aemet)
names(aemet)
names(aemet$df)
par(mfrow=c(3,1))
plot(aemet$temp)
plot(aemet$wind.speed)
plot(aemet$logprec)
```

---

anova.hetero

*ANOVA for heteroscedastic data*

---

### Description

Univariate ANOVA for heteroscedastic data.

**Usage**

```
## S3 method for class 'hetero'
anova(object=NULL, formula, pr=FALSE, contrast=NULL, ...)
```

**Arguments**

object	A data frame with dimension (n x p+1). In the first column contains the n response values and on the following p columns the explanatory variables specified in the formula.
formula	as <a href="#">formula</a> .
pr	If TRUE, print intermediate results.
contrast	List of special contrast to be used, by default no special contrasts are used (contrast=NULL).
...	Further arguments passed to or from other methods.

**Details**

This function fits a univariate analysis of variance model and allows calculate special contrasts defined by the user. The list of special contrast to be used for some of the factors in the formula. Each matrix of the list has  $r$  rows and  $r-1$  columns.

The user can also request special predetermined contrasts, for example using [contr.helmert](#), [contr.sum](#) or [contr.treatment](#) functions.

**Value**

Return:

ans	A list with components including: the Beta estimation Est, the factor degrees of freedom df1, the residual degrees of freedom df2 and p-value for each factor.
contrast	List of special contrasts.

**Note**

It only works with categorical variables.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Brunner, E., Dette, H., Munk, A. *Box-Type Approximations in Nonparametric Factorial Designs*. Journal of the American Statistical Association, Vol. 92, No. 440 (Dec., 1997), pp. 1494-1502.

**See Also**

See Also as: [anova.RPm](#)

**Examples**

```
data(phoneme)
ind=1 # beetwen 1:150
fdataobj=data.frame(phoneme$learn[["data"]][,ind])
n=dim(fdataobj)[1]
group<-factor(phoneme$classlearn)

#ex 1: real factor and random factor
group.rand=as.factor(sample(rep(1:3,n),n))
f=data.frame(group,group.rand)
mm=data.frame(fdataobj,f)
colnames(mm)=c("value","group","group.rand")
out1=anova.hetero(object=mm[,-2],value~group.rand,pr=FALSE)
out2=anova.hetero(object=mm[,-3],value~group,pr=FALSE)
out1
out2

#ex 2: real factor, random factor and special contrasts
cr5=contr.sum(5) #each level vs last level
cr3=c(1,0,-1) #first level vs last level
out.contrast=anova.hetero(object=mm[,-3],value~group,pr=FALSE,
contrast=list(group=cr5))
out.contrast
```

---

anova.onefactor

*One-way anova model for functional data*

---

**Description**

One-way anova model for k independent samples of functional data. The function contrasts the null hypothesis of equality of mean functions of functional data based on the an asymptotic version of the anova F-test.

$$H_0 : m_1 = \dots = m_k$$

**Usage**

```
## S3 method for class 'onefactor'
anova(object,group,nboot=100,plot=FALSE,verbose=FALSE,...)
```

**Arguments**

object	functional response data. fdata class object with n curves.
group	a factor specifying the class for each curve.
nboot	number of bootstrap samples.
plot	if TRUE, plot the mean of each factor level and the results of test.
verbose	if TRUE, print intermediate results.
...	further arguments passed to or from other methods.

**Details**

The function returns the p-value of test using one-way anova model over nboot runs.

**Value**

returns:

pvalue	p-value.
stat	statistic value of test.
wm	statistic values of bootstrap resamples.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Cuevas, A., Febrero, M., & Fraiman, R. (2004). *An anova test for functional data*. Computational statistics & data analysis, **47**(1), 111-122.

**See Also**

See Also as: [anova.RPm](#)

**Examples**

```
## Not run:
data(MCO)
grupo<-MCO$classintact
datos<-MCO$intact
res=anova.onefactor(datos,grupo,nboot=50,plot=TRUE)

grupo<-MCO$classpermea
datos<-MCO$permea
res=anova.onefactor(datos,grupo,nboot=50,plot=TRUE)

## End(Not run)
```

anova.RPm

*Functional ANOVA with Random Project.***Description**

The procedure is based on the analysis of randomly chosen one-dimensional projections. The function tests ANOVA models for functional data with continuous covariates and perform special contrasts for the factors in the formula.

**Usage**

```
## S3 method for class 'RPm'
anova(object, formula, data.fac, RP=min(30, ncol(object)),
alpha=0.95, hetero=TRUE, pr=FALSE, w=rep(1, ncol(object)),
nboot=0, contrast=NULL, ...)
```

**Arguments**

object	Functional response data. Object with class fdata with n curves discretized in m points.
formula	as <a href="#">formula</a> .
data.fac	Explanatory variables. Data frame with dimension (n x p), where p are the number of factors or covariate considered.
RP	Vector of random projections.
alpha	Alpha value, by default alpha=0.95.
hetero	=TRUE (by default) heteroskedastic ANOVA.
pr	If TRUE, print intermediate results.
w	Vector of weights.
nboot	Number of bootstrap samples, by default no bootstrap sample, nboot=0.
contrast	List of special contrast to be used ; by default no special contrasts are used (contrast=NULL).
...	Further arguments passed to or from other methods.

**Details**

The function allows user-defined contrasts. The list of contrast to be used for some of the factors in the formula. Each contrast matrix in the list has r rows, where r is the number of factor levels. The user can also request special predetermined contrasts, for example using the [contr.helmert](#), [contr.sum](#) or [contr.treatment](#) functions.

The function returns (by default) the significance of the variables using the Bonferroni test and the False Discovery Rate test. Bootstrap procedure provides more precision

**Value**

Shows:

proj	The projection value of each point on the curves. Matrix with dimension (RP x m), where RP is the number of projection and m are the points observed in each projection curve.
mins	minimum number for each random projection.
result	p-value for each random projection.
test.Bonf	significance (TRUE or FALSE) for vector of random projections RP in columns and factor (and special contrast) by rows.
p.Bonf	p-value for vector of random projections RP in columns and factor (and special contrast) by rows.
test.fdr	False Discovery Rate (TRUE or FALSE) for vector of random projections RP in columns and factor (and special contrast) by rows.
p.fdr	p-value of False Discovery Rate for vector of random projections RP in columns and factor (and special contrast) by rows.
test.Boot	False Discovery Rate (TRUE or FALSE) for vector of random projections RP in columns and factor (and special contrast) by rows.
p.Boot	p-value of Bootstrap sample for vector of random projections RP in columns and factor (and special contrast) by rows.

**Note**

If hetero=TRUE then all factors must be categorical.

**Author(s)**

Juan A. Cuesta-Albertos, Manuel Febrero-Bande, Manuel Oviedo de la Fuente  
<manuel.oviedo@usc.es>

**References**

Cuesta-Albertos, J.A., Febrero-Bande, M. *A simple multiway ANOVA for functional data*. Test 2010, DOI [10.1007/s11749-010-0185-3](https://doi.org/10.1007/s11749-010-0185-3).

**See Also**

See Also as: [anova.onefactor](#)

**Examples**

```
# ex anova.hetero
data(phoneme)
names(phoneme)
data=as.data.frame(phoneme$learn[["data"]])
```

```

group=phoneme$classlearn
n=nrow(data)
group.rand=as.factor(sample(rep(1:3,len=n),n))
RP=c(2,5,15,30)

#ex 1: real factor and random factor
m03=data.frame(group,group.rand)
resul1=anova.RPm(data,~group+group.rand,m03,RP=c(5,30))
summary.anova(resul1)

#ex 2: real factor with special contrast
m0=data.frame(group)
cr5=contr.sum(5) #each level vs last level
resul03c1=anova.RPm(data,~group,m0,contrast=list(group=cr5))
summary.anova(resul03c1)

#ex 3: random factor with special contrast
m0=data.frame(group.rand)
cr3=contr.sum(3) #each level vs last level
resul03c1=anova.RPm(data,~group.rand,m0,contrast=list(group.rand=cr3))
summary.anova(resul03c1)

```

---

classif.DD

*DD-Classifer Based on DD-plot*


---

### Description

Fits Nonparametric Classification Procedure Based on DD-plot (depth-versus-depth plot) for  $G$  dimensions ( $G = g \times h$ ,  $g$  levels and  $p$  data depth).

### Usage

```

classif.DD(group, fdataobj, depth="FM", classif="glm", w,
           par.classif=list(), par.depth=list(),
           control=list(verbose=FALSE, draw=TRUE, col=NULL, alpha=.25))

```

### Arguments

group	Factor of length $n$ with $g$ levels.
fdataobj	<a href="#">data.frame</a> , <a href="#">fdata</a> or list with the multivariate, functional or both covariates respectively.
depth	Character vector specifying the type of depth functions to use, see <a href="#">Details</a> .
classif	Character vector specifying the type of classifier method to use, see <a href="#">Details</a> .
w	Optional case weights, weights for each value of depth argument, see <a href="#">Details</a> .
par.depth	List of parameters for depth function.
par.classif	List of parameters for classif procedure.

control            List of parameters for controlling the process.  
 If verbose=TRUE, report extra information on progress.  
 If draw=TRUE print DD-plot of two samples based on data depth.  
 col, the colors for points in DD-plot.  
 alpha, the alpha transparency used in the background of DD-plot, a number in [0,1].

## Details

Make the group classification of a training dataset using DD-classifier estimation in the following steps.

1. The function computes the selected depth measure of the points in `fdataobj` w.r.t. a sub-sample of each  $g$  level group and  $p$  data dimension ( $G = g \times p$ ). The user can specify the parameters for depth function in `par.depth`.

(i) Type of depth function from functional data, see [Depth](#):

- "FM": Fraiman and Muniz depth.
- "mode": h-modal depth.
- "RT": random Tukey depth.
- "RP": random project depth.
- "RPD": double random project depth.

(ii) Type of depth function from multivariate functional data, see [Depth.pfdata](#):

- "FMp": Fraiman and Muniz depth with common support. Suppose that all  $p$ -fdata objects have the same support (same rangevals), see [depth.FMp](#).
- "modep": h-modal depth using a  $p$ -dimensional metric, see [depth.modep](#).
- "RPP": random project depth using a  $p$ -variate depth with the projections, see [depth.RPP](#).

If the procedure requires to compute a distance such as in "knn" or "np" classifier or "mode" depth, the user must use a proper distance function: [metric.lp](#) for functional data and [metric.dist](#) for multivariate data.

(iii) Type of depth function from multivariate data, see [Depth.Multivariate](#):

- "SD": Simplicial depth (for bivariate data).
- "HS": Half-space depth.
- "MhD": Mahalanobis dept.
- "RD": random projections depth.
- "LD": Likelihood depth.

2. The function calculates the misclassification rate based on data depth computed in step (1) using the following classifiers.

- "MaxD": Maximum depth.
- "DD1": Search the best separating polynomial of degree 1.
- "DD2": Search the best separating polynomial of degree 2.



- "DD3": Search the best separating polynomial of degree 3.
- "glm": Logistic regression is computed using Generalized Linear Models [classif.glm](#).
- "gam": Logistic regression is computed using Generalized Additive Models [classif.gsam](#).
- "lda": Linear Discriminant Analysis is computed using [lda](#).
- "qda": Quadratic Discriminant Analysis is computed using [qda](#).
- "knn": k-Nearest Neighbour classification is computed using [classif.knn](#).
- "np": Non-parametric Kernel classifier is computed using [classif.np](#).

The user can specify the parameters for classifier function in `par.classif` such as the smoothing parameter `par.classif[['h']]`, if `classif="np"` or the k-Nearest Neighbour `par.classif[['knn']]`, if `classif="knn"`.

In the case of polynomial classifier ("DD1", "DD2" and "DD3") uses the original procedure proposed by Li et al. (2012), by default rotating the DD-plot (to exchange abscise and ordinate) using in `par.classif` argument `rotate=TRUE`. Notice that the maximum depth classifier can be considered as a particular case of DD1, fixing the slope with a value of 1 (`par.classif=list(pol=1)`).

The number of possible different polynomials depends on the sample size  $n$  and increases polynomially with order  $k$ . In the case of  $g$  groups, so the procedure applies some multiple-start optimization scheme to save time:

- generate all combinations of the elements of  $n$  taken  $k$  at a time:  $g \times \text{combn}(N, k)$  candidate solutions, and, when this number is larger than `nmax=10000`, a random sample of 10000 combinations.
- smooth the empirical loss with the logistic function  $1/(1 + e^{-tx})$ . The classification rule is constructed optimizing the best `noptim` combinations in this random sample (by default `noptim=1` and `tt=50/range(depth values)`). Note that Li et al. found that the optimization results become stable for  $t \in [50, 200]$  when the depth is standardized with upper bound 1.

The original procedure (Li et al. (2012)) not need to try many initial polynomials (`nmax=1000`) and that the procedure optimize the best (`noptim=1`), but we recommended to repeat the last step for different solutions, as for example `nmax=250` and `noptim=25`. User can change the parameters `pol`, `rotate`, `nmax`, `noptim` and `tt` in the argument `par.classif`.

The `classif.DD` procedure extends to multi-class problems by incorporating the method of *majority voting* in the case of polynomial classifier and the method *One vs the Rest* in the logistic case ("glm" and "gam").

**Value**

<code>group.est</code>	Estimated vector groups by classified method selected.
<code>misclassification</code>	Probability of misclassification.
<code>prob.classification</code>	Probability of correct classification by group level.
<code>dep</code>	Data frame with the depth of the curves for functional data (or points for multi-variate data) in <code>fdataobj</code> w.r.t. each group level.

depth	Character vector specifying the type of depth functions used.
par.depth	List of parameters for depth function.
classif	Type of classifier used.
par.classif	List of parameters for classif procedure.
w	Optional case weights.
fit	Fitted object by classif method using the depth as covariate.

### Author(s)

This version was created by Manuel Oviedo de la Fuente and Manuel Febrero Bande and includes the original version for polynomial classifier created by Jun Li, Juan A. Cuesta-Albertos and Regina Y. Liu.

### References

Li, J., P.C., Cuesta-Albertos, J.A. and Liu, R. *DD-Classifier: Nonparametric Classification Procedure Based on DD-plot*. Journal of the American Statistical Association (2012), Vol. 107, 737–753.

Cuesta-Albertos, J.A., Febrero-Bande, M. and Oviedo de la Fuente, M. *The DDG-classifier in the functional setting*. Submitted.

### See Also

See Also as [predict.classif.DD](#)

### Examples

```
## Not run:

# DD-classif for functional data
data(tecator)
ab=tecator$absorp.fdata
ab1=fdata.deriv(ab,nderiv=1)
ab2=fdata.deriv(ab,nderiv=2)
gfat=factor(as.numeric(tecator$y$Fat>=15))

# DD-classif for p=1 functional data set
out01=classif.DD(gfat,ab,depth="mode",classif="np")
out02=classif.DD(gfat,ab2,depth="mode",classif="np")
# DD-plot in gray scale
ctrl<-list(draw=T,col=gray(c(0,.5)),alpha=.2)
out02bis=classif.DD(gfat,ab2,depth="mode",classif="np",control=ctrl)

# 2 depth functions (same curves)
out03=classif.DD(gfat,list(ab2,ab2),depth=c("RP","mode"),classif="np")
# DD-classif for p=2 functional data set
ldata<-list("ab"=ab2,"ab2"=ab2)
# Weighted version
out04=classif.DD(gfat,ldata,depth="mode",classif="np",w=c(0.5,0.5))
# Model version
```

```

out05=classif.DD(gfat,ldata,depth="mode",classif="np")
# Integrated version (for multivariate functional data)
out06=classif.DD(gfat,ldata,depth="modep",classif="np")

# DD-classif for multivariate data
data(iris)
group<-iris[,5]
x<-iris[,1:4]
out10=classif.DD(group,x,depth="RP",classif="lda")
summary.classif(out10)
out11=classif.DD(group,list(x,x),depth=c("MhD","RP"),classif="lda")
summary.classif(out11)

# DD-classif for functional data: g levels
data(phoneme)
mlearn<-phoneme[["learn"]]
glearn<-as.numeric(phoneme[["classlearn"]])-1
out20=classif.DD(glearn,mlearn,depth="FM",classif="glm")
out21=classif.DD(glearn,list(mlearn,mlearn),depth=c("FM","RP"),classif="glm")
summary.classif(out20)
summary.classif(out21)

## End(Not run)

```

---

classif.depth	<i>Classifier from Functional Data</i>
---------------	--

---

## Description

Classification of functional data using maximum depth.

## Usage

```
classif.depth(group, fdataobj, newfdataobj, depth="RP",
par.depth=list(), CV="none")
```

## Arguments

group	Factor of length $n$
fdataobj	fdata, matrix or data.frame class object of train data.
newfdataobj	fdata, matrix or data.frame class object of test data.
depth	Type of depth function from functional data: <ul style="list-style-type: none"> <li>• FM: Fraiman and Muniz depth.</li> <li>• mode: modal depth.</li> <li>• RT: random Tukey depth.</li> </ul>

- RP: random project depth.
- RPD: double random project depth.

par.depth	List of parameters for depth.
CV	=“none” group.est=group.pred, =TRUE group.est is estimated by cross-validation, =FALSE group.est is estimated.

**Value**

group.est	Vector of classes of train sample data.
group.pred	Vector of classes of test sample data.
prob.classification	Probability of correct classification by group.
max.prob	Highest probability of correct classification.
fdataobj	<a href="#">fdata</a> class object.
group	Factor of length $n$ .

**Author(s)**

Febrero-Bande, M. and Oviedo de la Fuente, M.

**References**

Cuevas, A., Febrero-Bande, M. and Fraiman, R. (2007). *Robust estimation and classification for functional data via projection-based depth notions*. Computational Statistics 22, 3, 481-496.

**Examples**

```
## Not run:
data(phoneme)
mlearn<-phoneme[["learn"]]
mtest<-phoneme[["test"]]
glearn<-phoneme[["classlearn"]]
gtest<-phoneme[["classtest"]]

a1<-classif.depth(glearn,mlearn,depth="RP")
table(a1$group.est,glearn)
a2<-classif.depth(glearn,mlearn,depth="RP",CV=TRUE)
a3<-classif.depth(glearn,mlearn,depth="RP",CV=FALSE)
a4<-classif.depth(glearn,mlearn,mtest,"RP")
a5<-classif.depth(glearn,mlearn,mtest,"RP",CV=TRUE)
table(a5$group.est,glearn)
a6<-classif.depth(glearn,mlearn,mtest,"RP",CV=FALSE)
table(a6$group.est,glearn)

## End(Not run)
```

---

classif.gkam

*Classification Fitting Functional Generalized Kernel Additive Models*


---

## Description

Computes functional classification using functional explanatory variables using backfitting algorithm.

## Usage

```
classif.gkam(formula, family = binomial(), data, weights = rep(1, nobs),
  par.metric = NULL, par.np=NULL, offset=NULL,
  control = list(maxit = 100, epsilon = 0.001, trace = FALSE ,
  inverse="solve"), ...)
```

## Arguments

formula	an object of class formula (or one that can be coerced to that class): a symbolic description of the model to be fitted. The procedure only considers functional covariates (not implemented for non-functional covariates). The details of model specification are given under Details.
data	List that containing the variables in the model.
family	a description of the error distribution and link function to be used in the model. This can be a character string naming a family function, a family function or the result of a call to a family function. (See <a href="#">family</a> for details of family functions.)
weights	weights
par.metric	List of arguments by covariable to pass to the metric function by covariable.
par.np	List of arguments to pass to the <code>fregre.np.cv</code> function
offset	this can be used to specify an a priori known component to be included in the linear predictor during fitting.
control	a list of parameters for controlling the fitting process, by default: maxit, epsilon, trace and inverse.
...	Further arguments passed to or from other methods.

## Details

The first item in the data list is called "*df*" and is a data frame with the response, as [glm](#). Functional covariates of class `fdata` are introduced in the following items in the data list.

**Value**

Return gam object plus:

formula	formula.
data	List that containing the variables in the model.
group	Factor of length $n$
group.est	Estimated vector groups
prob.classification	Probability of correct classification by group.
prob.group	Matrix of predicted class probabilities. For each functional point shows the probability of each possible group membership.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

- Febrero-Bande M. and Gonzalez-Manteiga W. (2012). *Generalized Additive Models for Functional Data*. TEST. Springer-Velag. <http://dx.doi.org/10.1007/s11749-012-0308-0>
- McCullagh and Nelder (1989), *Generalized Linear Models* 2nd ed. Chapman and Hall.
- Opsomer J.D. and Ruppert D.(1997). *Fitting a bivariate additive model by local polynomial regression*. Annals of Statistics, 25, 186-211.

**See Also**

See Also as: [fregre.gkam](#).  
Alternative method: [classif.glm](#).

**Examples**

```
## Time-consuming: selection of 2 levels
data(phoneme)
mlearn<-phoneme[["learn"]][1:100]
glearn<-as.numeric(phoneme[["classlearn"]][1:100])
dataf<-data.frame(glearn)
dat=list("df"=dataf,"x"=mlearn)
# a1<-classif.gkam(glearn~x,data=dat)
# summary(a1)
mtest<-phoneme[["test"]][1:100]
gtest<-as.numeric(phoneme[["classtest"]][1:100])
newdat<-list("x"=mtest)
# p1<-predict.classif(a1,newdat)
# table(gtest,p1)
```

classif.glm

*Classification Fitting Functional Generalized Linear Models***Description**

Computes functional classification using functional (and non functional) explanatory variables by basis representation.

**Usage**

```
classif.glm(formula,data,family = binomial(),
            basis.x=NULL,basis.b=NULL,CV=FALSE,...)
```

**Arguments**

formula	an object of class formula (or one that can be coerced to that class): a symbolic description of the model to be fitted. The details of model specification are given under Details.
data	List that containing the variables in the model.
family	a description of the error distribution and link function to be used in the model. This can be a character string naming a family function, a family function or the result of a call to a family function. (See <a href="#">family</a> for details of family functions).
basis.x	List of basis for functional explanatory data estimation.
basis.b	List of basis for functional beta parameter estimation.
CV	=TRUE, Cross-validation (CV) is done.
...	Further arguments passed to or from other methods.

**Details**

The first item in the data list is called "*df*" and is a data frame with the response and non functional explanatory variables, as [glm](#).

Functional covariates of class *fdata* or *fd* are introduced in the following items in the data list. *basis.x* is a list of basis for represent each functional covariate. The basis object can be created by the function: [create.pc.basis](#), [pca.fd](#) [create.pc.basis](#), [create.fdata.basis](#) o [create.basis](#). *basis.b* is a list of basis for represent each functional beta parameter. If *basis.x* is a list of functional principal components basis (see [create.pc.basis](#) or [pca.fd](#)) the argument *basis.b* is ignored.

**Value**

Return *glm* object plus:

formula	formula.
---------	----------

data	List that containing the variables in the model.
group	Factor of length $n$
group.est	Estimated vector groups
prob.classification	Probability of correct classification by group.
prob.group	Matrix of predicted class probabilities. For each functional point shows the probability of each possible group membership.

**Note**

If the formula only contains a non functional explanatory variables (multivariate covariates), the function compute a standard `glm` procedure.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ramsay, James O., and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.

McCullagh and Nelder (1989), *Generalized Linear Models* 2nd ed. Chapman and Hall.

Venables, W. N. and Ripley, B. D. (2002) *Modern Applied Statistics with S*, New York: Springer.

**See Also**

See Also as: [fregre.glm](#).

**Examples**

```
data(phoneme)
mlearn<-phoneme[["learn"]]
glearn<-phoneme[["classlearn"]]
mtest<-phoneme[["test"]]
gtest<-phoneme[["classtest"]]
dataf<-data.frame(glearn)
dat=list("df"=dataf,"x"=mlearn)
a1<-classif.glm(glearn~x, data = dat)
newdat<-list("x"=mtest)
p1<-predict.classif(a1,newdat)
table(gtest,p1)
sum(p1==gtest)/250
```



---

classif.gsam

*Classification Fitting Functional Generalized Additive Models*


---

## Description

Computes functional classification using functional (and non functional) explanatory variables by basis representation.

## Usage

```
classif.gsam(formula,data,family = binomial(),weights=NULL,
basis.x=NULL,basis.b=NULL,CV=FALSE,...)
```

## Arguments

formula	an object of class formula (or one that can be coerced to that class): a symbolic description of the model to be fitted. The details of model specification are given under Details.
family	a description of the error distribution and link function to be used in the model. This can be a character string naming a family function, a family function or the result of a call to a family function. (See <a href="#">family</a> for details of family functions.)
data	List that containing the variables in the model.
weights	weights
basis.x	List of basis for functional explanatory data estimation.
basis.b	List of basis for functional beta parameter estimation.
CV	=TRUE, Cross-validation (CV) is done.
...	Further arguments passed to or from other methods.

## Details

The first item in the data list is called "*df*" and is a data frame with the response and non functional explanatory variables, as [glm](#).

Functional covariates of class `fdata` or `fd` are introduced in the following items in the data list. `basis.x` is a list of basis for represent each functional covariate. The basis object can be created by the function: [create.pc.basis](#), [pca.fd](#) [create.pc.basis](#), [create.fdata.basis](#) o [create.basis](#). `basis.b` is a list of basis for represent each functional beta parameter. If `basis.x` is a list of functional principal components basis (see [create.pc.basis](#) or [pca.fd](#)) the argument `basis.b` is ignored.

**Value**

Return gam object plus:

formula	formula.
data	List that containing the variables in the model.
group	Factor of length $n$
group.est	Estimated vector groups
prob.classification	Probability of correct classification by group.
prob.group	Matrix of predicted class probabilities. For each functional point shows the probability of each possible group membership.

**Note**

If the formula only contains a non functional explanatory variables (multivariate covariates), the function compute a standard `glm` procedure.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ramsay, James O., and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.

McCullagh and Nelder (1989), *Generalized Linear Models* 2nd ed. Chapman and Hall.

Venables, W. N. and Ripley, B. D. (2002) *Modern Applied Statistics with S*, New York: Springer.

**See Also**

See Also as: [fregre.gsam](#).

Alternative method: [classif.np](#), [classif.glm](#) and [classif.gkam](#).

**Examples**

```
data(phoneme)
mlearn<-phoneme[["learn"]]
glearn<-phoneme[["classlearn"]]
mtest<-phoneme[["test"]]
gtest<-phoneme[["classtest"]]
dataf<-data.frame(glearn)
dat=list("df"=dataf,"x"=mlearn)
a1<-classif.gsam(glearn~s(x,k=3),data=dat)
summary(a1)
newdat<-list("x"=mtest)
p1<-predict.classif(a1,newdat)
```

```
table(gtest,p1)
sum(p1==gtest)/250
```

---

classif.np

*Kernel Classifier from Functional Data*


---

## Description

Fits Nonparametric Supervised Classification for Functional Data.

## Usage

```
classif.np(group, fdataobj, h=NULL, Ker=AKer.norm, metric,
  type.CV = GCV.S, type.S=S.NW, par.CV=list(trim=0), par.S=list(), ...)
classif.knn(group, fdataobj, knn=NULL, metric, type.CV = GCV.S,
  par.CV=list(cv = TRUE, trim = 0), par.S=list(), ...)
classif.kernel(group, fdataobj, h=NULL, Ker=AKer.norm, metric,
  type.CV = GCV.S, par.CV=list(trim=0), par.S=list(), ...)
```

## Arguments

group	Factor of length $n$
fdataobj	<a href="#">fdata</a> class object.
h	Vector of smoothing parameter or bandwidth.
knn	Vector of number of nearest neighbors considered.
Ker	Type of kernel used.
metric	Metric function, by default <a href="#">metric.lp</a> .
type.CV	Type of cross-validation. By default generalized cross-validation <a href="#">GCV.S</a> method.
type.S	Type of smothing matrix $S$ . By default $S$ is calculated by Nadaraya-Watson kernel estimator ( $S.NW$ ).
par.CV	List of parameters for type.CV: trim, the alpha of the trimming and draw=TRUE.
par.S	List of parameters for type.S: w, the weights.
...	Arguments to be passed for <a href="#">metric.lp</a> o other metric function and <a href="#">Kernel</a> function.

## Details

Make the group classification of a training dataset using kernel or KNN estimation: [Kernel](#). Different types of metric funtions can be used.

**Value**

<code>fdataobj</code>	<code>fdata</code> class object.
<code>group</code>	Factor of length <code>n</code> .
<code>group.est</code>	Estimated vector groups
<code>prob.group</code>	Matrix of predicted class probabilities. For each functional point shows the probability of each possible group membership.
<code>max.prob</code>	Highest probability of correct classification.
<code>h.opt</code>	Optimal smoothing parameter or bandwidth estimated.
<code>D</code>	Matrix of distances of the optimal quantile distance <code>hh.opt</code> .
<code>prob.classification</code>	Probability of correct classification by group.
<code>misclassification</code>	Vector of probability of misclassification by number of neighbors <code>knn</code> .
<code>h</code>	Vector of smoothing parameter or bandwidth.
<code>C</code>	A call of function <code>classif.kernel</code> .

**Note**

If `fdataobj` is a `data.frame` the function considers the case of multivariate covariates. `metric.dist` function is used to compute the distances between the rows of a data matrix (as `dist` function).

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

- Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.
- Ferraty, F. and Vieu, P. (2006). *NPFDA in practice*. Free access on line at <http://www.lsp.ups-tlse.fr/staph/npfda/>

**See Also**

See Also as `predict.classif`

**Examples**

```
data(phoneme)
mlearn<-phoneme[["learn"]]
glearn<-phoneme[["classlearn"]]

h=9:19
out=classif.np(glearn,mlearn,h=h)
summary.classif(out)
#round(out$prob.group,4)
```

---

classif.tree	<i>Classification Fitting Functional Recursive Partitioning and Regression Trees</i>
--------------	--

---

**Description**

Computes functional classification using functional (and non functional) explanatory variables by rpart model

**Usage**

```
classif.tree(formula,data,basis.x=NULL,basis.b=NULL,CV=FALSE,...)
```

**Arguments**

formula	an object of class formula (or one that can be coerced to that class): a symbolic description of the model to be fitted. The details of model specification are given under Details.
data	List that containing the variables in the model.
basis.x	List of basis for functional explanatory data estimation.
basis.b	List of basis for functional beta parameter estimation.
CV	=TRUE, Cross-validation (CV) is done .
...	Further arguments passed to or from other methods.

**Details**

The first item in the data list is called "*df*" and is a data frame with the response and non functional explanatory variables, as [glm](#).

Functional covariates of class `fdata` or `fd` are introduced in the following items in the data list. `basis.x` is a list of basis for represent each functional covariate. The basis object can be created by the function: [create.pc.basis](#), [pca.fd](#) [create.pc.basis](#), [create.fdata.basis](#) o [create.basis](#). `basis.b` is a list of basis for represent each functional beta parameter. If `basis.x` is a list of functional principal components basis (see [create.pc.basis](#) or [pca.fd](#)) the argument `basis.b` is ignored.

**Value**

Return rpart object plus:

basis.x	Basis used for fdata or fd covariates.
basis.b	Basis used for beta parameter estimation.
beta.l	List of estimated beta parameter of functional covariates.
data	List that containing the variables in the model.
formula	formula.
CV	\\$y.pred predicted response by cross-validation.

**Author(s)**

Febrero-Bande, M. and Oviedo de la Fuente, M.

**References**

Ramsay, James O., and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.  
 McCullagh and Nelder (1989), *Generalized Linear Models* 2nd ed. Chapman and Hall.  
 Venables, W. N. and Ripley, B. D. (2002) *Modern Applied Statistics with S*, New York: Springer.

**See Also**

See Also as: [rpart](#).  
 Alternative method: [classif.glm](#).

**Examples**

```
data(phoneme)
mlearn<-phoneme[["learn"]]
glearn<-phoneme[["classlearn"]]
mtest<-phoneme[["test"]]
gtest<-phoneme[["classtest"]]
dataf<-data.frame(glearn)
dat=list("df"=dataf, "x"=mlearn)
a1<-classif.tree(glearn~x,data=dat)
summary(a1)
newdat<-list("x"=mtest)
p1<-predict.classif(a1,newdat,type="class")
table(gtest,p1)
sum(p1==gtest)/250
```

---

 cond.F

---

*Conditional Distribution Function*


---

**Description**

Calculate the conditional distribution function of a scalar response with functional data.

**Usage**

```
cond.F(fdata0,y0,fdataobj,y,h=0.15,g=0.15,metric=metric.lp,
      Ker=list(AKer=AKer.epa,IKer=IKer.epa),...)
```

**Arguments**

<code>fdata0</code>	Conditional explanatory functional data of <code>fdata</code> class.
<code>y0</code>	Vector of conditional response with length <code>n</code> .
<code>fdataobj</code>	<code>fdata</code> class object.
<code>y</code>	Vector of scalar response with length <code>nn</code> .
<code>h</code>	Smoothing parameter or bandwidth of response <code>y</code> .
<code>g</code>	Smoothing parameter or bandwidth of explanatory functional data <code>fdataobj</code> .
<code>metric</code>	Metric function, by default <code>metric.lp</code> .
<code>Ker</code>	List of 2 arguments. The first argument is a character string that determines the type of asymmetric kernel (see <code>Kernel.asymmetric</code> ). Asymmetric Epanechnikov kernel is selected by default. The second argument is a string that determines the type of integrated kernel (see <code>Kernel.integrate</code> ). Integrate Epanechnikov kernel is selected by default.
<code>.</code>	.
<code>...</code>	Further arguments passed to or from other methods.

**Details**

If `x.dist=NULL` the distance matrix between `fdata` objects is calculated by function passed in `metric` argument.

**Value**

<code>Fc</code>	Conditional distribution function.
<code>y0</code>	Vector of conditional response.
<code>g</code>	Smoothing parameter or bandwidth of explanatory functional data ( <code>fdataobj</code> ).
<code>h</code>	Smoothing parameter or bandwidth of response, <code>y</code> .
<code>x.dist</code>	Distance matrix between curves of <code>fdataobj</code> object.
<code>xy.dist</code>	Distance matrix between curves of <code>fdataobj</code> and <code>fdata0</code> objects.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.

**See Also**

See Also as: `cond.mode` and `cond.quantile`.

## Examples

```
# Read data
n= 500
t= seq(0,1,len=101)
beta = t*sin(2*pi*t)^2
x = matrix(NA, ncol=101, nrow=n)
y=numeric(n)
x0<-rproc2fdata(n,seq(0,1,len=101),sigma="wiener")
x1<-rproc2fdata(n,seq(0,1,len=101),sigma=0.1)
x<-x0*3+x1
fbeta = fdata(beta,t)
y<-inprod.fdata(x,fbeta)+rnorm(n,sd=0.1)

prx=x[1:100];pry=y[1:100]
ind=101;ind2=102:110
pr0=x[ind];pr10=x[ind2,]
ndist=61
gridy=seq(-1.598069,1.598069, len=ndist)
# Conditional Function
res1 = cond.F(pr10, gridy, prx, pry,p=1)
# res2 = cond.F(pr10, gridy, prx, pry,h=0.3)
# res3 = cond.F(pr10, gridy, prx, pry,g=0.25,h=0.3)

# plot(res1$Fc[,1],type="l",ylim=c(0,1))
# lines(res2$Fc[,1],type="l",col=2)
# lines(res3$Fc[,1],type="l",col=3)
```

---

cond.mode

*Conditional mode*

---

## Description

Computes the mode for conditional distribution function.

## Usage

```
cond.mode(Fc, method = "monoH.FC", draw=TRUE)
```

## Arguments

Fc	Object estimated by cond.F function.
method	Specifies the type of spline to be used. Possible values are <i>"diff"</i> , <i>"fmm"</i> , <i>"natural"</i> , <i>"periodic"</i> and <i>"monoH.FC"</i> .
draw	=TRUE, plots the conditional distribution and density function.



**Details**

The conditional mode is calculated as the maximum argument of the derivative of the conditional distribution function (density function  $f$ ).

**Value**

Return the mode for conditional distribution function.

mode.cond	Conditional mode.
x	Grid of length n where the the conditional density function is evaluated.
f	The conditional density function evaluated in x.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.

**See Also**

See Also as: [cond.F](#), [cond.quantile](#) and [splinefun](#) .

**Examples**

```
## Not run:
n= 500
t= seq(0,1,len=101)
beta = t*sin(2*pi*t)^2
x = matrix(NA, ncol=101, nrow=n)
y=numeric(n)
x0<-rproc2fdata(n,seq(0,1,len=101),sigma="wiener")
x1<-rproc2fdata(n,seq(0,1,len=101),sigma=0.1)
x<-x0*3+x1
fbeta = fdata(beta,t)
y<-inprod.fdata(x,fbeta)+rnorm(n,sd=0.1)
prx=x[1:100];pry=y[1:100]
ind=101;ind2=101:110
pr0=x[ind];pr10=x[ind2]
ndist=161
gridy=seq(-1.598069,1.598069, len=ndist)
# Conditional Function
I=5
# Time consuming
res = cond.F(pr10[I], gridy, prx, pry, h=1)
mcond=cond.mode(res)
mcond2=cond.mode(res,method="diff")

## End(Not run)
```

---

cond.quantile	<i>Conditional quantile</i>
---------------	-----------------------------

---

**Description**

Computes the quantile for conditional distribution function.

**Usage**

```
cond.quantile(qua=0.5, fdata0, fdataobj, y, fn, a=min(y), b=max(y),
             tol=10^floor(log10(max(y)-min(y))-3), iter.max=100, ...)
```

**Arguments**

qua	Quantile value, by default the median (qua=0.5).
fdata0	Conditional functional explanatory data of <a href="#">fdata</a> class object.
fdataobj	Functional explanatory data of <a href="#">fdata</a> class object.
y	Scalar Response.
fn	Conditional distribution function.
a	Lower limit.
b	Upper limit.
tol	Tolerance.
iter.max	Maximum iterations allowed, by default 100.
...	Further arguments passed to or from other methods.

**Value**

Return the quantile for conditional distribution function.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.

**See Also**

See Also as: [cond.F](#) and [cond.mode](#).

**Examples**

```

n= 100
t= seq(0,1,len=101)
beta = t*sin(2*pi*t)^2
x = matrix(NA, ncol=101, nrow=n)
y=numeric(n)
x0<-rproc2fdata(n,seq(0,1,len=101),sigma="wiener")
x1<-rproc2fdata(n,seq(0,1,len=101),sigma=0.1)
x<-x0*3+x1
fbeta = fdata(beta,t)
y<-inprod.fdata(x,fbeta)+rnorm(n,sd=0.1)

prx=x[1:50];pry=y[1:50]
ind=50+1;ind2=51:60
pr0=x[ind];pr10=x[ind2]
ndist=161
gridy=seq(-1.598069,1.598069, len=ndist)
ind4=5
y0 = gridy[ind4]

## Conditional median
med=cond.quantile(qua=0.5,fdata0=pr0,fdataobj=prx,y=pry,fn=cond.F,h=1)

## Not run
## Conditional CI 95% conditional
# lo=cond.quantile(qua=0.025,fdata0=pr0,fdataobj=prx,y=pry,fn=cond.F,h=1)
# up=cond.quantile(qua=0.975,fdata0=pr0,fdataobj=prx,y=pry,fn=cond.F,h=1)
# print(c(lo,med,up))

```

---

create.fdata.basis      *Create Basis Set for Functional Data of fdata class*

---

**Description**

Compute basis for functional data.

**Usage**

```

create.fdata.basis(fdataobj,l=1:5,maxl=max(1),type.basis="bspline",
rangeval=fdataobj$rangeval,class.out="fd")
create.pc.basis(fdataobj,l=1:5,norm=TRUE,basis=NULL,lambda=0,
P=c(0,0,1),...)
create.pls.basis(fdataobj, y, l = 1:5,norm=TRUE,lambda=0,
P=c(0,0,1),...)
create.raw.fdata(fdataobj, l = 1:ncol(fdataobj))

```

**Arguments**

fdataobj	fdata class object.
y	Vector of response (scalar).
l	Vector of basis index.
maxl	maximum number of basis
type.basis	Type of basis (see create.basis function).
rangeval	A vector of length 2 giving the lower and upper limits of the range of permissible values for the function argument.
norm	=TRUE the norm of eigenvectors basis is 1.
class.out	=="fd" basisfd class, ==fdata" fdata class.
basis	"fd" basis object.
lambda	Amount of penalization. Default value is 0, i.e. no penalization is used.
P	If P is a vector: coefficients to define the penalty matrix object. By default $P=c(0,0,1)$ penalize the second derivative (curvature) or acceleration. If P is a matrix: the penalty matrix object.
...	Further arguments passed to or from other methods.

**Value**

basis	basis
x	Is true the value of the rotated data (the centred data multiplied by the rotation matrix) is returned
mean	functional mean of fdataobj
df	degree of freedom
type	type of basis

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ramsay, James O. and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.

N. Kraemer, A.-L. Boulsteix, and G. Tutz (2008). Penalized Partial Least Squares with Applications to B-Spline Transformations and Functional Data. *Chemometrics and Intelligent Laboratory Systems*, 94, 60 - 69. <http://dx.doi.org/10.1016/j.chemolab.2008.06.009>

**See Also**

See Also as [create.basis](#) and [fdata2pc](#).

**Examples**

```

data(tecator)
basis.pc<-create.pc.basis(tecator$absorp.fdata,c(1,4,5))
plot(basis.pc$basis,col=1)
basis.pls<-create.pls.basis(tecator$absorp.fdata,y=tecator$y[,1],c(1,4,5))
lines(basis.pls$basis,col=2)

basis.fd<-create.fdata.basis(tecator$absorp.fdata,c(1,4,5),
type.basis="fourier")
plot(basis.pc$basis)
basis.fdata<-create.fdata.basis(tecator$absorp.fdata,c(1,4,5),
type.basis="fourier",class.out="fdata")
plot(basis.fd,col=2,lty=1)
lines(basis.fdata,col=3,lty=1)

```

CV.S

*The cross-validation (CV) score***Description**

The cross-validation (CV) score.

**Usage**

```
CV.S(y,S,W=NULL,trim=0,draw=FALSE,metric=metric.lp,...)
```

**Arguments**

y	Matrix of set cases with dimension (n x m), where n is the number of curves and m are the points observed in each curve.
S	Smoothing matrix, see <a href="#">S.NW</a> , <a href="#">S.LLR</a> or <a href="#">S.KNN</a> .
W	Matrix of weights.
trim	The alpha of the trimming.
draw	=TRUE, draw the curves, the sample median and trimmed mean.
metric	Metric function, by default <a href="#">metric.lp</a> .
...	Further arguments passed to or from other methods.

**Details**

Compute the leave-one-out cross-validation score.

A.-If trim=0:

$$CV(h) = \frac{1}{n} \sum_{i=1}^n \left( \frac{y_i - r_i(x_i)}{(1 - S_{ii})} \right)^2 w(x_i)$$

$S_{ii}$  is the  $i$ th diagonal element of the smoothing matrix  $S$ .

B.-If trim>0:

$$CV(h) = \frac{1}{l} \sum_{i=1}^l \left( \frac{y_i - r_i(x_i)}{(1 - S_{ii})} \right)^2 w(x_i)$$

$S_{ii}$  is the  $i$ th diagonal element of the smoothing matrix  $S$  and  $l$  the index of  $(1-\text{trim})$  curves with less error.

### Value

res Returns CV score calculated for input parameters.

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

### References

Wasserman, L. *All of Nonparametric Statistics*. Springer Texts in Statistics, 2006.

### See Also

See Also as [min.np](#)  
Alternative method: [GCV.S](#)

### Examples

```
data(tecator)
x<-tecator$absorp.fdata
np<-ncol(x)
tt<-1:np
S1 <- S.NW(tt,3,Ker.epa)
S2 <- S.LLR(tt,3,Ker.epa)
S3 <- S.NW(tt,5,Ker.epa)
S4 <- S.LLR(tt,5,Ker.epa)
cv1 <- CV.S(x, S1)
cv2 <- CV.S(x, S2)
cv3 <- CV.S(x, S3)
cv4 <- CV.S(x, S4)
cv5 <- CV.S(x, S4,trim=0.1,draw=TRUE)
cv1;cv2;cv3;cv4;cv5
S6 <- S.KNN(tt,1,Ker.unif,cv=TRUE)
S7 <- S.KNN(tt,5,Ker.unif,cv=TRUE)
cv6 <- CV.S(x, S6)
cv7 <- CV.S(x, S7)
cv6;cv7
```

dcor.xy

*Distance Correlation Statistic and t-Test***Description**

Distance correlation t-test of multivariate and functional independence (wrapper functions of energy package).

**Usage**

```
dcor.xy(x,y,test=TRUE,metric.x,metric.y,par.metric.x,
par.metric.y,n)
dcor.test(D1,D2,n)
bcdcor.dist(D1,D2,n)
dcor.dist(D1, D2)
```

**Arguments**

x	data (fdata, matrix or data.frame class) of first sample.
y	data (fdata, matrix or data.frame class) of second sample.
test	if TRUE, compute bias corrected distance correlation statistic and the corresponding t-test, else compute distance correlation statistic.
metric.x,metric.y	Name of metric or semi-metric function used for compute the distances of x and y object respectively. By default, <a href="#">metric.lp</a> for functional data and <a href="#">metric.dist</a> for multivariate data.
par.metric.x,par.metric.y	List of parameters for the corresponding metric function.
n	The sample size used in bias corrected version of distance correlation, by default is the number of rows of x.
D1	Distances of first sample.
D2	Distances of second sample.

**Details**

These wrapper functions extend the functions of the energy package for multivariate data to functional data. Distance correlation is a measure of dependence between random vectors introduced by Szekely, Rizzo, and Bakirov (2007).

dcor.xy performs a nonparametric t-test of multivariate or functional independence in high dimension. The distribution of the test statistic is approximately Student t with  $n(n-3)/2 - 1$  degrees of freedom and for  $n \geq 10$  the statistic is approximately distributed as standard normal. Wrapper function of energy: `dcor.ttest`. The t statistic is a transformation of a bias corrected version of distance correlation (see SR 2013 for details). Large values (upper tail) of the t statistic are significant.

dcor.test similar to dcor.xy but only for distance matrix.

`dcor.dist` compute distance correlation statistic. Wrapper function of `energy::dcor` but only for distance matrix.

`bcdcor.dist` compute bias corrected distance correlation statistic. Wrapper function of `energy::bcdcor` but only for distance matrix.

### Value

`dcor.test` returns a list with class `htest` containing

<code>method</code>	description of test
<code>statistic</code>	observed value of the test statistic
<code>parameter</code>	degrees of freedom
<code>estimate</code>	bias corrected distance correlation <code>bcdcor(x, y)</code>
<code>p.value</code>	p-value of the t-test
<code>data.name</code>	description of data

`dcor.xy` returns the previous list with class `htest` and

<code>D1</code>	the distance matrix of <code>x</code>
<code>D2</code>	the distance matrix of <code>y</code>

`dcor.dist` returns the distance correlation statistic.

`bcdcor.dist` returns the bias corrected distance correlation statistic.

### Author(s)

Manuel Oviedo de la Fuente <[manuel.oviedo@usc.es](mailto:manuel.oviedo@usc.es)> and Manuel Febrero Bande

### References

Szekely, G.J. and Rizzo, M.L. (2013). The distance correlation t-test of independence in high dimension. *Journal of Multivariate Analysis*, Volume 117, pp. 193-213.

<http://dx.doi.org/10.1016/j.jmva.2013.02.012>

Szekely, G.J., Rizzo, M.L., and Bakirov, N.K. (2007), Measuring and Testing Dependence by Correlation of Distances, *Annals of Statistics*, Vol. 35 No. 6, pp. 2769-2794.

<http://dx.doi.org/10.1214/009053607000000505>

### See Also

[metric.lp](#) and [metric.dist](#).

### Examples

```
x<-rproc2fdata(100,1:50)
y<-rproc2fdata(100,1:50)
dcor.xy(x, y, test=TRUE)
dx <- metric.lp(x)
dy <- metric.lp(y)
dcor.test(dx, dy)
```



```

bcdcor.dist(dx, dy)
dcor.xy(x, y, test=FALSE)
dcor.dist(dx, dy)

```

---

Depth for a multivariate dataset

*Provides the depth measure for multivariate data*

---

## Description

Compute measure of centrality of the multivariate data. Type of depth function: simplicial depth (SD), Mahalanobis depth (MhD), Random Half-Space depth (HS), random projection depth (RP) and Likelihood Depth (LD).

- The `mdepth.SD` function provides the simplicial depth measure for bivariate data.
- The `mdepth.MhD` function implements a Mahalanobis depth measure.
- The `mdepth.RP` function provides the depth measure using random projections for multivariate data.
- The `mdepth.LD` function provides the Likelihood depth measure for multivariate data.
- The `mdepth.TD` function provides the Tukey depth measure for multivariate data.

## Usage

```

mdepth.SD(x, xx = NULL, scale=FALSE)
mdepth.HS(x, xx=x, proj=50, scale=FALSE, xeps=1e-15, random=FALSE)
mdepth.MhD(x, xx=x, scale=FALSE)
mdepth.RP(x, xx = x, proj = 50, scale=FALSE)
mdepth.TD(x, xx=x, xeps=1e-15, scale=FALSE)
mdepth.LD(x, xx=x, metric=metric.dist, h=NULL, scale=FALSE, ...)

```

## Arguments

<code>x</code>	is a set of points, a d-column matrix.
<code>xx</code>	is a d-dimension multivariate sample, a d-column matrix.
<code>proj</code>	are the directions for random projections, by default 500 random projections generated from a scaled <code>runif(500, -1, 1)</code> .
<code>scale</code>	=TRUE, scale the depth, see <a href="#">scale</a> .
<code>metric</code>	Metric function, by default <code>metric.dist</code> . Distance matrix between <code>x</code> and <code>xx</code> is computed.
<code>xeps</code>	Accuracy. The left limit of the empirical distribution function.
<code>random</code>	=TRUE for random projections. =FALSE for deterministic projections.
<code>h</code>	Bandwidth, $h > 0$ . Default argument values are provided as the 15%-quantile of the distance between <code>x</code> and <code>xx</code> .
<code>...</code>	Further arguments passed to or from other methods.

## Details

Type of depth measures,

- The `mdepth.SD` calculates the simplicial depth (HD) of the points in  $x$  w.r.t.  $xx$ .
- The `mdepth.HS` function calculates the random half-space depth (HS) of the points in  $x$  w.r.t.  $xx$  based on random projections `proj`.
- The `mdepth.MhD` function calculates the Mahalanobis depth (MhD) of the points in  $x$  w.r.t.  $xx$ .
- The `mdepth.RP` calculates the random projection depth (RP) of the points in  $x$  w.r.t.  $xx$  based on random projections `proj`.
- The `mdepth.LD` calculates the Likelihood depth (LD) of the points in  $x$  w.r.t.  $xx$ .

## Value

<code>lmed</code>	Index of deepest element median of $xx$ .
<code>ltrim</code>	Index of set of points $x$ with trimmed mean <code>mtrim</code> .
<code>dep</code>	Depth of each point $x$ w.r.t. $xx$ .
<code>proj</code>	The projection value of each point on set of points.

## Author(s)

`mdepth.RP`, `mdepth.MhD` and `mdepth.HS` are versions created by Manuel Febrero Bande and Manuel Oviedo de la Fuente of the original version created by Jun Li, Juan A. Cuesta Albertos and Regina Y. Liu for polynomial classifier.

## References

Liu, R. Y., Parelius, J. M., and Singh, K. (1999). Multivariate analysis by data depth: descriptive statistics, graphics and inference,(with discussion and a rejoinder by Liu and Singh). *The Annals of Statistics*, 27(3), 783-858.

## See Also

Functional depth functions: `depth.FM`, `depth.mode`, `depth.RP`, `depth.RPD` and `depth.RT`.

## Examples

```
data(iris)
group<-iris[,5]
x<-iris[,1:2]

MhD<-mdepth.MhD(x)
PD<-mdepth.RP(x)
HD<-mdepth.HS(x)
SD<-mdepth.SD(x)

x.setosa<-x[group=="setosa",]
x.versicolor<-x[group=="versicolor",]
x.virginica<-x[group=="virginica",]
```

```
d1<-mdepth.SD(x,x.setosa)$dep
d2<-mdepth.SD(x,x.versicolor)$dep
d3<-mdepth.SD(x,x.virginica)$dep
```

---

Depth for multivariate fdata

*Provides the depth measure for a list of p–functional data objects*

---

### Description

This function computes the depth measure for a list of p–functional data objects. The procedure extends the Fraiman and Muniz (FM), modal, and random project depth functions from 1 functional dataset to p functional datasets.

### Usage

```
depth.FMp(lfdata, lfdataref = lfdata, trim = 0.25,
dfunc = "mdepth.MhD",par.dfunc = list(scale = FALSE),
draw = FALSE, ask = FALSE, ...)

depth.RPp(lfdata, lfdataref = lfdata, nproj = 50,
proj = "vexponential",trim = 0.25, dfunc = "mdepth.TD",
par.dfunc = list(scale = TRUE), draw = FALSE,ask = FALSE)

depth.modep(lfdata, lfdataref = lfdata, h = NULL, metric,
par.metric = list(), method = "euclidean", scale = FALSE,
trim = 0.25, draw = FALSE,ask = FALSE)
```

### Arguments

lfdata	A list of new curves (list of fdata ojects) to evaluate the depth.
lfdataref	A set of reference curves (list of fdata ojects) w.r.t. the depth of lfdata is computed.
trim	The alpha of the trimming.
dfunc	Type of multivariate depth (of order p) function used in Framiman and Muniz depth, depth.FMp or in Random Projection depth,depth.FMp: : <ul style="list-style-type: none"> <li>• The <a href="#">mdepth.SD</a> function provides the simplicial depth measure for bivariate data.</li> <li>• The <a href="#">mdepth.LD</a> function provides the Likelihood depth measure based on Nadaraya–Watson estimator of empirical density function.</li> <li>• The <a href="#">mdepth.HS</a> function implements a half-space depth measure based on random projections.</li> <li>• The <a href="#">mdepth.TD</a> function implements a Tukey depth measure.</li> </ul>

- The `mdepth.MhD` function implements a Mahalanobis depth measure.
- The `mdepth.RP` function provides the depth measure using random projections for multivariate data.

<code>par.dfunc</code>	list of parameters for the <code>dfunc</code> depth function, see <a href="#">Depth.Multivariate</a> .
<code>nproj</code>	The number projection.
<code>proj</code>	if is a character: create the random projection using a covariance matrix by process indicated in the argument (by default, <code>proj=1</code> , <code>sigma=diag(ncol(fdataobj))</code> ), else if is a matrix of random projection provided by the user.
<code>h</code>	Bandwidth, $h > 0$ . Default argument values are provided as the 15%-quantile of the distance between <code>fdataobj</code> and <code>fdataori</code> .
<code>metric</code>	Metric or semi-metric function used for compute the distance between each element in <code>ldata</code> w.r.t. <code>ldataref</code> , by default <a href="#">metric.lp</a> .
<code>par.metric</code>	list of parameters for the metric function.
<code>method</code>	Type of the distance measure (by default <code>euclidean</code> ) to compute the metric between the $p$ -distance matrix computed from the $p$ functional data elements.
<code>scale</code>	=TRUE, scale the depth.
<code>draw</code>	=TRUE, draw the curves, the sample median and trimmed mean.
<code>ask</code>	logical. If TRUE (and the R session is interactive) the user is asked for input, before a new figure is drawn.
<code>...</code>	Further arguments passed to or from other methods.

### Details

- `depth.FMp`, this procedure supposes that each curve of the `ldataobj` have the same support  $[0, T]$  (same `argvals` and `rangeval`). The FMp depth is defined as:  $FM_i^p = \int_0^T Z_i^p(t) dt$  where  $Z_i^p(t)$  is a  $p$ -variate depth of the vector  $(x_i^1(t), \dots, x_i^p(t))$  w.r.t. the sample at  $t$ .
- The `depth.RPp` function calculates the depth in two steps. It builds random projections for the each curve of the `ldata` w.r.t. each curve of the `ldataref` object. Then it applies a multivariate depth function specified in `dfunc` argument to the set of random projections. This procedure is a generalization of Random Projection with derivatives (RPD) implemented in `depth.RPD` function. Now, the procedure computes a  $p$ -variate depth with the projections using the  $p$  functional dataset.
- The modal depth `depth.modep` function calculates the depth in three steps. First, the function calculates a suitable metrics or semi-metrics  $m_1 + \dots + m_p$  for each curve of the `ldata` w.r.t. each curve in the `ldataref` object using the `metric` and `par.metric` arguments, see [metric.lp](#) or [semimetric.NPFDA](#) for more details. Second, the function uses the  $p$ -dimensional metrics to construct a new `metric`, specified in `method` argument, by default if `method="euclidean"`, i.e.  $m := \sqrt{m_1^2 + \dots + m_p^2}$ . Finally, the empirical  $h$ -depth is computed as:

$$\hat{f}_h(x_0) = N^{-1} \sum_{i=1}^N K(m/h)$$

where  $x$  is dataset with  $p$  observed functional data,  $m$  is a suitable metric or semi-metric,  $K(t)$  is an asymmetric kernel function and  $h$  is the bandwidth parameter.

**Value**

lmed	Index deepest element median.
ltrim	Index of curves with trimmed mean mtrim.
dep	Depth of each curve of fdataobj w.r.t. fdataori.
dfunc	second depth function used as multivariate depth, see details section.
par.dfunc	list of parameters for the dfunc depth function.
proj	The projection value of each point on the curves.
dist	Distance matrix between curves or functional data.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Cuevas, A., Febrero-Bande, M. and Fraiman, R. (2007). *Robust estimation and classification for functional data via projection-based depth notions*. Computational Statistics 22, 3, 481-496.

**See Also**

See Also as [Descriptive](#).

**Examples**

```
## Not run:
data(tecator)
xx<-tecator$absorp
xx1<-fdata.deriv(xx,1)
lx<-list(xx=xx,xx=xx1)
# Fraiman-Muniz Depth
par.df<-list(scale =TRUE)
out.FM1p=depth.FMp(lx,trim=0.1,draw=TRUE, par.dfunc = par.df)
out.FM2p=depth.FMp(lx,trim=0.1,dfunc="mdepth.LD",
par.dfunc = par.df, draw=TRUE)

# Random Project Depth
out.RP1p=depth.RPp(lx,trim=0.1,dfunc="mdepth.TD",
draw=TRUE,par.dfunc = par.df)
out.RP2p=depth.RPp(lx,trim=0.1,dfunc="mdepth.LD",
draw=TRUE,par.dfunc = par.df)

#Modal Depth
out.mode1p=depth.modep(lx,trim=0.1,draw=T,scale=T)
out.mode2p=depth.modep(lx,trim=0.1,method="manhattan",
draw=T,scale=T)

par(mfrow=c(2,3))
plot(out.FM1p$dep,out.FM2p$dep)
plot(out.RP1p$dep,out.RP2p$dep)
```

```

plot(out.mode1p$dep,out.mode2p$dep)
plot(out.FM1p$dep,out.RP1p$dep)
plot(out.RP1p$dep,out.mode1p$dep)
plot(out.FM1p$dep,out.mode1p$dep)

## End(Not run)

```

---

Depth for univariate fdata

*Provides the depth measure for functional data*

---

### Description

Compute measure of centrality of the functional data. Type of depth function: Fraiman and Muniz (FM) depth, modal depth, random tukey (RT), random project (RP) depth and double random project depth (RPD).

- `depth.FM` computes the integration of an univariate depth along the axis  $x$  (see Fraiman and Muniz 2001). It is also known as Integrated Depth.
- `depth.mode` implements the modal depth (see Cuevas et al 2007).
- `depth.RT` implements the Random Tukey depth (see Cuesta–Albertos and Nieto–Reyes 2008).
- `depth.RP` computes the Random Projection depth (see Cuevas et al. 2007).
- `depth.RPD` implements a depth measure based on random projections possibly using several derivatives (see Cuevas et al. 2007).
- `depth.FSD` computes the Functional Spatial Depth (see Sguera et al. 2014).
- `depth.KFSD` implements the Kernelized Functional Spatial Depth (see Sguera et al. 2014).

### Usage

```
depth.FM(fdataobj, fdataori=fdataobj, trim=0.25, scale=FALSE,
dfunc="FM1", par.dfunc = list(scale = TRUE), draw = FALSE)
```

```
depth.mode(fdataobj, fdataori=fdataobj, trim=0.25,
metric=metric.lp, h=NULL, scale=FALSE, draw=FALSE, ...)
```

```
depth.RT(fdataobj, fdataori=fdataobj, trim = 0.25, nproj = 10,
proj = 1, xeps = 1e-07, draw = FALSE, ...)
```

```
depth.RP(fdataobj, fdataori=fdataobj, trim=0.25, nproj=50,
proj="vexponential", dfunc="TD1", par.dfunc=list(), scale=FALSE,
draw=FALSE, ...)
```

```
depth.RPD(fdataobj, fdataori=fdataobj, nproj=50, proj=1, deriv=c(0,1),
trim=0.25, dfunc2=depth.mode, method="fmm", draw=FALSE, ...)
```

```
depth.FSD(fdataobj, fdataori = fdataobj,
          trim = 0.25, scale = FALSE, draw = FALSE)
```

```
depth.KFSD(fdataobj, fdataori = fdataobj, trim = 0.25,
           h=NULL, scale = FALSE, draw = FALSE)
```

### Arguments

<code>fdataobj</code>	A set of new curves to evaluate the depth. <a href="#">fdata</a> class object.
<code>fdataori</code>	A set of original curves where the depth is computed. <a href="#">fdata</a> class object.
<code>trim</code>	The alpha of the trimming.
<code>nproj</code>	The number of projections.
<code>proj</code>	matrix or <a href="#">fdata</a> class object with a random projection provided by the user. If is a character: create the random projection using a covariance matrix by process indicated in the argument. Otherwise, it is a sigma parameter of <a href="#">rproc2fdata</a> function.
<code>dfunc</code>	type of univariate depth function used inside depth function: "FM1" refers to the original Fraiman and Muniz univariate depth (default), "TD1" Tukey (Half-space), "Liu1" for simplicial depth, "LD1" for Likelihood depth and "MhD1" for Mahalanobis 1D depth. Also, any user function fulfilling the following pattern <i>FUN.USER(x,xx,...)</i> and returning a dep component can be included.
<code>par.dfunc</code>	List of parameters for <i>dfunc</i> .
<code>dfunc2</code>	Second depth function in RPD depth, by default <a href="#">depth.mode</a> .
<code>deriv</code>	Number of derivatives described in integer vector <code>deriv</code> . =0 means no derivative.
<code>method</code>	Type of derivative method. See <a href="#">fdata.deriv</a> for more details.
<code>h</code>	Bandwidth parameter. <ol style="list-style-type: none"> <li>1. If <code>h</code> is a numerical value, the procedure considers the argument value as bandwidth.</li> <li>2. If is NULL (by default) the bandwidth is provided as the 15%-quantile of the distance between <code>fdataobj</code> and <code>fdataori</code>.</li> <li>3. If <code>h</code> is a character string (like "q-0.15"), the procedure reads the numeric value from the third position of the character to the end and uses it to compute the quantile of the distance between <code>fdataobj</code> and <code>fdataori</code> (as in the second case).</li> </ol>
<code>metric</code>	Metric function, by default <a href="#">metric.lp</a> . Distance matrix between <code>fdataobj</code> and <code>fdataori</code> .
<code>scale</code>	=TRUE, scale the depth.
<code>xeps</code>	Accuracy. The left limit of the empirical distribution function.
<code>draw</code>	=TRUE, draw the curves, the sample median and trimmed mean.
<code>...</code>	Further arguments passed to or from other methods.

## Details

- The modal depth `depth.mode` function calculates the depth of a datum accounting the number of curves in its neighbourhood. By default, the distance is calculated using `metric.lp` function although any other distance could be employed through argument `metric` (with the general pattern `USER.DIST(fdataobj, fdataori)`).
- The `depth.RP` function summarizes the random projections through averages whereas the `depth.RT` function uses the minimum of all projections.
- The `depth.RPD` function involves the original trajectories and the derivatives of each curve in two steps. It builds random projections for the function and their derivatives (indicated in the parameter `deriv`) and then applies a depth function (by default `depth.mode`) to this set of random projections (by default the Tukey one).
- The `depth.FSD` and `depth.KFSD` are the implementations of the default versions of the functional spatial depths proposed in Sguera et al 2014. At this moment, it is not possible to change the kernel in the second one.

## Value

<code>median</code>	Deepest curve.
<code>lmed</code>	Index deepest element median.
<code>mtrim</code>	<code>fdata</code> class object with the average from the $(1-\text{trim})\%$ deepest curves.
<code>ltrim</code>	Indexes of curves that conform the trimmed mean <code>mtrim</code> .
<code>dep</code>	Depth of each curve of <code>fdataobj</code> w.r.t. <code>fdataori</code> .
<code>dep.ori</code>	Depth of each curve of <code>fdataori</code> w.r.t. <code>fdataori</code> .
<code>proj</code>	The projection value of each point on the curves.
<code>dist</code>	Distance matrix between curves or functional data.

## Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

## References

- Cuevas, A., Febrero-Bande, M., Fraiman, R. (2007). Robust estimation and classification for functional data via projection-based depth notions. *Computational Statistics* 22, 3, 481-496.
- Fraiman R, Muniz G. (2001). Trimmed means for functional data. *Test* 10: 419-440.
- Cuesta-Albertos, JA, Nieto-Reyes, A. (2008) The Random Tukey Depth. *Computational Statistics and Data Analysis* Vol. 52, Issue 11, 4979-4988.
- Febrero-Bande, M, Oviedo de la Fuente, M. (2012). Statistical Computing in Functional Data Analysis: The R Package `fda.usc`. *Journal of Statistical Software*, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>
- Sguera C, Galeano P, Lillo R (2014). Spatial depth based classification for functional data. *TEST* 23(4):725–750.



**See Also**

See Also as [Descriptive](#).

**Examples**

```
## Not run:
#Ex: CanadianWeather data
tt=1:365
fdataobj<-fdata(t(CanadianWeather$dailyAv[, , 1]), tt)
# Fraiman-Muniz Depth
out.FM=depth.FM(fdataobj, trim=0.1, draw=TRUE)
#Modal Depth
out.mode=depth.mode(fdataobj, trim=0.1, draw=TRUE)
out.RP=depth.RP(fdataobj, trim=0.1, draw=TRUE)
out.RT=depth.RT(fdataobj, trim=0.1, draw=TRUE)
out.FSD=depth.FSD(fdataobj, trim=0.1, draw=TRUE)
out.KFSD=depth.KFSD(fdataobj, trim=0.1, draw=TRUE)
## Double Random Projections
out.RPD=depth.RPD(fdataobj, deriv=c(0, 1), dfunc2=depth.FM,
trim=0.1, draw=TRUE)
out<-c(out.FM$mtrim, out.mode$mtrim, out.RP$mtrim, out.RPD$mtrim)
plot(fdataobj, col="grey")
lines(out)
cdep<-cbind(out.FM$dep, out.mode$dep, out.RP$dep, out.RT$dep, out.FSD$dep, out.KFSD$dep)
colnames(cdep)<-c("FM", "mode", "RP", "RT", "FSD", "KFSD")
pairs(cdep)
round(cor(cdep), 2)

## End(Not run)
```

---

Descriptive

*Descriptive measures for functional data.*

---

**Description**

Central and dispersion measures for functional data.

**Usage**

```
## S3 method for class 'formula'
func.mean(formula, data = NULL, ..., drop=FALSE)
func.mean(fdataobj)
func.var(fdataobj)
func.trim.FM(fdataobj, ...)
func.trim.mode(fdataobj, ...)
func.trim.RP(fdataobj, ...)
func.trim.RT(fdataobj, ...)
func.trim.RPD(fdataobj, ...)
```

```

func.med.FM(fdataobj,...)
func.med.mode(fdataobj,...)
func.med.RP(fdataobj,...)
func.med.RT(fdataobj,...)
func.med.RPD(fdataobj,...)
func.trimvar.FM(fdataobj,...)
func.trimvar.mode(fdataobj,...)
func.trimvar.RP(fdataobj,...)
func.trimvar.RT(fdataobj,...)
func.trimvar.RPD(fdataobj,...)

```

### Arguments

formula	a formula, such as $y \sim \text{group}$ , where $y$ is a <code>fdata</code> object to be split into groups according to the grouping variable <code>group</code> (usually a factor).
data	List that containing the variables in the formula. The item called " <i>df</i> " is a data frame with the grouping variable. The item called " <i>y</i> " is a <code>fdata</code> object.
drop	logical indicating if levels that do not occur should be dropped (if $f$ is a factor or a list).
fdataobj	<code>fdata</code> class object.
...	Further arguments passed to or from other methods. If the argument <code>p</code> is passed, it used <code>metric.lp</code> function, by default <code>p=2</code> . If the argument <code>trim</code> (alpha of the trimming) is passed, it used <code>metric.lp</code> function. If the argument <code>deriv</code> (number of derivatives to use) is passed. This parameter is used in <code>depth.RPD</code> function, by default it uses <code>deriv = (0, 1)</code> .

### Value

`func.mean.formula` The value returned from `split` is a list of `fdata` containing the mean curves for the groups. The components of the list are named by the levels of  $f$  (after converting to a factor, or if already a factor and `drop = TRUE`, dropping unused levels).

`func.mean` gives mean curve.

`func.var` gives variance curve.

`func.trim.FM` Returns the average from the  $(1-\text{trim})\%$  deepest curves following FM criteria.

`func.trim.mode` Returns the average from the  $(1-\text{trim})\%$  deepest curves following mode criteria.

`func.trim.RP` Returns the average from the  $(1-\text{trim})\%$  deepest curves following RP criteria.

`func.trim.RT` Returns the average from the  $(1-\text{trim})\%$  deepest curves following RT criteria.

`func.trim.RPD` Returns the average from the  $(1-\text{trim})\%$  deepest curves following RPD criteria.

`func.med.FM` Returns the deepest curve following FM criteria.

`func.med.mode` Returns the deepest curve following mode criteria.

`func.med.RP` Returns the deepest curve following RP criteria.

`func.med.RPD` Returns the deepest curve following RPD criteria.

`func.trimvar.FM` Returns the marginal variance from the deepest curves following FM criteria.

`func.trimvar.mode` Returns the marginal variance from the deepest curves following mode criteria.

`func.trimvar.RP` Returns the marginal variance from the deepest curves following RP criteria.  
`func.trimvar.RT` Returns the marginal variance from the deepest curves following RT criteria.  
`func.trimvar.RPD` Returns the marginal variance from the deepest curves following RPD criteria.

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

### References

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

### Examples

```
# Example with Montreal Daily Temperature (fda-package)
fdataobj<-fdata(MontrealTemp)

# Measures of central tendency by group
fac<-factor(c(rep(1,len=17),rep(2,len=17)))
ldata=list("df"=data.frame(fac),"fdataobj"=fdataobj)
a1<-func.mean.formula(fdataobj~fac,data=ldata)
plot(a1)

## Not run:
# Measures of central tendency
a1<-func.mean(fdataobj)
a2<-func.trim.FM(fdataobj)
a3<-func.trim.mode(fdataobj)
a4<-func.trim.RP(fdataobj)
# a5<-func.trim.RPD(fdataobj,deriv=c(0,1)) # Time-consuming
a6<-func.med.FM(fdataobj)
a7<-func.med.mode(fdataobj)
a8<-func.med.RP(fdataobj)
# a9<-func.med.RPD(fdataobj,deriv=c(0,1)) # Time-consuming
# a10<-func.med.RT(fdataobj)

dev.new()
par(mfrow=c(1,2))
plot(c(a1,a2,a3,a4),ylim=c(-26,29),main="Central tendency: trimmed mean")
plot(c(a1,a6,a7,a8),ylim=c(-26,29),main="Central tendency: median")

## Measures of dispersion
b1<-func.var(fdataobj)
b2<-func.trimvar.FM(fdataobj)
b3<-func.trimvar.FM(fdataobj,trim=0.1)
b4<-func.trimvar.mode(fdataobj)
b5<-func.trimvar.mode(fdataobj,p=1)
```

```

b6<-func.trimvar.RP(fdataobj)
b7<-func.trimvar.RPD(fdataobj)
b8<-func.trimvar.RPD(fdataobj)
b9<-func.trimvar.RPD(fdataobj,deriv=c(0,1))
dev.new()
par(mfrow=c(1,2))
plot(c(b1,b2,b3,b4,b5),ylim=c(0,79),main="Measures of dispersion I")
plot(c(b1,b6,b7,b8,b9),ylim=c(0,79),main="Measures of dispersion II")

## End(Not run)

```

---

dev.S

*The deviance score .*


---

### Description

Returns the deviance of a fitted model object by GCV score.

### Usage

```

dev.S(y, S, obs, family = gaussian(), off, offdf, criteria="GCV",
W = diag(1, ncol = ncol(S), nrow = nrow(S)), trim = 0,
draw = FALSE, ...)

```

### Arguments

y	Matrix of set cases with dimension (n x m), where n is the number of curves and m are the points observed in each curve.
obs	observed response.
S	Smoothing matrix.
family	a description of the error distribution and link function to be used in the model. This can be a character string naming a family function, a family function or the result of a call to a family function. (See <a href="#">family</a> for details of family functions.)
off	off
offdf	off, degrees of freedom
criteria	The penalizing function. By default "Rice" criteria. Possible values are "GCV", "AIC", "FPE", "Shibata", "Rice".
W	Matrix of weights.
trim	The alpha of the trimming.
draw	=TRUE, draw the curves, the sample median and trimmed mean.
...	Further arguments passed to or from other methods.

**Details**

up to a constant, minus twice the maximized log-likelihood. Where sensible, the constant is chosen so that a saturated model has deviance zero.

$$GCV(h) = p(h)\Xi(n^{-1}h^{-1})$$

Where

$$p(h) = \frac{1}{n} \sum_{i=1}^n \left( y_i - r_i(x_i) \right)^2 w(x_i)$$

and penalty function

$$\Xi()$$

can be selected from the following criteria:

Generalized Cross-validation (GCV):

$$\Xi_{GCV}(n^{-1}h^{-1}) = (1 - n^{-1}S_{ii})^{-2}$$

Akaike's Information Criterion (AIC):

$$\Xi_{AIC}(n^{-1}h^{-1}) = \exp(2n^{-1}S_{ii})$$

Finite Prediction Error (FPE)

$$\Xi_{FPE}(n^{-1}h^{-1}) = \frac{(1 + n^{-1}S_{ii})}{(1 - n^{-1}S_{ii})}$$

Shibata's model selector (Shibata):

$$\Xi_{Shibata}(n^{-1}h^{-1}) = (1 + 2n^{-1}S_{ii})$$

Rice's bandwidth selector (Rice):

$$\Xi_{Rice}(n^{-1}h^{-1}) = (1 - 2n^{-1}S_{ii})^{-1}$$

**Value**

res Returns GCV score calculated for input parameters.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

- Wasserman, L. *All of Nonparametric Statistics*. Springer Texts in Statistics, 2006.
- Hardle, W. *Applied Nonparametric Regression*. Cambridge University Press, 1994.
- Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

**See Also**

See Also as [GCV.S](#).  
Alternative method: [CV.S](#)

**Examples**

```
data(phoneme)
mlearn<-phoneme$learn
np<-ncol(mlearn)
tt<-mlearn[["argvals"]]
S1 <- S.NW(tt,2.5)
gcv1 <- dev.S(mlearn$data[1,],obs=(sample(150)),
S1,off=rep(1,150),offdf=3)
gcv2 <- dev.S(mlearn$data[1,],obs=sort(sample(150)),
S1,off=rep(1,150),offdf=3)
```

---

dfv.test

*Delsol, Ferraty and Vieu test for no functional-scalar interaction*


---

**Description**

The function `dfv.test` tests the null hypothesis of no interaction between a functional covariate and a scalar response in a general framework. The null hypothesis is

$$H_0 : m(X) = 0,$$

where  $m(\cdot)$  denotes the regression function of the functional variate  $X$  over the centred scalar response  $Y$  ( $E[Y] = 0$ ). The null hypothesis is tested by the smoothed integrated square error of the response (see Details).

**Usage**

```
dfv.statistic (X.fdata, Y, h=quantile(x=metric.lp(X.fdata),
  probs=c(0.05,0.10,0.15,0.25,0.50)),
  K=function(x)2*dnorm(abs(x)),
  weights=rep(1,dim(X.fdata$data)[1]),d=metric.lp,
  dist=NULL)
```

```
dfv.test (X.fdata, Y, B=5000, h=quantile(x=metric.lp(X.fdata),
  probs=c(0.05,0.10,0.15,0.25,0.50)),
  K=function(x)2*dnorm(abs(x)),
  weights=rep(1,dim(X.fdata$data)[1]),d=metric.lp,
  verbose=TRUE)
```

**Arguments**

X.fdata	Functional covariate. The object must be in the class <code>fdata</code> .
Y	Scalar response. Must be a vector with the same number of elements as functions are in X.fdata.
h	Bandwidth parameter for the kernel smoothing. This is a crucial parameter that affects the power performance of the test. One possibility to choose it is considering the Cross-validators bandwidth of the nonparametric functional regression, given by the function <code>fregre.np</code> (see Examples). Other possibility is to consider a grid of bandwidths. This is the default option, considering the grid given by the quantiles 0.05, 0.10, 0.15, 0.25 and 0.50 of the functional $L^2$ distances of the data.
B	Number of bootstrap replicates to calibrate the distribution of the test statistic. B=5000 replicates are the recommended for carry out the test, although for exploratory analysis ( <b>not inferential</b> ), an acceptable less time-consuming option is B=500.
K	Kernel function. If no specified it is taken to be the rescaled right part of the normal density.
weights	A vector of weights for the sample data. The default is the uniform weights <code>rep(1,dim(X.fdata\$data)[1])</code> .
d	Semimetric to use in the kernel smoothers. By default is the $L^2$ distance given by <code>metric.lp</code> .
dist	Matrix of distances of the functional data, used to save time in the bootstrap calibration. If not given, the matrix is automatically computed using the semimetric d.
verbose	Either to show or not information about computing progress.

**Details**

The Delsol, Ferraty and Vieu statistic is defined as

$$T_n = \int \left( \sum_{i=1}^n (Y_i - m(X_i)) K \left( \frac{d(X, X_i)}{h} \right) \right)^2 \omega(X) dP_X(X)$$

and in the case of no interaction with centred scalar response (when  $H_0 : m(X) = 0$  holds), its sample version is computed from

$$T_n = \frac{1}{n} \sum_{j=1}^n \left( \sum_{i=1}^n Y_i K \left( \frac{d(X_j, X_i)}{h} \right) \right)^2 \omega(X_j).$$

The sample version implemented here does not consider a splitting of the sample, as the authors comment in their paper. The statistic is computed by the function `dfv.statistic` and, before applying the test, the response  $Y$  is centred. The distribution of the test statistic is approximated by a wild bootstrap on the residuals, using the *golden section bootstrap*.

Please note that if a grid of bandwidths is passed, a harmless warning message will prompt at the end of the test (it comes from returning several p-values in the `hstest` class).

## Value

The value of `dfv.statistic` is a vector of length `length(h)` with the values of the statistic for each bandwidth. The value of `dfv.test` is an object with class "hstest" whose underlying structure is a list containing the following components:

<code>statistic</code>	The value of the Delsol, Ferraty and Vieu test statistic.
<code>boot.statistics</code>	A vector of length <code>B</code> with the values of the bootstrap test statistics.
<code>p.value</code>	The p-value of the test.
<code>method</code>	The character string "Delsol, Ferraty and Vieu test for no functional-scalar interaction".
<code>B</code>	The number of bootstrap replicates used.
<code>h</code>	Bandwidth parameters for the test.
<code>K</code>	Kernel function used.
<code>weights</code>	The weights considered.
<code>d</code>	Matrix of distances of the functional data.
<code>data.name</code>	The character string "Y=0+e"

## Note

No NA's are allowed neither in the functional covariate nor in the scalar response.

## Author(s)

Eduardo Garcia-Portugues. Please, report bugs and suggestions to `<egarcia@math.ku.dk>`

## References

- Delsol, L., Ferraty, F. and Vieu, P. (2011). Structural test in regression on functional variables. *Journal of Multivariate Analysis*, 102, 422-447. <http://dx.doi.org/10.1016/j.jmva.2010.10.003>
- Delsol, L. (2013). No effect tests in regression on functional variable and some applications to spectrometric studies. *Computational Statistics*, 28(4), 1775-1811. <http://dx.doi.org/10.1007/s00180-012-0378-1>



**See Also**

[rwild](#), [flm.test](#), [flm.Ftest](#), [fregre.np](#)

**Examples**

```
## Simulated example ##

X=rproc2fdata(n=50,t=seq(0,1,l=101),sigma="OU")

beta0=fdata(mdata=rep(0,length=101)+rnorm(101,sd=0.05),
  argvals=seq(0,1,l=101),rangeval=c(0,1))
beta1=fdata(mdata=cos(2*pi*seq(0,1,l=101))-(seq(0,1,l=101)-0.5)^2+
  rnorm(101,sd=0.05),argvals=seq(0,1,l=101),rangeval=c(0,1))

# Null hypothesis holds
Y0=drop(inprod.fdata(X,beta0)+rnorm(50,sd=0.1))

# Null hypothesis does not hold
Y1=drop(inprod.fdata(X,beta1)+rnorm(50,sd=0.1))

# We use the CV bandwidth given by fregre.np
# Do not reject H0
dfv.test(X,Y0,h=fregre.np(X,Y0)$h.opt,B=100)
# dfv.test(X,Y0,B=5000)

# Reject H0
dfv.test(X,Y1,B=100)
# dfv.test(X,Y1,B=5000)
```

**Description**

Computes the cosine correlation distance between two functional dataset.

**Usage**

```
dis.cos.cor(fdata1,fdata2=NULL,as.dis=FALSE)
```

**Arguments**

fdata1	Functional data 1 or curve 1.
fdata2	Functional data 2 or curve 2.
as.dis	Returns the distance matrix from class dist.

**Details**

Computes the cosine correlation distance between two functional dataset of class fdata.

**Value**

Returns a proximities matrix between functional data.

**References**

Kemmeren P, van Berkum NL, Vilo J, et al. (2002). *Protein Interaction Verification and Functional Annotation by Integrated Analysis of Genome-Scale Data* . Mol Cell. 2002 9(5):1133-43.

**See Also**

See also [metric.lp](#) and [semimetric.NPFDA](#)

**Examples**

```
r1<-rnorm(1001,sd=.01)
r2<-rnorm(1001,sd=.01)
x<-seq(0,2*pi,length=1001)
fx<-fdata(sin(x)/sqrt(pi)+r1,x)
dis.cos.cor(fx,fx)
dis.cos.cor(c(fx,fx),as.dis=TRUE)
fx0<-fdata(rep(0,length(x))+r2,x)
plot(c(fx,fx0))
dis.cos.cor(c(fx,fx0),as.dis=TRUE)
```

---

fda.usc.internal

*fda.usc internal functions*

---

**Description**

Internal undocumented functions for fda.usc package.

**Usage**

```
## S3 method for class 'fdata'
fdataobj[i = TRUE, j = TRUE,drop=FALSE]
## S3 method for class 'fdist'
fdataobj[i = TRUE, j = TRUE,drop=FALSE]
## S3 method for class 'fdata'
fdata1 == fdata2
## S3 method for class 'fdata'
fdata1 != fdata2
## S3 method for class 'fdata'
```

```

fdata1 + fdata2
## S3 method for class 'fdata'
fdata1 - fdata2
## S3 method for class 'fdata'
fdata1 * fdata2
## S3 method for class 'fdata'
fdata1 / fdata2
## S3 method for class 'fdata'
fdataobj ^ pot
## S3 method for class 'fdata'
c(...)
## S3 method for class 'fdata'
dim(x)
## S3 method for class 'fdata'
nrow(x)
## S3 method for class 'fdata'
ncol(x)
## S3 method for class 'fdata'
NROW(x)
## S3 method for class 'fdata'
NCOL(x)
## S3 method for class 'fdata'
length(x)
## S3 method for class 'fdata'
is(fdataobj)
## S3 method for class 'fdata'
omit(fdataobj,y=NULL)
## S3 method for class 'fdata'
omit2(fdataobj,index.na=FALSE)
## S3 method for class 'fdata'
missing(fdataobj,basis=NULL)
## S3 method for class 'fdata'
is.na(x)
## S3 method for class 'fdata'
anyNA(x, recursive = FALSE)
count.na.fdata( x )
argvals(fdataobj)
rangeval(fdataobj)

```

### Arguments

x, fdataobj, fdata1, fdata2	fdata class object.
i, j	Indices specifying elements to extract, replace. Indices are numeric or character vectors or empty
pot	Numeric value for exponentiation.
drop	For fdata class object. If TRUE the result is coerced to the lowest possible

	dimension of element data. This only works for extracting elements, not for the replacement.
...	fdata objects to be concatenated.
y	Vector
basis	fd basis
index.na	Return the index of NA elements
recursive	should anyNA be applied recursively to lists and pairlists?

### Note

In "Ops" functions "+.fdata", "-.fdata", "\*.fdata" and "/.fdata": The lengths of the objects fdata1 and fdata2 may be different because operates recycled into minimum size as necessary.

### References

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

---

fdata	<i>Converts raw data or other functional data classes into fdata class.</i>
-------	---

---

### Description

Create a functional data object of class fdata from (matrix, data.frame, numeric, integer, fd, fds, fts or sfts) class data.

### Usage

```
fdata(mdata, argvals=NULL, rangeval=NULL, names=NULL, fdata2d=FALSE)
```

### Arguments

mdata	Matrix of set cases with dimension (n x m), where n is the number of curves and m are the points observed in each curve.
argvals	Argvals, by default: 1:m.
rangeval	(optional) Range of discretization points, by default: range(argvals).
names	(optional) list with tree components: main an overall title, xlab title for x axis and ylab title for y axis.
fdata2d	TRUE class fdata2d, the functional data is observed in at least a two grids (the argvals is a list of vectors). By default fdata2d=FALSE the functional data is observed in a single grid (the argvals is a vector).

**Value**

Return fdata class object with:

- "data": matrix of set cases with dimension (n x m), where n is the number of curves and m are the points observed in each curve
- "rangeval": the discretizations points values, if not provided: 1:m
- "rangeval": range of the discretizations points values, by default: range(argvals)
- "names": (optional) list with main an overall title, xlab title for x axis and ylab title for y axis.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

**See Also**

See Also as [plot.fdata](#)

**Examples**

```
data(phoneme)
mlearn<-phoneme$learn[1:4,1:150]
# Center curves
fdata.c=fdata.cen(mlearn)$xcen
par(mfrow=c(2,1))
plot.fdata(mlearn,type="l")
plot.fdata(fdata.c,type="l")

# Convert from class fda to fdata
bsp1 <- create.bspline.basis(c(1,150),21)
fd1 <- Data2fd(1:150,y=t(mlearn$data),basisobj=bsp1)
fdataobj=fdata(fd1)

# Convert from class fds, fts or sfts to fdata
#require(fds)
#a=fds(x = 1:20, y = Simulationdata$y, xname = "x",
# yname = "Simulated value")
#b=fts(x = 15:49, y = Australiasmoothfertility$y, xname = "Age",
# yname = "Fertility rate")
#c=sfts(ts(as.numeric(ElNino$y), frequency = 12), xname = "Month",
#yname = "Sea surface temperature")
#class(a);class(b);class(c)
#fdataobj=fdata(b)
```

---

fdata.bootstrap	<i>Bootstrap samples of a functional statistic</i>
-----------------	--

---

## Description

fdata.bootstrap provides bootstrap samples for functional data.

## Usage

```
fdata.bootstrap(fdataobj, statistic=func.mean, alpha=0.05,
nb=200, smo=0.0, draw=FALSE, draw.control=NULL, ...)
```

## Arguments

fdataobj	fdata class object.
statistic	Sample statistic. It must be a function that returns an object of class fdata. By default, it uses sample mean <code>func.mean</code> . See <a href="#">Descriptive</a> for other statistics.
alpha	Significance value.
nb	Number of bootstrap resamples.
smo	The smoothing parameter for the bootstrap samples as a proportion of the sample variance matrix.
draw	=TRUE, plot the bootstrap samples and the statistic.
draw.control	list that it specifies the col, lty and lwd for objects: fdataobj, statistic, IN and OUT.
...	Further arguments passed to or from other methods.

## Details

The `fdata.bootstrap()` computes a confidence ball using bootstrap in the following way:

- Let  $X_1(t), \dots, X_n(t)$  the original data and  $T = T(X_1(t), \dots, X_n(t))$  the sample statistic.
- Calculate the nb bootstrap resamples  $\{X_1^*(t), \dots, X_n^*(t)\}$ , using the following scheme  $X_i^*(t) = X_i(t) + Z(t)$  where  $Z(t)$  is normally distributed with mean 0 and covariance matrix  $\gamma \Sigma_x$ , where  $\Sigma_x$  is the covariance matrix of  $\{X_1(t), \dots, X_n(t)\}$  and  $\gamma$  is the smoothing parameter.
- Let  $T^{*j} = T(X_1^{*j}(t), \dots, X_n^{*j}(t))$  the estimate using the  $j$  resample.
- Compute  $d(T, T^{*j})$ ,  $j = 1, \dots, nb$ . Define the bootstrap confidence ball of level  $1 - \alpha$  as  $CB(\alpha) = X \in E$  such that  $d(T, X) \leq d_\alpha$  being  $d_\alpha$  the quantile  $(1 - \alpha)$  of the distances between the bootstrap resamples and the sample estimate.

The `fdata.bootstrap` function allows us to define a statistic calculated on the nb resamples, control the degree of smoothing by `smo` argument and represent the confidence ball with level  $1 - \alpha$  as those resamples that fulfill the condition of belonging to  $CB(\alpha)$ . The `statistic` used by default is the mean (`func.mean`) but also other depth-based functions can be used (see `help(Descriptive)`).

**Value**

statistic	fdata class object with the statistic estimate from nb bootstrap samples.
dband	Bootstrap estimate of $(1-\alpha)\%$ distance.
rep.dist	Distance from every replicate.
resamples	fdata class object with the bootstrap resamples.
fdataobj	fdata class object.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Cuevas A., Febrero-Bande, M. and Fraiman, R. (2007). *Robust estimation and classification for functional data via projection-based depth notions*. Computational Statistics 22, 3: 481-496.

Cuevas A., Febrero-Bande, M., Fraiman R. 2006. *On the use of bootstrap for estimating functions with functional data*. Computational Statistics and Data Analysis 51: 1063-1074.

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

**See Also**

See Also as [Descriptive](#)

**Examples**

```
## Not run:
data(tecator)
absorp<-tecator$absorp.fdata
# Time consuming
#Bootstrap for Trimmed Mean with depth mode
out.boot=fdata.bootstrap(absorp,statistic=func.trim.FM,nb=200,draw=TRUE)
names(out.boot)
#Bootstrap for Median with with depth mode
control=list("col"=c("grey","blue","cyan"),"lty"=c(2,1,1),"lwd"=c(1,3,1))
out.boot=fdata.bootstrap(absorp,statistic=func.med.mode,
draw=TRUE,draw.control=control)

## End(Not run)
```

---

fdata.cen	<i>Functional data centred (subtract the mean of each discretization point)</i>
-----------	---

---

## Description

The function `fdata.cen` centres the curves by subtracting the functional mean.

## Usage

```
fdata.cen(fdataobj, meanX=func.mean(fdataobj))
```

## Arguments

fdataobj	<a href="#">fdata</a> class object.
meanX	The functional mean subtracted in the <code>fdataobj</code> .

## Value

Return:  
two `fdata` class objects with:

Xcen	The centered <code>fdata</code> .
meanX	Functional mean subtracted.

## Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <[manuel.oviedo@usc.es](mailto:manuel.oviedo@usc.es)>

## See Also

See Also as [fdata](#)

## Examples

```
data(phoneme)
mlearn<-phoneme[["learn"]][13:15,]
fdata.c=fdata.cen(mlearn)$Xcen
par(mfrow=c(1,2))
plot.fdata(mlearn, type="l")
plot.fdata(fdata.c, type="l")
```



---

fdata.deriv	<i>Computes the derivative of functional data object.</i>
-------------	---

---

### Description

Computes the derivative of functional data.

### Usage

```
fdata.deriv(fdataobj,nderiv=1,method="bspline",class.out='fdata',
,nbasis=NULL,...)
```

### Arguments

fdataobj	<a href="#">fdata</a> class object.
nderiv	Order of derivation, by default nderiv=1.
method	Type of derivative method, for more information see <b>details</b> .
class.out	Class of functional data returned: fdata or fd class.
nbasis	Number of Basis for fdataobj\$DATA. It is only used if method = <i>"bspline"</i> , <i>"exponential"</i> , <i>"fourier"</i> , <i>"monomial"</i> or <i>"polynomial"</i>
...	Further arguments passed to or from other methods.

### Details

- If method =*"bspline"*, *"exponential"*, *"fourier"*, *"monomial"* or *"polynomial"* fdata.deriv function creates a basis to represent the functional data. The functional data are converted to class fd using the [Data2fd](#) function and the basis indicated in the method. Finally, the function calculates the derivative of order nderiv of curves using [deriv.fd](#) function.
- If method=*"fmm"*, *"periodic"*, *"natural"* or *"monoH.FC"* is used [splinefun](#) function.
- If method=*"diff"*, raw derivation is applied. Not recommended to use this method when the values are not equally spaced.

### Value

Returns the derivative of functional data of fd class if class.out=*"fd"* or fdata class if class.out=*"fdata"*.

### See Also

See also [deriv.fd](#), [splinefun](#) and [fdata](#)

**Examples**

```

data(tecator)
absorp=tecator$absorp.fdata
tecator.fd1=fdata2fd(absorp)
tecator.fd2=fdata2fd(absorp,"fourier",9)
tecator.fd3=fdata2fd(absorp,"fourier",nbasis=9,nderiv=1)
#tecator.fd1;tecator.fd2;tecator.fd3
tecator.fdata1=fdata(tecator.fd1)
tecator.fdata2=fdata(tecator.fd2)
tecator.fdata3=fdata(tecator.fd3)
tecator.fdata4=fdata.deriv(absorp,nderiv=1,method="bspline",
class.out='fdata',nbasis=9)
tecator.fd4=fdata.deriv(tecator.fd3,nderiv=0,class.out='fd',nbasis=9)
plot(tecator.fdata4)
plot(fdata.deriv(absorp,nderiv=1,method="bspline",class.out='fd',nbasis=11))

```

fdata.methods

*fdata S3 Group Generic Functions***Description**

fdata Group generic methods defined for four specified groups of functions, Math, Ops, Summary and Complex.

**Usage**

```

## S3 method for class 'fdata'
Math(x, ...)
## S3 method for class 'fdata'
Ops(e1, e2)
## S3 method for class 'fdata'
Summary(..., na.rm = FALSE)

```

**Arguments**

x, e1, e2	fdata class object.
...	Further arguments passed to methods.
na.rm	logical: should missing values be removed?

**See Also**

See [Summary](#) and [Complex](#).

**Examples**

```
#####
#####
data(tecator)
absor<-tecator$absorp.fdata
absor2<-fdata.deriv(absor,1)
absor<-absor2[1:5,1:4]
absor2<-absor2[1:5,1:4]
sum(absor)
round(absor,4)
log1<-log(absor)
```

---

fdata2fd	<i>Converts fdata class object into fd class object</i>
----------	---

---

**Description**

Converts fdata class object into fd class object using Data2fd function.

**Usage**

```
fdata2fd(fdataobj,type.basis=NULL,nbasis=NULL,nderiv=0,
lambda=NULL,...)
```

**Arguments**

fdataobj	fdata class object.
type.basis	Type of basis. A function create."type.basis".basis must exists. By default, bspline basis is used.
nbasis	Number of basis which is used in create.basis function.
nderiv	Order of derivation which is used in deriv.fd function (optional).
lambda	Weight on the smoothing operator specified by nderiv.
...	Further arguments passed to or from other methods.

**Value**

Return an object of the fd class.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

**See Also**

See Also as [fdata](#) and [Data2fd](#)

**Examples**

```
data(phoneme)
mlearn<-phoneme$learn[1,]
fdata2=fdata2fd(mlearn)
class(mlearn)
class(fdata2)
fdata3=fdata2fd(mlearn,type.basis="fourier",nbasis=7)
plot(mlearn)
lines(fdata2,col=2)
lines(fdata3,col=3)
fdata5=fdata2fd(mlearn,nderiv=1)
```

---

fdata2pc

*Principal components for functional data*


---

**Description**

Compute (penalized) principal components for functional data. `fdata2ppc` is deprecated.

**Usage**

```
fdata2pc(fdataobj, ncomp = 2, norm = TRUE, lambda=0, P=c(0,0,1), ...)
fdata2ppc(fdataobj, ncomp = 2, norm = TRUE, lambda=0, P=c(0,0,1), ...)
```

**Arguments**

<code>fdataobj</code>	<code>fdata</code> class object.
<code>ncomp</code>	Number of principal components.
<code>norm</code>	=TRUE the norm of eigenvectors (rotation) is 1.
<code>lambda</code>	Amount of penalization. Default value is 0, i.e. no penalization is used.
<code>P</code>	If <code>P</code> is a vector: coefficients to define the penalty matrix object. By default <code>P=c(0,0,1)</code> penalize the second derivative (curvature) or acceleration. If <code>P</code> is a matrix: the penalty matrix object.
<code>...</code>	Further arguments passed to or from other methods.

## Details

Smoothing is achieved by penalizing the integral of the square of the derivative of order  $m$  over `rangeval`:

- $m = 0$  penalizes the squared difference from 0 of the function
- $m = 1$  penalize the square of the slope or velocity
- $m = 2$  penalize the squared acceleration
- $m = 3$  penalize the squared rate of change of acceleration

## Value

<code>d</code>	The standard deviations of the functional principal components.
<code>rotation</code>	are also known as loadings. A <code>fdata</code> class object whose rows contain the eigenvectors.
<code>x</code>	are also known as scores. The value of the rotated functional data is returned.
<code>fdataobj.cen</code>	The centered <code>fdataobj</code> object.
<code>mean</code>	The functional mean of <code>fdataobj</code> object.
<code>l</code>	Vector of index of principal components.
<code>C</code>	The matched call.
<code>lambda</code>	Amount of penalization.
<code>P</code>	Penalty matrix.

## Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <[manuel.oviedo@usc.es](mailto:manuel.oviedo@usc.es)>

## References

- Venables, W. N. and B. D. Ripley (2002). *Modern Applied Statistics with S*. Springer-Verlag.
- N. Kraemer, A.-L. Boulsteix, and G. Tutz (2008). Penalized Partial Least Squares with Applications to B-Spline Transformations and Functional Data. *Chemometrics and Intelligent Laboratory Systems*, 94, 60 - 69. <http://dx.doi.org/10.1016/j.chemolab.2008.06.009>
- Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. *Journal of Statistical Software*, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

## See Also

See Also as [svd](#) and [varimax](#).

## Examples

```
## Not run:
n= 100;tt= seq(0,1,len=51)
x0<-rproc2fdata(n,tt,sigma="wiener")
x1<-rproc2fdata(n,tt,sigma=0.1)
x<-x0*3+x1
pc=fdata2ppc(x,lambda=1)
summary(pc)
## End(Not run)
```

---

fdata2pls

*Partial least squares components for functional data.*

---

## Description

Compute penalized partial least squares (PLS) components for functional data. `fdata2ppls` is deprecated.

## Usage

```
fdata2pls(fdataobj, y, ncomp = 2, lambda = 0, P = c(0, 0, 1),
norm=TRUE,...)
fdata2ppls(fdataobj, y, ncomp = 2, lambda = 0, P = c(0, 0, 1),
norm=TRUE,...)
```

## Arguments

<code>fdataobj</code>	<code>fdata</code> class object.
<code>y</code>	Scalar response with length <code>n</code> .
<code>ncomp</code>	The number of components to include in the model.
<code>lambda</code>	Amount of penalization. Default value is 0, i.e. no penalization is used.
<code>P</code>	If <code>P</code> is a vector: coefficients to define the penalty matrix object. By default <code>P=c(0,0,1)</code> penalize the second derivative (curvature) or acceleration. If <code>P</code> is a matrix: the penalty matrix object.
<code>norm</code>	<code>=TRUE</code> the <code>fdataobj</code> are centered and scaled.
<code>...</code>	Further arguments passed to or from other methods.

## Details

If `norm=TRUE`, computes the PLS by NIPALS algorithm and the Degrees of Freedom using the Krylov representation of PLS, see Kraemer and Sugiyama (2011).

If `norm=FALSE`, computes the PLS by Orthogonal Scores Algorithm and the Degrees of Freedom are the number of components `ncomp`, see Martens and Naes (1989).

**Value**

fdata2pls function return:

df	degree of freedom
rotation	<a href="#">fdata</a> class object.
x	Is true the value of the rotated data (the centred data multiplied by the rotation matrix) is returned.
fdataobj.cen	The centered fdataobj object.
mean	mean of fdataobj.
l	Vector of index of principal components.
C	The matched call.
lambda	Amount of penalization.
P	Penalty matrix.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

- Kraemer, N., Sugiyama M. (2011). *The Degrees of Freedom of Partial Least Squares Regression*. Journal of the American Statistical Association. Volume 106, 697-705.
- Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>
- Martens, H., Naes, T. (1989) *Multivariate calibration*. Chichester: Wiley.

**See Also**

Used in: [fregre.pls](#), [fregre.pls.cv](#).  
Alternative method: [fdata2pc](#).

**Examples**

```
## Not run:
n= 500;tt= seq(0,1,len=101)
x0<-rproc2fdata(n,tt,sigma="wiener")
x1<-rproc2fdata(n,tt,sigma=0.1)
x<-x0*3+x1
beta = tt*sin(2*pi*tt)^2
fbeta = fdata(beta,tt)
y<-inprod.fdata(x,fbeta)+rnorm(n,sd=0.1)
pls1=fdata2pls(x,y)
norm.fdata(pls1$rotation)

## End(Not run)
```

---

**FDR** *False Discovery Rate (FDR)*

---

**Description**

Compute the False Discovery Rate for a vector of p-values and alpha value.

**Usage**

```
FDR(pvalues, alpha=0.95, dep=1)
pvalue.FDR(pvalues, dep=1)
```

**Arguments**

pvalues	Vector of p-values
alpha	Alpha value (level of significance).
dep	Parameter dependence test. By default dep = 1, direct dependence between tests.

**Details**

FDR method is used for multiple hypothesis testing to correct problems of multiple contrasts. If dep = 1, the tests are positively correlated, for example when many tests are the same contrast. If dep < 1 the tests are negatively correlated.

**Value**

Return:

out.FDR	=TRUE. If there are significative differences.
pv.FDR	p-value for False Discovery Rate test.

**Author(s)**

Febrero-Bande, M. and Oviedo de la Fuente, M.

**References**

Benjamini, Y., Yekutieli, D. (2001). *The control of the false discovery rate in multiple testing under dependency*. Annals of Statistics. 29 (4): 1165-1188. DOI:10.1214/aos/1013699998.

**See Also**

Function used in [anova.RPm](#)



**Examples**

```

p=seq(1:50)/1000
FDR(p)
pvalue.FDR(p)
FDR(p,alpha=0.9999)
FDR(p,alpha=0.9)
FDR(p,alpha=0.9,dep=-1)

```

flm.Ftest

*F-test for the Functional Linear Model with scalar response***Description**

The function `flm.Ftest` tests the null hypothesis of no interaction between a functional covariate and a scalar response inside the Functional Linear Model (FLM):  $Y = \langle X, \beta \rangle + \epsilon$ . The null hypothesis is  $H_0 : \beta = 0$  and the alternative is  $H_1 : \beta \neq 0$ . The null hypothesis is tested by a functional extension of the classical F-test (see Details).

**Usage**

```

Ftest.statistic(X.fdata, Y)
flm.Ftest(X.fdata, Y, B=5000, verbose=TRUE)

```

**Arguments**

<code>X.fdata</code>	Functional covariate for the FLM. The object must be in the class <code>fdata</code> .
<code>Y</code>	Scalar response for the FLM. Must be a vector with the same number of elements as functions are in <code>X.fdata</code> .
<code>B</code>	Number of bootstrap replicates to calibrate the distribution of the test statistic. <code>B=5000</code> replicates are the recommended for carry out the test, although for exploratory analysis ( <b>not inferential</b> ), an acceptable less time-consuming option is <code>B=500</code> .
<code>verbose</code>	Either to show or not information about computing progress.

**Details**

The Functional Linear Model with scalar response (FLM), is defined as  $Y = \langle X, \beta \rangle + \epsilon$ , for a functional process  $X$  such that  $E[X(t)] = 0$ ,  $E[X(t)\epsilon] = 0$  for all  $t$  and for a scalar variable  $Y$  such that  $E[Y] = 0$ . The *functional F-test* is defined as

$$T_n = \left\| \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) \right\|,$$

where  $\bar{X}$  is the functional mean of  $X$ ,  $\bar{Y}$  is the ordinary mean of  $Y$  and  $\| \cdot \|$  is the  $L^2$  functional norm. The statistic is computed with the function `Ftest.statistic`. The distribution of the test statistic is approximated by a wild bootstrap resampling on the residuals, using the *golden section bootstrap*.

**Value**

The value for `Ftest.statistic` is simply the F-test statistic. The value for `flm.Ftest` is an object with class "htest" whose underlying structure is a list containing the following components:

<code>statistic</code>	The value of the F-test statistic.
<code>boot.statistics</code>	A vector of length B with the values of the bootstrap F-test statistics.
<code>p.value</code>	The p-value of the test.
<code>method</code>	The character string "Functional Linear Model F-test".
<code>B</code>	The number of bootstrap replicates used.
<code>data.name</code>	The character string "Y=<X,0>+e"

**Note**

No NA's are allowed neither in the functional covariate nor in the scalar response.

**Author(s)**

Eduardo Garcia-Portugues. Please, report bugs and suggestions to <egarcia@math.ku.dk>

**References**

Garcia-Portugues, E., Gonzalez-Manteiga, W. and Febrero-Bande, M. (2014). A goodness-of-fit test for the functional linear model with scalar response. *Journal of Computational and Graphical Statistics*, 23(3), 761-778. <http://dx.doi.org/10.1080/10618600.2013.812519>

Gonzalez-Manteiga, W., Gonzalez-Rodriguez, G., Martinez-Calvo, A. and Garcia-Portugues, E. Bootstrap independence test for functional linear models. arXiv:1210.1072. <http://arxiv.org/abs/1210.1072>

**See Also**

[rwild](#), [flm.test](#), [dfv.test](#)

**Examples**

```
## Simulated example ##

X=rproc2fdata(n=50,t=seq(0,1,l=101),sigma="OU")

beta0=fdata(mdata=rep(0,length=101)+rnorm(101,sd=0.05),
  argvals=seq(0,1,l=101),rangeval=c(0,1))
beta1=fdata(mdata=cos(2*pi*seq(0,1,l=101))-(seq(0,1,l=101)-0.5)^2+
  rnorm(101,sd=0.05),argvals=seq(0,1,l=101),rangeval=c(0,1))

# Null hypothesis holds
Y0=drop(inprod.fdata(X,beta0)+rnorm(50,sd=0.1))
```

```

# Null hypothesis does not hold
Y1=drop(inprod.fdata(X,beta1)+rnorm(50,sd=0.1))

## Not run:
# Do not reject H0
flm.Ftest(X,Y0,B=100)
flm.Ftest(X,Y0,B=5000)

# Reject H0
flm.Ftest(X,Y1,B=100)
flm.Ftest(X,Y1,B=5000)

## End(Not run)

```

---

flm.test	<i>Goodness-of-fit test for the Functional Linear Model with scalar response</i>
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### Description

The function `flm.test` tests the composite null hypothesis of a Functional Linear Model with scalar response (FLM),

$$H_0 : Y = \langle X, \beta \rangle + \epsilon,$$

versus a general alternative. If  $\beta = \beta_0$  is provided, then the simple hypothesis  $H_0 : Y = \langle X, \beta_0 \rangle + \epsilon$  is tested. The testing of the null hypothesis is done by a Projected Cramer-von Mises statistic (see Details).

### Usage

```

flm.test (X.fdata, Y, beta0.fdata = NULL, B = 5000,
         est.method = "pls", p = NULL, type.basis = "bspline",
         verbose = TRUE, plot.it = TRUE, B.plot = 100,
         G = 200, ...)

```

### Arguments

<code>X.fdata</code>	Functional covariate for the FLM. The object must be in the class <code>fdata</code> .
<code>Y</code>	Scalar response for the FLM. Must be a vector with the same number of elements as functions are in <code>X.fdata</code> .
<code>beta0.fdata</code>	Functional parameter for the simple null hypothesis, in the <code>fdata</code> class. Recall that the <code>argvals</code> and <code>rangeval</code> arguments of <code>beta0.fdata</code> must be the same of <code>X.fdata</code> . A possibility to do this is to consider, for example for $\beta_0 = 0$ (the simple null hypothesis of no interaction), <code>beta0.fdata=fdata(mdata=rep(0,length(X.fdata\$argvals)),  argvals=X.fdata\$argvals,rangeval=X.fdata\$rangeval).</code> If <code>beta0.fdata=NULL</code> (default), the function will test for the composite null hypothesis.

B	Number of bootstrap replicates to calibrate the distribution of the test statistic. B=5000 replicates are the recommended for carry out the test, although for exploratory analysis ( <b>not inferential</b> ), an acceptable less time-consuming option is B=500.
est.method	<p>Estimation method for the unknown parameter <math>\beta</math>, only used in the composite case. Mainly, there are two options: specify the number of basis elements for the estimated <math>\beta</math> by p or optimally select p by a data-driven criteria (see Details section for discussion). Then, it must be one of the following methods:</p> <ul style="list-style-type: none"> <li>• "pc" If p, the number of basis elements, is given, then <math>\beta</math> is estimated by <code>fregre.pc</code>. Otherwise, an optimum p is chosen using <code>fregre.pc.cv</code> and the "SICc" criteria.</li> <li>• "pls" If p is given, <math>\beta</math> is estimated by <code>fregre.pls</code>. Otherwise, an optimum p is chosen using <code>fregre.pls.cv</code> and the "SICc" criteria. This is the default argument as it has been checked empirically that provides a good balance between the performance of the test and the estimation of <math>\beta</math>.</li> <li>• "basis" If p is given, <math>\beta</math> is estimated by <code>fregre.basis</code>. Otherwise, an optimum p is chosen using <code>fregre.basis.cv</code> and the "GCV.S" criteria. In these functions, the same basis for the arguments <code>basis.x</code> and <code>basis.b</code> is considered. The type of basis used will be the given by the argument <code>type.basis</code> and must be one of the class of <code>create.basis</code>. Further arguments passed to <code>create.basis</code> (not <code>rangeval</code> that is taken as the <code>rangeval</code> of <code>X.fdata</code>), can be passed throughout . . . .</li> </ul>
p	Number of elements of the basis considered. If it is not given, an optimal p will be chosen using a specific criteria (see <code>est.method</code> and <code>type.basis</code> arguments).
type.basis	<p>Type of basis used to represent the functional process. Depending on the hypothesis it will have a different interpretation:</p> <ul style="list-style-type: none"> <li>• Simple hypothesis. One of these options: <ul style="list-style-type: none"> <li>– "bspline" If p is given, the functional process is expressed in a basis of p B-splines. If not, an optimal p will be chosen by <code>min.basis</code>, using the "GCV.S" criteria.</li> <li>– "fourier" If p is given, the functional process is expressed in a basis of p fourier functions. If not, an optimal p will be chosen by <code>min.basis</code>, using the "GCV.S" criteria.</li> <li>– "pc" p must be given. Expresses the functional process in a basis of p PC.</li> <li>– "pls" p must be given. Expresses the functional process in a basis of p PLS.</li> </ul> <p>Although other of the basis supported by <code>create.basis</code> are possible too, "bspline" and "fourier" are recommended. Other basis may cause incompatibilities.</p> </li> <li>• Composite hypothesis. This argument is only used when <code>est.method="basis"</code> and, in this case, claims for the type of basis used in the basis estimation method of the functional parameter. Again, basis "bspline" and "fourier" are recommended, as other basis may cause incompatibilities.</li> </ul>

verbose	Either to show or not information about computing progress.
plot.it	Either to show or not a graph of the observed trajectory, and the bootstrap trajectories under the null composite hypothesis, of the process $R_n(\cdot)$ (see Details). Note that if <code>plot.it=TRUE</code> , the function takes more time to run.
B.plot	Number of bootstrap trajectories to show in the resulting plot of the test. As the trajectories shown are the first B.plot of B, B.plot must be lower or equal to B.
G	Number of projections used to compute the trajectories of the process $R_n(\cdot)$ by Monte Carlo.
...	Further arguments passed to <code>create.basis</code> .

## Details

The Functional Linear Model with scalar response (FLM), is defined as  $Y = \langle X, \beta \rangle + \epsilon$ , for a functional process  $X$  such that  $E[X(t)] = 0$ ,  $E[X(t)\epsilon] = 0$  for all  $t$  and for a scalar variable  $Y$  such that  $E[Y] = 0$ . Then, the test assumes that  $Y$  and  $X$ .fdata are **centred** and will automatically center them. So, bear in mind that when you apply the test for  $Y$  and  $X$ .fdata, actually, you are applying it to  $Y - \text{mean}(Y)$  and `fdata.cen(X.fdata)$Xcen`.

The test statistic corresponds to the Cramer-von Mises norm of the *Residual Marked empirical Process based on Projections*  $R_n(u, \gamma)$  defined in Garcia-Portugues *et al.* (2014). The expression of this process in a  $p$ -truncated basis of the space  $L^2[0, T]$  leads to the  $p$ -multivariate process  $R_{n,p}(u, \gamma^{(p)})$ , whose Cramer-von Mises norm is computed.

The choice of an appropriate  $p$  to represent the functional process  $X$ , in case that is not provided, is done via the estimation of  $\beta$  for the composite hypothesis. For the simple hypothesis, as no estimation of  $\beta$  is done, the choice of  $p$  depends only on the functional process  $X$ . As the result of the test may change for different  $p$ 's, we recommend to use an automatic criterion to select  $p$  instead of provide a fixed one. The distribution of the test statistic is approximated by a wild bootstrap resampling on the residuals, using the *golden section bootstrap*.

Finally, the graph shown if `plot.it=TRUE` represents the observed trajectory, and the bootstrap trajectories under the null, of the process RMPP *integrated on the projections*:

$$R_n(u) \approx \frac{1}{G} \sum_{g=1}^G R_n(u, \gamma_g),$$

where  $\gamma_g$  are simulated as Gaussians processes. This gives a graphical idea of how *distant* is the observed trajectory from the null hypothesis.

## Value

An object with class "htest" whose underlying structure is a list containing the following components:

statistic	The value of the test statistic.
boot.statistics	A vector of length B with the values of the bootstrap test statistics.
p.value	The p-value of the test.
method	The method used.

B	The number of bootstrap replicates used.
type.basis	The type of basis used.
beta.est	The estimated functional parameter $\beta$ in the composite hypothesis. For the simple hypothesis, the given <code>beta0.fdata</code> .
p	The number of basis elements passed or automatically chosen.
ord	The optimal order for PC and PLS given by <code>fregre.pc.cv</code> and <code>fregre.pls.cv</code> . For other methods is setted to 1:p.
data.name	The character string "Y=<X,b>+e"

**Note**

No NA's are allowed neither in the functional covariate nor in the scalar response.

**Author(s)**

Eduardo Garcia-Portugues. Please, report bugs and suggestions to <egarcia@math.ku.dk>

**References**

- Escanciano, J. C. (2006). A consistent diagnostic test for regression models using projections. *Econometric Theory*, 22, 1030-1051. <http://dx.doi.org/10.1017/S0266466606060506>
- Garcia-Portugues, E., Gonzalez-Manteiga, W. and Febrero-Bande, M. (2014). A goodness-of-fit test for the functional linear model with scalar response. *Journal of Computational and Graphical Statistics*, 23(3), 761-778. <http://dx.doi.org/10.1080/10618600.2013.812519>

**See Also**

[Adot](#), [PCvM.statistic](#), [rwild](#), [flm.Ftest](#), [dfv.test](#), [fregre.pc](#), [fregre.pls](#), [fregre.basis](#), [fregre.pc.cv](#), [fregre.pls.cv](#), [fregre.basis.cv](#), [min.basis](#), [create.basis](#)

**Examples**

```
# Simulated example #

X=rproc2fdata(n=100,t=seq(0,1,l=101),sigma="OU")
beta0=fdata(mdata=cos(2*pi*seq(0,1,l=101))-(seq(0,1,l=101)-0.5)^2+
rnorm(101,sd=0.05),argvals=seq(0,1,l=101),rangeval=c(0,1))
Y=inprod.fdata(X,beta0)+rnorm(100,sd=0.1)

dev.new(width=21,height=7)
par(mfrow=c(1,3))
plot(X,main="X")
plot(beta0,main="beta0")
plot(density(Y),main="Density of Y",xlab="Y",ylab="Density")
rug(Y)
```

```

## Not run:
# Composite hypothesis: do not reject FLM
pcvm.sim=flm.test(X,Y,B=50,B.plot=50,G=100,plot.it=TRUE)
pcvm.sim
flm.test(X,Y,B=5000)

# Estimated beta
dev.new()
plot(pcvim.sim$beta.est)

# Simple hypothesis: do not reject beta=beta0
flm.test(X,Y,beta0.fdata=beta0,B=50,B.plot=50,G=100)
flm.test(X,Y,beta0.fdata=beta0,B=5000)

# AEMET dataset #

data(aemet)

# Remove the 5% of the curves with less depth (i.e. 4 curves)
dev.new()
res.FM=depth.FM(aemet$temp,draw=TRUE)
qu=quantile(res.FM$dep,prob=0.05)
l=which(res.FM$dep<=qu)
lines(aemet$temp[l],col=3)
aemet$df$name[l]

# Data without outliers
wind.speed=apply(aemet$wind.speed$data,1,mean)[-1]
temp=aemet$temp[-1]

# Exploratory analysis: accept the FLM
pcvm.aemet=flm.test(temp,wind.speed,est.method="pls",B=100,B.plot=50,G=100)
pcvm.aemet

# Estimated beta
dev.new()
plot(pcvim.aemet$beta.est,lwd=2,col=2)

# B=5000 for more precision on calibration of the test: also accept the FLM
flm.test(temp,wind.speed,est.method="pls",B=5000)

# Simple hypothesis: rejection of beta0=0? Limiting p-value...
dat=rep(0,length(temp$argvals))
flm.test(temp,wind.speed, beta0.fdata=fdata(mdata=dat,argvals=temp$argvals,
rangeval=temp$rangeval),B=100)
flm.test(temp,wind.speed, beta0.fdata=fdata(mdata=dat,argvals=temp$argvals,
rangeval=temp$rangeval),B=5000)

# Tecator dataset #

data(tecator)

```

```

names(tecator)
absorp=tecator$absorp.fdata
ind=1:129 # or ind=1:215
x=absorp[ind,]
y=tecator$y$Fat[ind]
tt=absorp[["argvals"]]

# Exploratory analysis for composite hypothesis with automatic choose of p
pcvm.tecat=film.test(x,y,B=100,B.plot=50,G=100)
pcvm.tecat

# B=5000 for more precision on calibration of the test: also reject the FLM
film.test(x,y,B=5000)

# Distribution of the PCvM statistic
plot(density(pcvm.tecat$boot.statistics),lwd=2,xlim=c(0,10),
main="PCvM distribution", xlab="PCvM*",ylab="Density")
rug(pcvm.tecat$boot.statistics)
abline(v=pcvm.tecat$statistic,col=2,lwd=2)
legend("top",legend=c("PCvM observed"),lwd=2,col=2)

# Simple hypothesis: fixed p
dat=rep(0,length(x$argvals))
film.test(x,y,beta0.fdata=fdata(mdata=dat,argvals=x$argvals,
rangeval=x$rangeval),B=100,p=11)

# Simple hypothesis, automatic choose of p
film.test(x,y,beta0.fdata=fdata(mdata=dat,argvals=x$argvals,
rangeval=x$rangeval),B=100)
film.test(x,y,beta0.fdata=fdata(mdata=dat,argvals=x$argvals,
rangeval=x$rangeval),B=5000)

## End(Not run)

```

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fregre.basis

*Functional Regression with scalar response using basis representation.*


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### Description

Computes functional regression between functional explanatory variable  $X(t)$  and scalar response  $Y$  using basis representation.

$$Y = \langle X, \beta \rangle + \epsilon = \int_T X(t)\beta(t)dt + \epsilon$$

where  $\langle \cdot, \cdot \rangle$  denotes the inner product on  $L_2$  and  $\epsilon$  are random errors with mean zero, finite variance  $\sigma^2$  and  $E[X(t)\epsilon] = 0$ .



**Usage**

```
fregre.basis(fdataobj,y,basis.x=NULL,basis.b=NULL,
lambda=0,Lfdobj=vec2Lfd(c(0,0),rtt), weights = rep(1,n),...)
```

**Arguments**

fdataobj	<a href="#">fdata</a> class object.
y	Scalar response with length n.
basis.x	Basis for functional explanatory data fdataobj.
basis.b	Basis for functional beta parameter.
lambda	A roughness penalty. By default, no penalty lambda=0.
Lfdobj	See <a href="#">eval.penalty</a> .
weights	weights
...	Further arguments passed to or from other methods.

**Details**

The function uses the basis representation proposed by Ramsay and Silverman (2005) to model the relationship between the scalar response and the functional covariate by basis representation of the observed functional data  $X(t) \approx \sum_{k=1}^{k_{n1}} c_k \xi_k(t)$  and the unknown functional parameter  $\beta(t) \approx \sum_{k=1}^{k_{n2}} b_k \phi_k(t)$ .

The functional linear models estimated by the expression:

$$\hat{y} = \langle X, \hat{\beta} \rangle = C^T \psi(t) \phi^T(t) \hat{b} = \tilde{X} \hat{b}$$

where  $\tilde{X}(t) = C^T \psi(t) \phi^T(t)$ , and  $\hat{b} = (\tilde{X}^T \tilde{X})^{-1} \tilde{X}^T y$  and so,  $\hat{y} = \tilde{X} \hat{b} = \tilde{X} (\tilde{X}^T \tilde{X})^{-1} \tilde{X}^T y = Hy$  where  $H$  is the hat matrix with degrees of freedom:  $df = tr(H)$ .

If  $\lambda > 0$  then `fregre.basis` incorporates a roughness penalty:

$$\hat{y} = \tilde{X} \hat{b} = \tilde{X} (\tilde{X}^T \tilde{X} + \lambda R_0)^{-1} \tilde{X}^T y = H_\lambda y \text{ where } R_0 \text{ is the penalty matrix.}$$

This function allows covariates of class `fdata`, `matrix`, `data.frame` or directly covariates of class `fd`. The function also gives default values to arguments `basis.x` and `basis.b` for representation on the basis of functional data  $X(t)$  and the functional parameter  $\beta(t)$ , respectively.

If `basis=NULL` creates the `bspline` basis by [create.bspline.basis](#).

If the functional covariate `fdataobj` is a `matrix` or `data.frame`, it creates an object of class "fdata" with default attributes, see [fdata](#).

If `basis.x$type="fourier"` and `basis.b$type="fourier"`, the basis are orthonormal and the function decreases the number of fourier basis elements on the  $\min(k_{n1}, k_{n2})$ , where  $k_{n1}$  and  $k_{n2}$  are the number of basis element of `basis.x` and `basis.b` respectively.

**Value**

Return:

call	The matched call.
coefficients	A named vector of coefficients
residuals	y minus fitted values.
fitted.values	Estimated scalar response.
beta.est	beta parameter estimated of class fd
weights	(only for weighted fits) the specified weights.
df	The residual degrees of freedom.
r2	Coefficient of determination.
sr2	Residual variance.
Vp	Estimated covariance matrix for the parameters.
H	Hat matrix.
y	Response.
fdataobj	Functional explanatory data of class fdata.
a.est	Intercept parameter estimated
x.fd	Centered functional explanatory data of class fd.
basis.b	Basis used for beta parameter estimation.
lambda.opt	A roughness penalty.
Lfdobj	Order of a derivative or a linear differential operator.
P	Penalty matrix.
lm	Return lm object

**Author(s)**

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**References**

- Ramsay, James O., and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.
- Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

**See Also**

See Also as: [fregre.basis.cv](#), [summary.fregre.fd](#) and [predict.fregre.fd](#).  
Alternative method: [fregre.pc](#) and [fregre.np](#).

## Examples

```
# fregre.basis
data(tecator)
names(tecator)
absorp=tecator$absorp.fdata
ind=1:129
x=absorp[ind,]
y=tecator$y$Fat[ind]
tt=absorp[["argvals"]]
res1=fregre.basis(x,y)
summary(res1)
basis1=create.bspline.basis(rangeval=range(tt),nbasis=19)
basis2=create.bspline.basis(rangeval=range(tt),nbasis=9)
res5=fregre.basis(x,y,basis1,basis2)
summary(res5)
x.d2=fdata.deriv(x,nbasis=19,nderiv=1,method="bspline",class.out="fdata")
res7=fregre.basis(x.d2,y,basis1,basis2)
summary(res7)
```

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fregre.basis.cv	<i>Cross-validation Functional Regression with scalar response using basis representation.</i>
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## Description

Computes functional regression between functional explanatory variables and scalar response using basis representation. The function `fregre.basis.cv()` uses validation criterion defined by argument type.CV to estimate the number of basis elements and/or the penalized parameter ( $\lambda$ ) that best predicts the response.

## Usage

```
fregre.basis.cv(fdataobj,y,basis.x=NULL,basis.b=NULL,
type.basis=NULL,lambda=0,Lfdobj=vec2Lfd(c(0,0),rtt),
type.CV=GCV.S,par.CV=list(trim=0),weights= rep(1,n),
verbose=FALSE,...)
```

## Arguments

fdataobj	fdata class object.
y	Scalar response with length n.
basis.x	Basis for functional explanatory data fdataobj.
basis.b	Basis for functional beta parameter.
type.basis	A vector of character string which determines type of basis. By default "bspline". It is only used when basis.x or basis.b are a vector of number of basis considered.

lambda	A roughness penalty. By default, no penalty lambda=0.
Lfdobj	See <a href="#">eval.penalty</a> .
type.CV	Type of cross-validation. By default generalized cross-validation <a href="#">GCV.S</a> method.
par.CV	List of parameters for type.CV: trim, the alpha of the trimming and draw.
weights	weights
verbose	If TRUE information about the procedure is printed. Default is FALSE.
...	Further arguments passed to or from other methods.

### Details

If `basis = NULL` creates bspline basis.

If the functional covariate `fdataobj` is in a format raw data, such as matrix or data.frame, creates an object of class `fdata` with default attributes, see [fdata](#).

If `basis.x` is a vector of number of basis elements and `basis.b=NULL`, the function force the same number of elements in the basis of `x` and `beta`.

If `basis.x$type="fourier"` and `basis.b$type="fourier"`, the function decreases the number of fourier basis elements on the  $\min(k_{n1}, k_{n2})$ , where  $k_{n1}$  and  $k_{n2}$  are the number of basis element of `basis.x` and `basis.b` respectively.

### Value

Return:

call	The matched call.
coefficients	A named vector of coefficients
residuals	y minus fitted values.
fitted.values	Estimated scalar response.
beta.est	beta parameter estimated of class <code>fd</code>
weights	(only for weighted fits) the specified weights.
df	The residual degrees of freedom.
r2	Coefficient of determination.
sr2	Residual variance.
H	Hat matrix.
y	Scalar response.
fdataobj	Functional explanatory data of class <code>fdata</code> .
x.fd	Centered functional explanatory data of class <code>fd</code> .
lambda.opt	lambda value that minimizes CV or GCV method.
gcv.opt	Minimum value of CV or GCV method.

basis.x.opt	Basis used for functional explanatory data estimation fdata.
basis.b.opt	Basis used for for functional beta parameter estimation.
a.est	Intercept parameter estimated
lm	Return lm object.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ramsay, James O. and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

**See Also**

See Also as: [fregre.basis](#), [summary.fregre.fd](#) and [predict.fregre.fd](#).  
Alternative method: [fregre.pc.cv](#) and [fregre.np.cv](#).

**Examples**

```
data(tecator)
x<-tecator$absorp.fdata[1:129]
y=tecator$y$Fat[1:129]
b1<-c(15,21,31)
b2<-c(7,9)
res1=fregre.basis.cv(x,y,basis.x=b1)
res2=fregre.basis.cv(x,y,basis.x=b1,basis.b=b2)
res1$gcv
res2$gcv
## Not run:
l=2^(-4:10)
res3=fregre.basis.cv(x,y,basis.b=b1,type.basis="fourier",
lambda=1,type.CV=GCV.S,par.CV=list(trim=0.15))
res3$gcv

## End(Not run)
```

fregre.basis.fr

*Functional Regression with functional response using basis representation.***Description**

Computes functional regression between functional explanatory variable  $X(s)$  and functional response  $Y(t)$  using basis representation.

$$Y(t) = \alpha(t) + \int_T X(s)\beta(s,t)ds + \epsilon(t)$$

where  $\alpha(t)$  is the intercept function,  $\beta(s,t)$  is the bivariate regression function and  $\epsilon(t)$  are the error term with mean zero.

**Usage**

```
fregre.basis.fr(x,y,basis.s=NULL,basis.t=NULL,lambda.s=0,
lambda.t=0,Lfdobj.s=vec2Lfd(c(0,0),range.s),
Lfdobj.t=vec2Lfd(c(0,0),range.t),weights=NULL,...)
```

**Arguments**

x	Functional explanatory variable.
y	Functional response variable.
basis.s	Basis related with s and it is used in the estimation of $\beta(s,t)$ .
basis.t	Basis related with t and it is used in the estimation of $\beta(s,t)$ .
lambda.s	A roughness penalty with respect to s to be applied in the estimation of $\beta(s,t)$ . By default, no penalty lambda.s=0.
lambda.t	A roughness penalty with respect to t to be applied in the estimation of $\beta(s,t)$ . By default, no penalty lambda.t=0.
Lfdobj.s	A linear differential operator object with respect to s . See <a href="#">eval.penalty</a> .
Lfdobj.t	A linear differential operator object with respect to t. See <a href="#">eval.penalty</a> .
weights	Weights.
...	Further arguments passed to or from other methods.

**Details**

The function is a wrapped of [linmod](#) function proposed by Ramsay and Silverman (2005) to model the relationship between the functional response  $Y(t)$  and the functional covariate  $X(t)$  by basis representation of both.

The unknown bivariate functional parameter  $\beta(s,t)$  can be expressed as a double expansion in terms of  $K$  basis function  $\nu_k$  and  $L$  basis functions  $\theta_l$ ,

$$\beta(s,t) = \sum_{k=1}^K \sum_{l=1}^L b_{kl} \nu_k(s) \theta_l(t) = \nu(s)^\top \mathbf{B} \theta(t)$$

Then, the model can be re-written in a matrix version as,

$$Y(t) = \alpha(t) + \int_T X(s)\nu(s)^\top \mathbf{B}\theta(t)ds + \epsilon(t) = \alpha(t) + \mathbf{X}\mathbf{B}\theta(t) + \epsilon(t)$$

where  $\mathbf{X} = \int X(s)\nu^\top(t)ds$

This function allows objects of class `fdata` or directly covariates of class `fd`. If `x` is a `fdata` class, `basis.s` is also the basis used to represent `x` as `fd` class object. If `y` is a `fdata` class, `basis.t` is also the basis used to represent `y` as `fd` class object. The function also gives default values to arguments `basis.s` and `basis.t` for construct the `bifd` class object used in the estimation of  $\beta(s, t)$ . If `basis.s=NULL` or `basis.t=NULL` the function creates a `bspline` basis by `create.bspline.basis`.

`fregre.basis.fr` incorporates a roughness penalty using an appropriate linear differential operator; `{lambda.s,Lfdobj.s}` for penalization of  $\beta$ 's variations with respect to `s` and `{lambda.t,Lfdobj.t}` for penalization of  $\beta$ 's variations with respect to `t`.

## Value

Return:

<code>call</code>	The matched call.
<code>a.est</code>	Intercept parameter estimated.
<code>coefficients</code>	the matrix of the coefficients.
<code>beta.est</code>	A bivariate functional data object of class <code>bifd</code> with the estimated parameters of $\beta(s, t)$ .
<code>fitted.values</code>	Estimated response.
<code>residuals</code>	<code>y</code> minus fitted values.
<code>y</code>	Functional response.
<code>x</code>	Functional explanatory data.
<code>lambda.s</code>	A roughness penalty with respect to <code>s</code> .
<code>lambda.t</code>	A roughness penalty with respect to <code>t</code> .
<code>Lfdobj.s</code>	A linear differential operator with respect to <code>s</code> .
<code>Lfdobj.t</code>	A linear differential operator with respect to <code>t</code> .
<code>weights</code>	Weights.

## Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

## References

Ramsay, James O., and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.

**See Also**

See Also as: [predict.fregre.fr](#).

Alternative method: [linmod](#).

**Examples**

```
## Not run:
rtt<-c(0, 365)
basis.alpha <- create.constant.basis(rtt)
basisx <- create.bspline.basis(rtt,11)
basisy <- create.bspline.basis(rtt,11)
basisz <- create.bspline.basis(rtt,7)
basist <- create.bspline.basis(rtt,9)

# fd class
dayfd<-Data2fd(day.5,CanadianWeather$dailyAv,basisx)
tempfd<-dayfd[,1]
log10precfd<-dayfd[,3]
res1 <- fregre.basis.fr(tempfd, log10precfd,
basis.s=basisz,basis.t=basist)

# fdata class
tt<-1:365
tempfddata<-fdata(t(CanadianWeather$dailyAv[, ,1]),tt,rtt)
log10precfddata<-fdata(t(CanadianWeather$dailyAv[, ,3]),tt,rtt)
res2<-fregre.basis.fr(tempfddata,log10precfddata,
basis.s=basisz,basis.t=basist)

# penalization
Lfdobjt <- Lfdobjz <- vec2Lfd(c(0,0), rtt)
Lfdobjt <- vec2Lfd(c(0,0), rtt)
lambdat<-lambdas <- 100
res1.pen <- fregre.basis.fr(tempfddata,log10precfddata,basis.s=basisz,
basis.t=basist,lambdas=lambdas,lambdat=lambdat,
Lfdobj.s=Lfdobjz,Lfdobj.t=Lfdobjt)

res2.pen <- fregre.basis.fr(tempfd, log10precfd,
basis.s=basisz,basis.t=basist,lambdas=lambdas,
lambdat=lambdat,Lfdobj.s=Lfdobjz,Lfdobj.t=Lfdobjt)

plot(log10precfd,col=1)
lines(res1$fitted.values,col=2)
plot(res1$residuals)
plot.bifd(res1$beta.est,tt,tt)
plot.bifd(res1$beta.est,tt,tt,type="persp",theta=45,phi=30)

## End(Not run)
```



---

fregre.bootstrap      *Bootstrap regression*

---

### Description

Estimate the beta parameter by wild or smoothed bootstrap procedure

### Usage

```
fregre.bootstrap(model, nb = 500, wild = TRUE, type.wild="golden",
                 newX = NULL, smo = 0.1, smoX = 0.05, alpha = 0.95,
                 kmax.fix = FALSE, draw = TRUE, ...)
```

### Arguments

model	fregre.pc, fregre.pls or fregre.basis object.
nb	Number of bootstrap samples.
wild	Naive or smoothed bootstrap depending of the smo and smoX parameters.
type.wild	Type of distribution of V in wild bootstrap procedure, see <a href="#">rwild</a> .
smo	If > 0, smoothed bootstrap on the residuals (proportion of response variance).
smoX	If > 0, smoothed bootstrap on the explanatory functional variable fdata (proportion of variance-covariance matrix of fdata object).
newX	A fdata class containing the values of the model covariates at which predictions are required (only for smoothed bootstrap).
kmax.fix	The number of maximum components to consider in each bootstrap iteration. =TRUE, the bootstrap procedure considers the same number of components used in the previous fitted model. =FALSE, the bootstrap procedure estimates the best components in each iteration.
alpha	Significance level used for graphical option, draw=TRUE.
draw	=TRUE, plot the bootstrap estimated beta, and (optional) the CI for the predicted response values.
...	Further arguments passed to or from other methods.

### Details

Estimate the beta parameter by wild or smoothed bootstrap procedure using principal components representation [fregre.pc](#), Partial least squares components (PLS) representation [fregre.pls](#) or basis representation [fregre.basis](#).

If a new curves are in newX argument the bootstrap method estimates the response using the bootstrap resamples.

If the model exhibits heteroskedasticity, the use of wild bootstrap procedure is recommended (by default).

**Value**

Return:

model	fregre.pc, fregre.pls or fregre.basis object.
beta.boot	functional beta estimated by the nb bootstrap regressions.
norm.boot	norm of differences between the nboot betas estimated by bootstrap and beta estimated by regression model.
coefs.boot	matrix with the bootstrap estimated basis coefficients.
kn.boot	vector or list of length nb with index of the basis, PC or PLS factors selected in each bootstrap regression.
y.pred	predicted response values using newX covariates.
y.boot	matrix of bootstrap predicted response values using newX covariates.
newX	a fdata class containing the values of the model covariates at which predictions are required (only for smoothed bootstrap).

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente &lt;manuel.oviedo@usc.es&gt;

**References**

Febrero-Bande, M., Galeano, P. and Gonzalez-Manteiga, W. (2010). *Measures of influence for the functional linear model with scalar response*. Journal of Multivariate Analysis 101, 327-339.

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

**See Also**See Also as: [fregre.pc](#), [fregre.pls](#), [fregre.basis](#), .**Examples**

```
## Not run:
data(tecator)
iest<-1:129
x=tecator$absorp.fdata[iest]
y=tecator$y$Fat[iest]
nb<-5
## Time-consuming
res.pc=fregre.pc(x,y,1:6)
# Fix the components used in the each regression
res.boot1=fregre.bootstrap(res.pc,nb=nb,wild=FALSE,kmax.fix=TRUE)
# Select the "best" components used in the each regression
res.boot2=fregre.bootstrap(res.pc,nb=nb,wild=FALSE,kmax.fix=FALSE)
```

```

res.boot3=fregre.bootstrap(res.pc,nb=nb,wild=FALSE,kmax.fix=10)
## predicted responses and bootstrap confidence interval
newx=tecator$absorp.fdata[-iest]
res.boot4=fregre.bootstrap(res.pc,nb=nb,wild=FALSE,newX=newx,draw=TRUE)

## End(Not run)

```

fregre.gkam

*Fitting Functional Generalized Kernel Additive Models.***Description**

Computes functional regression between functional explanatory variables ( $X^1(t_1), \dots, X^q(t_q)$ ) and scalar response  $Y$  using backfitting algorithm.

**Usage**

```

fregre.gkam(formula, family = gaussian(), data, weights = rep(1, nobs),
            par.metric = NULL, par.np = NULL, offset = NULL,
            control = list(maxit = 100, epsilon = 0.001,
                          trace = FALSE, inverse = "solve"), ...)
kgam.H(object, inverse = "svd")

```

**Arguments**

formula	an object of class formula (or one that can be coerced to that class): a symbolic description of the model to be fitted. The procedure only considers functional covariates (not implemented for non-functional covariates). The details of model specification are given under Details.
data	List that containing the variables in the model.
family	a description of the error distribution and link function to be used in the model. This can be a character string naming a family function, a family function or the result of a call to a family function. (See <a href="#">family</a> for details of family functions).
weights	weights
par.metric	List of arguments by covariate to pass to the metric function by covariate.
par.np	List of arguments to pass to the fregre.np.cv function
offset	this can be used to specify an a priori known component to be included in the linear predictor during fitting.
control	a list of parameters for controlling the fitting process, by default: maxit, epsilon, trace and inverse
object	List that containing the Hat matrix for each variable in the model.
inverse	="svd" (by default) or ="solve" method.
...	Further arguments passed to or from other methods.

### Details

The smooth functions  $f(\cdot)$  are estimated nonparametrically using an iterative local scoring algorithm by applying Nadaraya-Watson weighted kernel smoothers using [fregre.np.cv](http://fregre.np.cv) in each step, see Febrero-Bande and Gonzalez-Manteiga (2011) for more details.

Consider the fitted response  $\hat{Y} = g^{-1}(H_Q y)$ , where  $H_Q$  is the weighted hat matrix. Opsomer and Ruppert (1997) solves a system of equations for fit the unknowns  $f(\cdot)$  computing the additive smoother matrix  $H_k$  such that  $\hat{f}_k(X^k) = H_k Y$  and  $H_Q = H_1 + \dots + H_q$ . The additive model is fitted as follows:

$$\hat{Y} = g^{-1}\left(\sum_i^q \hat{f}_i(X_i)\right)$$

### Value

result	List of non-parametric estimation by covariate.
fitted.values	Estimated scalar response.
residuals	y minus fitted values.
effects	The residual degrees of freedom.
alpha	Hat matrix.
family	Coefficient of determination.
linear.predictors	Residual variance.
deviance	Scalar response.
aic	Functional explanatory data.
null.deviance	Non functional explanatory data.
iter	Distance matrix between curves.
w	beta coefficient estimated
eqrnk	List that containing the variables in the model.
prior.weights	Asymmetric kernel used.
y	Scalar response.
H	Hat matrix, see Opsomer and Ruppert(1997) for more details.
converged	conv.

### Author(s)

Febrero-Bande, M. and Oviedo de la Fuente, M.

### References

- Febrero-Bande M. and Gonzalez-Manteiga W. (2012). *Generalized Additive Models for Functional Data*. TEST. Springer-Velag. <http://dx.doi.org/10.1007/s11749-012-0308-0>
- Opsomer J.D. and Ruppert D.(1997). *Fitting a bivariate additive model by local polynomial regression*.Annals of Statistics, 25, 186-211.

**See Also**

See Also as: [fregre.gsam](#), [fregre.glm](#) and [fregre.np.cv](#)

**Examples**

```
## Not run:
data(tecator)
ab=tecator$absorp.fdata[1:100]
ab2=fdata.deriv(ab,2)
yfat=tecator$y[1:100,"Fat"]

# Example 1: # Changing the argument par.np and family
yfat.cat=ifelse(yfat<15,0,1)
xlist=list("df"=data.frame(yfat.cat),"ab"=ab,"ab2"=ab2)
f2<-yfat.cat~ab+ab2

par.NP<-list("ab"=list(Ker=AKer.norm,type.S="S.NW"),
"ab2"=list(Ker=AKer.norm,type.S="S.NW"))
res2=fregre.gkam(f2,family=binomial(),data=xlist,
par.np=par.NP)
res2

# Example 2: Changing the argument par.metric and family link
par.metric=list("ab"=list(metric=semimetric.deriv,nderiv=2,nbasis=15),
"ab2"=list("metric"=semimetric.basis))
res3=fregre.gkam(f2,family=binomial("probit"),data=xlist,
par.metric=par.metric,control=list(maxit=2,trace=FALSE))
summary(res3)

# Example 3: Gaussian family (by default)
# Only 1 iteration (by default maxit=100)
xlist=list("df"=data.frame(yfat),"ab"=ab,"ab2"=ab2)
f<-yfat~ab+ab2
res=fregre.gkam(f,data=xlist,control=list(maxit=1,trace=FALSE))
res

## End(Not run)
```

**Description**

Computes functional generalized linear model between functional covariate  $X^j(t)$  (and non functional covariate  $Z^j$ ) and scalar response  $Y$  using basis representation.

This function is an extension of the linear regression models: [fregre.lm](#) where the  $E[Y|X, Z]$  is related to the linear prediction  $\eta$  via a link function  $g(\cdot)$ .

$$E[Y|X, Z] = \eta = g^{-1}\left(\alpha + \sum_{j=1}^p \beta_j Z^j + \sum_{k=1}^q \frac{1}{\sqrt{T_k}} \int_{T_k} X^k(t) \beta_k(t) dt\right)$$

where  $Z = [Z^1, \dots, Z^p]$  are the non functional covariates and  $X(t) = [X^1(t_1), \dots, X^q(t_q)]$  are the functional ones.

### Usage

```
fregre.glm(formula, family = gaussian(), data,
basis.x=NULL, basis.b=NULL, CV=FALSE, ...)
```

### Arguments

formula	an object of class formula (or one that can be coerced to that class): a symbolic description of the model to be fitted. The details of model specification are given under Details.
family	a description of the error distribution and link function to be used in the model. This can be a character string naming a family function, a family function or the result of a call to a family function. (See <a href="#">family</a> for details of family functions.)
data	List that containing the variables in the model.
basis.x	List of basis for functional explanatory data estimation.
basis.b	List of basis for $\beta(t)$ parameter estimation.
CV	=TRUE, Cross-validation (CV) is done .
...	Further arguments passed to or from other methods.

### Details

The first item in the data list is called "df" and is a data frame with the response and non functional explanatory variables, as [glm](#).

Functional covariates of class `fdata` or `fd` are introduced in the following items in the data list. `basis.x` is a list of basis for represent each functional covariate. The basis object can be created by the function: [create.pc.basis](#), [pca.fd](#) [create.pc.basis](#), [create.fdata.basis](#) o [create.basis](#). `basis.b` is a list of basis for represent each  $\beta(t)$  parameter. If `basis.x` is a list of functional principal components basis (see [create.pc.basis](#) or [pca.fd](#)) the argument `basis.b` is ignored.

### Value

Return `glm` object plus:

basis.x	Basis used for <code>fdata</code> or <code>fd</code> covariates.
basis.b	Basis used for beta parameter estimation.
beta.l	List of estimated beta parameter of functional covariates.

data	List that containing the variables in the model.
formula	formula.
CV	predicted response by cross-validation.

**Note**

If the formula only contains a non functional explanatory variables (multivariate covariates), the function compute a standard `glm` procedure.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

- Ramsay, James O., and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.
- McCullagh and Nelder (1989), *Generalized Linear Models* 2nd ed. Chapman and Hall.
- Venables, W. N. and Ripley, B. D. (2002) *Modern Applied Statistics with S*, New York: Springer.

**See Also**

See Also as: `predict.fregre.glm` and `summary.glm`.  
Alternative method if family=*gaussian*: `fregre.lm`.

**Examples**

```
data(tecator)
x=tecator$absorp.fdata
y=tecator$y$Fat
tt=x[["argvals"]]
dataf=as.data.frame(tecator$y)

nbasis.x=11
nbasis.b=7
basis1=create.bspline.basis(rangeval=range(tt),nbasis=nbasis.x)
basis2=create.bspline.basis(rangeval=range(tt),nbasis=nbasis.b)

f=Fat~Protein+x
basis.x=list("x"=basis1)
basis.b=list("x"=basis2)
ldata=list("df"=dataf,"x"=x)
res=fregre.glm(f,family=gaussian(),data=ldata,basis.x=basis.x,
basis.b=basis.b)
summary(res)
```

fregre.gls

*Fit Functional Linear Model Using Generalized Least Squares***Description**

This function fits a functional linear model using generalized least squares. The errors are allowed to be correlated and/or have unequal variances.

**Usage**

```
fregre.gls(formula, data, correlation = NULL, basis.x = NULL,
           basis.b = NULL, rn, lambda, weights = NULL, subset, method = c("REML",
                                "ML"), control = list(), verbose = FALSE, criteria="GCCV1",...)
```

**Arguments**

formula	a two-sided linear formula object describing the model, with the response on the left of a ~ operator and the terms, separated by + operators, on the right.
data	an optional data frame containing the variables named in model, correlation, weights, and subset. By default the variables are taken from the environment from which gls is called.
correlation	an optional <a href="#">corStruct</a> object describing the within-group correlation structure. See the documentation of <a href="#">corClasses</a> for a description of the available corStruct classes. If a grouping variable is to be used, it must be specified in the form argument to the corStruct constructor. Defaults to NULL, corresponding to uncorrelated errors.
basis.x	List of basis for functional explanatory data estimation.
basis.b	List of basis for $\beta(t)$ parameter estimation.
rn	List of Ridge parameter.
lambda	List of Roughness penalty parameter.
weights	an optional <a href="#">varFunc</a> object or one-sided formula describing the within-group heteroscedasticity structure. If given as a formula, it is used as the argument to <a href="#">varFixed</a> , corresponding to fixed variance weights. See the documentation on <a href="#">varClasses</a> for a description of the available <a href="#">varFunc</a> classes. Defaults to NULL, corresponding to homoscedastic errors.
subset	an optional expression indicating which subset of the rows of data should be used in the fit. This can be a logical vector, or a numeric vector indicating which observation numbers are to be included, or a character vector of the row names to be included. All observations are included by default.
method	a character string. If "REML" the model is fit by maximizing the restricted log-likelihood. If "ML" the log-likelihood is maximized. Defaults to "REML".
control	a list of control values for the estimation algorithm to replace the default values returned by the function <a href="#">glsControl</a> . Defaults to an empty list.



verbose	an optional logical value. If TRUE information on the evolution of the iterative algorithm is printed. Default is FALSE.
criteria	GCCV criteria, see <a href="#">GCCV.S</a> .
...	some methods for this generic require additional arguments. None are used in this methodl.

## Value

an object of class "gls" representing the functional linear model fit. Generic functions such as `print`, `plot`, and `summary` have methods to show the results of the fit.

See `glsObject` for the components of the fit. The functions `resid`, `coef` and `fitted`, can be used to extract some of its components.

Beside, the class(z) is "gls", "lm" and "fregre.lm" with the following objects:

sr2	Residual variance.
Vp	Estimated covariance matrix for the parameters.
lambda	A roughness penalty.
basis.x	Basis used for fdata or fd covariates.
basis.b	Basis used for beta parameter estimation.
beta.l	List of estimated beta parameter of functional covariates.
data	List that containing the variables in the model.
formula	formula used in ajusted model.
formula.ini	formula in call.
XX	desing matrix
W	inverse of covariance matrix
correlation	See <code>glsObject</code> for the components of the fit.

## References

Oviedo de la Fuente, M., Febrero-Bande, M., Pilar Munoz, and Dominguez, A. Predicting seasonal influenza transmission using Functional Regression Models with Temporal Dependence. arXiv:1610.08718. <https://arxiv.org/abs/1610.08718>

## Examples

```
## Not run:
data(tecator)
x=tecator$absorp.fdata
x.d2<-fdata.deriv(x,nderiv=)
tt<-x[["argvals"]]
dataf=as.data.frame(tecator$y)

# plot the response
plot(ts(tecator$y$Fat))
```

```

nbasis.x=11;nbasis.b=7
basis1=create.bspline.basis(rangeval=range(tt),nbasis=nbasis.x)
basis2=create.bspline.basis(rangeval=range(tt),nbasis=nbasis.b)
basis.x=list("x.d2"=basis1)
basis.b=list("x.d2"=basis2)
ldata=list("df"=dataf,"x.d2"=x.d2)
res.gls=fregre.gls(Fat~x.d2,data=ldata, correlation=corAR1(),
                  basis.x=basis.x,basis.b=basis.b)
summary(res.gls)

## End(Not run)

```

fregre.gsam

*Fitting Functional Generalized Spectral Additive Models***Description**

Computes functional GAM model between functional covariate  $(X^1(t_1), \dots, X^q(t_q))$  (and non functional covariate  $(Z^1, \dots, Z^p)$ ) and scalar response  $Y$ .

This function is an extension of the functional generalized linear regression models: [fregre.glm](#) where the  $E[Y|X, Z]$  is related to the linear prediction  $\eta$  via a link function  $g(\cdot)$  with integrated smoothness estimation by the smooth functions  $f(\cdot)$ .

$$E[Y|X, Z] = \eta = g^{-1}\left(\alpha + \sum_{i=1}^p f_i(Z^i) + \sum_{k=1}^q \sum_{j=1}^{k_q} f_j^k(\xi_j^k)\right)$$

where  $\xi_j^k$  is the coefficient of the basis function expansion of  $X^k$ , (in PCA analysis  $\xi_j^k$  is the score of the  $j$ -functional PC of  $X^k$ ).

The smooth functions  $f(\cdot)$  can be added to the right hand side of the formula to specify that the linear predictor depends on smooth functions of predictors using smooth terms **s** and **te** as in [gam](#) (or linear functionals of these as  $Z\beta$  and  $\langle X(t), \beta \rangle$  in [fregre.glm](#)).

**Usage**

```

fregre.gsam(formula, family = gaussian(), data=list(),
weights=NULL, basis.x=NULL, basis.b=NULL, CV=FALSE, ...)

```

**Arguments**

- |         |   |
|---------|---|
| formula | an object of class formula (or one that can be coerced to that class): a symbolic description of the model to be fitted. The details of model specification are given under <a href="#">Details</a> .   |
| family  | a description of the error distribution and link function to be used in the model. This can be a character string naming a family function, a family function or the result of a call to a family function. (See <a href="#">family</a> for details of family functions.) |

data	List that containing the variables in the model.
weights	weights
basis.x	List of basis for functional explanatory data estimation.
basis.b	List of basis for functional beta parameter estimation.
CV	=TRUE, Cross-validation (CV) is done.
...	Further arguments passed to or from other methods.

### Details

The first item in the data list is called "df" and is a data frame with the response and non functional explanatory variables, as [gam](#).

Functional covariates of class fdata or fd are introduced in the following items in the data list. basis.x is a list of basis for represent each functional covariate. The basis object can be created by the function: [create.pc.basis](#), [pca.fd](#) [create.pc.basis](#), [create.fdata.basis](#) o [create.basis](#). basis.b is a list of basis for represent each functional beta parameter. If basis.x is a list of functional principal components basis (see [create.pc.basis](#) or [pca.fd](#)) the argument basis.b is ignored.

### Value

Return gam object plus:

basis.x	Basis used for fdata or fd covariates.
basis.b	Basis used for beta parameter estimation.
data	List that containing the variables in the model.
formula	formula.
CV	\\$y.pred predicted response by cross-validation.

### Note

If the formula only contains a non functional explanatory variables (multivariate covariates), the function compute a standard [glm](#) procedure.

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <[manuel.oviedo@usc.es](mailto:manuel.oviedo@usc.es)>

### References

- Muller HG and Stadtmuller U. (2005). *Generalized functional linear models*. Ann. Statist.33 774-805.
- Wood (2001) *mgcv:GAMs and Generalized Ridge Regression for R*. R News 1(2):20-25.
- Ramsay, James O., and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.
- Venables, W. N. and Ripley, B. D. (2002) *Modern Applied Statistics with S*, New York: Springer.

**See Also**

See Also as: [predict.fregre.gsam](#) and [summary.gam](#).  
Alternative methods: [fregre.glm](#) and [fregre.gkam](#).

**Examples**

```

data(tecator)
x=tecator$absorp.fdata
x.d1<-fdata.deriv(x)
tt<-x[["argvals"]]
dataf=as.data.frame(tecator$y)
nbasis.x=11;nbasis.b=5
basis1=create.bspline.basis(rangeval=range(tt),nbasis=nbasis.x)
basis2=create.bspline.basis(rangeval=range(tt),nbasis=nbasis.b)
f=Fat~s(Protein)+s(x)
basis.x=list("x"=basis1,"x.d1"=basis1)
basis.b=list("x"=basis2,"x.d1"=basis2)
ldata=list("df"=dataf,"x"=x,"x.d1"=x.d1)
res=fregre.gsam(Fat~Water+s(Protein)+x+s(x.d1),ldata,family=gaussian(),
basis.x=basis.x,basis.b=basis.b)
res

## Not run:
res2=fregre.gsam(Fat~te(Protein,k=3)+x,data=ldata,family=gaussian())
summary(res2)

## dropind basis pc
basis.pc0=create.pc.basis(x,c(2,4,7))
basis.pc1=create.pc.basis(x.d1,c(1:3))
basis.x=list("x"=basis.pc0,"x.d1"=basis.pc1)
ldata=list("df"=dataf,"x"=x,"x.d1"=x.d1)
res.pc=fregre.gsam(f,data=ldata,family=gaussian(),
basis.x=basis.x,basis.b=basis.b)
summary(res.pc)

## Binomial family
x=tecator$absorp.fdata
tecator$y$Fat<-ifelse(tecator$y$Fat>20,1,0)
x.d1<-fdata.deriv(x)
dataf=as.data.frame(tecator$y)
ldata=list("df"=dataf,"x"=x,"x.d1"=x.d1)
res.bin=fregre.gsam(Fat~Protein+s(x),ldata,family=binomial())

## End(Not run)

```

**Description**

This function fits iteratively a functional linear model using generalized least squares. The errors are allowed to be correlated and/or have unequal variances.

1. Begin with a preliminary estimation of  $\hat{\theta} = \theta_0$  (for instance,  $\theta_0 = 0$ ). Compute  $\hat{W}$ .
2. Estimate  $b_{\Sigma} = (Z'\hat{W}Z)^{-1}Z'\hat{W}y$
3. Based on the residuals,  $\hat{e} = (y - Zb_{\Sigma})$ , update  $\hat{\theta} = \rho(\hat{e})$  where  $\rho$  depends on the dependence structure chosen.
4. Repeats steps 2 and 3 until convergence (small changes in  $b_{\Sigma}$  and/or  $\hat{\theta}$ ).

**Usage**

```
fregre.igls(formula,data,basis.x=NULL,basis.b=NULL,
correlation,maxit=100,rn,lambda,weights=rep(1,n),
control,...)
```

**Arguments**

formula	a two-sided linear formula object describing the model, with the response on the left of a $\sim$ operator and the terms, separated by + operators, on the right.
data	an optional data frame containing the variables named in model, correlation, weights, and subset. By default the variables are taken from the environment from which gls is called.
basis.x	List of basis for functional explanatory data estimation.
basis.b	List of basis for $\beta(t)$ parameter estimation.
rn	List of Ridge parameter.
lambda	List of Roughness penalty parameter.
correlation	an optional <a href="#">corStruct</a> object describing the within-group correlation structure. See the documentation of <a href="#">corClasses</a> for a description of the available corStruct classes. If a grouping variable is to be used, it must be specified in the form argument to the corStruct constructor. Defaults to NULL, corresponding to uncorrelated errors.
maxit	Number of maximum of interactions.
weights	an optional <a href="#">varFunc</a> object or one-sided formula describing the within-group heteroscedasticity structure. If given as a formula, it is used as the argument to <a href="#">varFixed</a> , corresponding to fixed variance weights. See the documentation on <a href="#">varClasses</a> for a description of the available varFunc classes. Defaults to NULL, corresponding to homoscedastic errors.
control	a list of control values for the estimation algorithm to replace the default values returned by the function <a href="#">glsControl</a> . Defaults to an empty list.
...	some methods for this generic require additional arguments. None are used in this method.

**Value**

an object of class "gls" representing the functional linear model fit. Generic functions such as `print`, `plot`, and `summary` have methods to show the results of the fit.

See `glsObject` for the components of the fit. The functions `resid`, `coef` and `fitted`, can be used to extract some of its components.

Beside, the class(z) is "gls", "lm" and "fregre.lm" with the following objects:

<code>sr2</code>	Residual variance.
<code>Vp</code>	Estimated covariance matrix for the parameters.
<code>lambda</code>	A roughness penalty.
<code>basis.x</code>	Basis used for fdata or fd covariates.
<code>basis.b</code>	Basis used for beta parameter estimation.
<code>beta.l</code>	List of estimated beta parameter of functional covariates.
<code>data</code>	List that containing the variables in the model.
<code>formula</code>	formula used in ajusted model.
<code>formula.ini</code>	formula in call.
<code>XX</code>	desing matrix
<code>W</code>	inverse of covariance matrix
<code>fdataob</code>	
<code>rn</code>	rn
<code>vs.list</code>	
<code>correlation</code>	See <code>glsObject</code> for the components of the fit.

**References**

Oviedo de la Fuente, M., Febrero-Bande, M., Pilar Munoz, and Dominguez, A. Predicting seasonal influenza transmission using Functional Regression Models with Temporal Dependence. arXiv:1610.08718. <https://arxiv.org/abs/1610.08718>

**Examples**

```
## Not run:
data(tecator)
x=tecator$absorp.fdata
x.d2<-fdata.deriv(x,nderiv=)
tt<-x[["argvals"]]
dataf=as.data.frame(tecator$y)
# plot the response
plot(ts(tecator$y$Fat))
ldata=list("df"=dataf,"x.d2"=x.d2)
res.gls=fregre.igls(Fat~x.d2,data=ldata,
  correlation=list("cor.ARMA"=list()),control=list("p"=1))
res.gls
```

```
res.gls$corStruct
## End(Not run)
```

fregre.lm

*Fitting Functional Linear Models***Description**

Computes functional regression between functional (and non functional) explanatory variables and scalar response using basis representation.

This section is presented as an extension of the linear regression models: [fregre.pc](#), [fregre.pls](#) and [fregre.basis](#). Now, the scalar response  $Y$  is estimated by more than one functional covariate  $X^j(t)$  and also more than one non functional covariate  $Z^j$ . The regression model is given by:

$$E[Y|X, Z] = \alpha + \sum_{j=1}^p \beta_j Z^j + \sum_{k=1}^q \frac{1}{\sqrt{T_k}} \int_{T_k} X^k(t) \beta_k(t) dt$$

where  $Z = [Z^1, \dots, Z^p]$  are the non functional covariates,  $X(t) = [X^1(t_1), \dots, X^q(t_q)]$  are the functional ones and  $\epsilon$  are random errors with mean zero, finite variance  $\sigma^2$  and  $E[X(t)\epsilon] = 0$ .

**Usage**

```
fregre.lm(formula, data, basis.x=NULL, basis.b=NULL, rn,
          lambda, weights=rep(1, n), ...)
```

**Arguments**

formula	an object of class formula (or one that can be coerced to that class): a symbolic description of the model to be fitted. The details of model specification are given under Details.
data	List that containing the variables in the model.
basis.x	List of basis for functional explanatory data estimation.
basis.b	List of basis for functional beta parameter estimation.
rn	List of Ridge parameter.
lambda	List of Roughness penalty parameter.
weights	weights
...	Further arguments passed to or from other methods.

## Details

The first item in the data list is called "df" and is a data frame with the response and non functional explanatory variables, as `lm`. Functional covariates of class `fdata` or `fd` are introduced in the following items in the data list.

`basis.x` is a list of basis for represent each functional covariate. The basis object can be created by the function: `create.pc.basis`, `pca.fd` `create.pc.basis`, `create.fdata.basis` or `create.basis`.

`basis.b` is a list of basis for represent each functional  $\beta_k$  parameter. If `basis.x` is a list of functional principal components basis (see `create.pc.basis` or `pca.fd`) the argument `basis.b` (*is unnecessary and*) is ignored.

The user can penalty the basis elements by: (i) `lambda` is a list of rough penalty values for the second derivative of each functional covariate, see `fregre.basis` for more details.

(ii) `rn` is a list of Ridge penalty value for each functional covariate, see `fregre.pc`, `fregre.pls` and `P.penalty` for more details.

Note: For the case of the Functional Principal Components basis two penalties are allowed (but not the two together).

## Value

Return `lm` object plus:

<code>sr2</code>	Residual variance.
<code>Vp</code>	Estimated covariance matrix for the parameters.
<code>lambda</code>	A roughness penalty.
<code>basis.x</code>	Basis used for <code>fdata</code> or <code>fd</code> covariates.
<code>basis.b</code>	Basis used for beta parameter estimation.
<code>beta.l</code>	List of estimated beta parameter of functional covariates.
<code>data</code>	List that containing the variables in the model.
<code>formula</code>	formula.

## Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

## References

Ramsay, James O., and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>



**See Also**

See Also as: [predict.fregre.lm](#) and [summary.lm](#).  
Alternative method: [fregre.glm](#).

**Examples**

```

data(tecator)
x=tecator$absorp.fdata
y=tecator$y$Fat
tt=x[["argvals"]]
dataf=as.data.frame(tecator$y)

nbasis.x=11
nbasis.b=7
basis1=create.bspline.basis(rangeval=range(tt),nbasis=nbasis.x)
basis2=create.bspline.basis(rangeval=range(tt),nbasis=nbasis.b)

f=Fat~Protein+x
basis.x=list("x"=basis1)
basis.b=list("x"=basis2)
ldata=list("df"=dataf,"x"=x)
res=fregre.lm(f,ldata,basis.x=basis.x,basis.b=basis.b)
summary(res)

f2=Fat~Protein+xd
xd=fdata.deriv(x,nderiv=2,class.out='fdata',nbasis=nbasis.x)
ldata2=list("df"=dataf,"xd"=xd)
basis.x2=list("xd"=basis1)
basis.b2=list("xd"=basis2)
res2=fregre.lm(f2,ldata2,basis.x=basis.x2,basis.b=basis.b2)
summary(res2)

par(mfrow=c(2,1))
plot(res$beta.l$x,main="functional beta estimation")
plot(res2$beta.l$xd,col=2)

```

fregre.np

*Functional regression with scalar response using non-parametric kernel estimation*

**Description**

Computes functional regression between functional explanatory variables and scalar response using kernel estimation. The non-parametric functional regression model can be written as follows

$$y_i = r(X_i) + \epsilon_i$$

where the unknown smooth real function  $r$  is estimated using kernel estimation by means of

$$\hat{r}(X) = \frac{\sum_{i=1}^n K(h^{-1}d(X, X_i))y_i}{\sum_{i=1}^n K(h^{-1}d(X, X_i))}$$

where  $K$  is an kernel function (see `Ker` argument),  $h$  is the smoothing parameter and  $d$  is a metric or a semi-metric (see `metric` argument).

### Usage

```
fregre.np(fdataobj, y, h=NULL, Ker=AKer.norm, metric=metric.lp,
type.S=S.NW, par.S=list(w=1), ...)
```

### Arguments

<code>fdataobj</code>	<code>fdata</code> class object.
<code>y</code>	Scalar response with length <code>n</code> .
<code>h</code>	Bandwidth, $h > 0$ . Default argument values are provided as the 5%-quantile of the distance between <code>fdataobj</code> curves, see <a href="#">h.default</a> .
<code>Ker</code>	Type of asymmetric kernel used, by default asymmetric normal kernel.
<code>metric</code>	Metric function, by default <a href="#">metric.lp</a> .
<code>type.S</code>	Type of smothing matrix <code>S</code> . By default <code>S</code> is calculated by Nadaraya-Watson kernel estimator ( <code>S.NW</code> ).
<code>par.S</code>	List of parameters for <code>type.S</code> : <code>w</code> , the weights.
<code>...</code>	Arguments to be passed for <a href="#">metric.lp</a> or other metric function.

### Details

The distance between curves is calculated using the [metric.lp](#) although any other semimetric could be used (see [semimetric.basis](#) or [semimetric.NPFDA](#) functions). The kernel is applied to a metric or semi-metrics that provides non-negative values, so it is common to use asymmetric kernels. Different asymmetric kernels can be used, see [Kernel.asymmetric](#).

### Value

Return:

<code>call</code>	The matched call.
<code>fitted.values</code>	Estimated scalar response.
<code>H</code>	Hat matrix.
<code>residuals</code>	<code>y</code> minus fitted values.
<code>df</code>	The residual degrees of freedom.
<code>r2</code>	Coefficient of determination.
<code>sr2</code>	Residual variance.

y	Response.
fdataobj	Functional explanatory data.
mdist	Distance matrix between x and newx.
Ker	Asymmetric kernel used.
h.opt	smoothing parameter or bandwidth.

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

### References

Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

Hardle, W. *Applied Nonparametric Regression*. Cambridge University Press, 1994.

### See Also

See Also as: [fregre.np.cv](#), [summary.fregre.fd](#) and [predict.fregre.fd](#).  
Alternative method: [fregre.basis](#), [cand](#) and [fregre.pc](#).

### Examples

```
## Not run:
data(tecator)
absorp=tecator$absorp.fdata
ind=1:129
x=absorp[ind,]
y=tecator$y$Fat[ind]

res.np=fregre.np(x,y,Ker=AKer.epa)
summary.fregre.fd(res.np)
res.np2=fregre.np(x,y,Ker=AKer.tri)
summary.fregre.fd(res.np2)

# with other semimetrics.
res.pca1=fregre.np(x,y,Ker=AKer.tri,metri=semimetric.pca,q=1)
summary.fregre.fd(res.pca1)
res.deriv=fregre.np(x,y,metri=semimetric.deriv)
summary.fregre.fd(res.deriv)
x.d2=fdata.deriv(x,nderiv=1,method="fmm",class.out='fdata')
res.deriv2=fregre.np(x.d2,y)
summary.fregre.fd(res.deriv2)
x.d3=fdata.deriv(x,nderiv=1,method="bspline",class.out='fdata')
```

```
res.deriv3=fregre.np(x.d3,y)
summary.fregre.fd(res.deriv3)

## End(Not run)
```

---

fregre.np.cv                      *Cross-validation functional regression with scalar response using kernel estimation.*

---

## Description

Computes functional regression between functional explanatory variables and scalar response using asymmetric kernel estimation by cross-validation method.

The non-parametric functional regression model can be written as follows

$$y_i = r(X_i) + \epsilon_i$$

where the unknown smooth real function  $r$  is estimated using kernel estimation by means of

$$\hat{r}(X) = \frac{\sum_{i=1}^n K(h^{-1}d(X, X_i))y_i}{\sum_{i=1}^n K(h^{-1}d(X, X_i))}$$

where  $K$  is an kernel function (see `Ker` argument),  $h$  is the smoothing parameter and  $d$  is a metric or a semi-metric (see `metric` argument).

The function estimates the value of smoothing parameter (also called bandwidth)  $h$  through Generalized Cross-validation GCV criteria, see [GCV.S](#) or [CV.S](#).

## Usage

```
fregre.np.cv(fdataobj,y,h=NULL,Ker=AKer.norm,metric=metric.lp,
type.CV = GCV.S,type.S=S.NW,par.CV=list(trim=0),par.S=list(w=1),...)
```

## Arguments

<code>fdataobj</code>	<a href="#">fdata</a> class object.
<code>y</code>	Scalar response with length <code>n</code> .
<code>h</code>	Bandwidth, $h > 0$ . Default argument values are provided as the sequence of length 25 from 2.5%-quantile to 25%-quantile of the distance between <code>fdataobj</code> curves, see <a href="#">h.default</a> .
<code>Ker</code>	Type of asymmetric kernel used, by default asymmetric normal kernel.
<code>metric</code>	Metric function, by default <a href="#">metric.lp</a> .
<code>type.CV</code>	Type of cross-validation. By default generalized cross-validation <a href="#">GCV.S</a> method.
<code>type.S</code>	Type of smothing matrix $S$ . By default $S$ is calculated by Nadaraya-Watson kernel estimator ( $S.NW$ ).
<code>par.CV</code>	List of parameters for <code>type.CV</code> : <code>trim</code> , the alpha of the trimming and <code>draw=TRUE</code> .
<code>par.S</code>	List of parameters for <code>type.S</code> : <code>w</code> , the weights.
<code>...</code>	Arguments to be passed for <a href="#">metric.lp</a> or other metric function.

## Details

The function estimates the value of smoothing parameter or the bandwidth through the cross validation methods: `GCV.S` or `CV.S`. It computes the distance between curves using the `metric.lp`, although any other semimetric could be used (see `semimetric.basis` or `semimetric.NPFDA` functions). Different asymmetric kernels can be used, see `Kernel.asymmetric`.

## Value

Return:

<code>call</code>	The matched call.
<code>residuals</code>	<code>y</code> minus fitted values.
<code>fitted.values</code>	Estimated scalar response.
<code>df</code>	The residual degrees of freedom.
<code>r2</code>	Coefficient of determination.
<code>sr2</code>	Residual variance.
<code>H</code>	Hat matrix.
<code>y</code>	Response.
<code>fdataobj</code>	Functional explanatory data.
<code>mdist</code>	Distance matrix between <code>x</code> and <code>newx</code> .
<code>Ker</code>	Asymmetric kernel used.
<code>gcv</code>	CV or GCV values.
<code>h.opt</code>	smoothing parameter or bandwidth that minimizes CV or GCV method.
<code>h</code>	Vector of smoothing parameter or bandwidth.
<code>cv</code>	List with the fitted values and residuals estimated by CV, without the same curve.

## Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

## References

- Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.
- Hardle, W. *Applied Nonparametric Regression*. Cambridge University Press, 1994.
- Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

## See Also

See Also as: `fregre.np`, `summary.fregre.fd` and `predict.fregre.fd`.  
Alternative method: `fregre.basis.cv` and `fregre.np.cv`.

## Examples

```
## Not run:
data(tecator)
absorp=tecator$absorp.fdata
ind=1:129
x=absorp[ind,]
y=tecator$y$Fat[ind]
Ker=AKer.tri
res.np=fregre.np.cv(x,y,Ker=Ker)
summary.fregre.fd(res.np)
res.np2=fregre.np.cv(x,y,type.CV=GCV.S,criteria="Shibata")
summary.fregre.fd(res.np2)

## Example with other semimetrics (not run)
res.pca1=fregre.np.cv(x,y,Ker=Ker,metric=semimetric.pca,q=1)
summary.fregre.fd(res.pca1)
res.deriv=fregre.np.cv(x,y,Ker=Ker,metric=semimetric.deriv)
summary.fregre.fd(res.deriv)

x.d2=fdata.deriv(x,nderiv=1,method="fmm",class.out='fdata')
res.deriv2=fregre.np.cv(x.d2,y,Ker=Ker)
summary.fregre.fd(res.deriv2)
x.d3=fdata.deriv(x,nderiv=1,method="bspline",class.out='fdata')
res.deriv3=fregre.np.cv(x.d3,y,Ker=Ker)
summary.fregre.fd(res.deriv3)

## End(Not run)
```

---

fregre.pc

*Functional Regression with scalar response using Principal Components Analysis.*

---

## Description

Computes functional (ridge or penalized) regression between functional explanatory variable  $X(t)$  and scalar response  $Y$  using Principal Components Analysis.

$$Y = \langle X, \beta \rangle + \epsilon = \int_T X(t)\beta(t)dt + \epsilon$$

where  $\langle \cdot, \cdot \rangle$  denotes the inner product on  $L_2$  and  $\epsilon$  are random errors with mean zero, finite variance  $\sigma^2$  and  $E[X(t)\epsilon] = 0$ .

## Usage

```
fregre.pc(fdataobj, y, l = NULL, lambda=0, P=c(1,0,0),
weights = rep(1, len = n),...)
```

**Arguments**

fdataobj	fdata class object or fdata.comp class object created by <code>create.pc.basis</code> function.
y	Scalar response with length n.
l	Index of components to include in the model. If is null 1 (by default), 1=1:3.
lambda	Amount of penalization. Default value is 0, i.e. no penalization is used.
P	If P is a vector: P are coefficients to define the penalty matrix object, see <code>P.penalty</code> . If P is a matrix: P is the penalty matrix object.
weights	weights
...	Further arguments passed to or from other methods.

**Details**

The function computes the  $\{\nu_k\}_{k=1}^{\infty}$  orthonormal basis of functional principal components to represent the functional data as  $X_i(t) = \sum_{k=1}^{\infty} \gamma_{ik} \nu_k$  and the functional parameter as  $\beta(t) = \sum_{k=1}^{\infty} \beta_k \nu_k$ , where  $\gamma_{ik} = \langle X_i(t), \nu_k \rangle$  and  $\beta_k = \langle \beta, \nu_k \rangle$ .

The response can be fitted by:

- $\lambda = 0$ , no penalization,

$$\hat{y} = \nu_k^\top (\nu_k^\top \nu_k)^{-1} \nu_k^\top y$$

- Ridge regression,  $\lambda > 0$  and  $P = 1$ ,

$$\hat{y} = \nu_k^\top (\nu_k^\top \nu_k + \lambda I)^{-1} \nu_k^\top y$$

- Penalized regression,  $\lambda > 0$  and  $P \neq 0$ . For example,  $P = c(0, 0, 1)$  penalizes the second derivative (curvature) by `P=P.penalty(fdataobj["argvals"],P)`,

$$\hat{y} = \nu_k^\top (\nu_k^\top \nu_k + \lambda \nu_k^\top \mathbf{P} \nu_k)^{-1} \nu_k^\top y$$

**Value**

Return:

call	The matched call of <code>fregre.pc</code> function.
coefficients	A named vector of coefficients.
residuals	y-fitted values.
fitted.values	Estimated scalar response.
beta.est	beta coefficient estimated of class fdata
df	The residual degrees of freedom. In ridge regression, <code>df(rn)</code> is the effective degrees of freedom.
r2	Coefficient of determination.

sr2	Residual variance.
Vp	Estimated covariance matrix for the parameters.
H	Hat matrix.
l	Index of principal components selected.
lambda	Amount of shrinkage.
P	Penalty matrix.
fdata.comp	Fitted object in <code>fdata2pc</code> function.
lm	lm object.
fdataobj	Functional explanatory data.
y	Scalar response.

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

### References

- Cai TT, Hall P. 2006. *Prediction in functional linear regression*. Annals of Statistics 34: 2159-2179.
- Cardot H, Ferraty F, Sarda P. 1999. *Functional linear model*. Statistics and Probability Letters 45: 11-22.
- Hall P, Hosseini-Nasab M. 2006. *On properties of functional principal components analysis*. Journal of the Royal Statistical Society B 68: 109-126.
- Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>
- N. Kraemer, A.-L. Boulsteix, and G. Tutz (2008). *Penalized Partial Least Squares with Applications to B-Spline Transformations and Functional Data*. Chemometrics and Intelligent Laboratory Systems, 94, 60 - 69. <http://dx.doi.org/10.1016/j.chemolab.2008.06.009>

### See Also

See Also as: [fregre.pc.cv](#), [summary.fregre.fd](#) and [predict.fregre.fd](#).  
Alternative method: [fregre.basis](#) and [fregre.np](#).

### Examples

```
## Not run:
data(tecator)
absorp=tecator$absorp.fdata
ind=1:129
x=absorp[ind,]
y=tecator$y$Fat[ind]
res=fregre.pc(x,y)
summary(res)
res2=fregre.pc(x,y,l=c(1,3,4))
summary(res2)
```



```

# Functional Ridge Regression
res3=fregre.pc(x,y,l=c(1,3,4),lambda=1,P=1)
summary(res3)
# Functional Regression with 2nd derivative penalization
res4=fregre.pc(x,y,l=c(1,3,4),lambda=1,P=c(0,0,1))
summary(res4)
betas<-c(res$beta.est,res2$beta.est,res3$beta.est,res4$beta.est)
plot(betas)

## End(Not run)

```

---

fregre.pc.cv	<i>Functional penalized PC regression with scalar response using selection of number of PC components</i>
--------------	---

---

## Description

Functional Regression with scalar response using selection of number of (penalized) principal components PC through cross-validation. The algorithm selects the PC with best estimates the response. The selection is performed by cross-validation (CV) or Model Selection Criteria (MSC). After is computing functional regression using the best selection of principal components.

## Usage

```
fregre.pc.cv(fdataobj, y, kmax=8, lambda = 0, P = c(1, 0, 0),
  criteria = "SIC",weights=rep(1,len=n),...)
```

## Arguments

fdataobj	<a href="#">fdata</a> class object.
y	Scalar response with length n.
kmax	The number of components to include in the model.
lambda	Vector with the amounts of penalization. Default value is 0, i.e. no penalization is used. If lambda=TRUE the algorithm computes a sequence of lambda values.
P	The vector of coefficients to define the penalty matrix object. For example, if $P=c(1,0,0)$ , ridge regression is computed and if $P=c(0,0,1)$ , penalized regression is computed penalizing the second derivative (curvature).
criteria	Type of cross-validation (CV) or Model Selection Criteria (MSC) applied. Possible values are "CV", "AIC", "AICc", "SIC", "SICc".
weights	weights
...	Further arguments passed to <a href="#">fregre.pc</a> or <a href="#">fregre.pls</a>

## Details

The algorithm selects the best principal components `pc.opt` from the first `kmax` PC and (optionally) the best penalized parameter `lambda.opt` from a sequence of non-negative numbers `lambda`.

If `kmax` is a integer (by default and recommended) the procedure is as follows (see example 1):

- Calculate the best principal component (`pc.order[1]`) between `kmax` by `fregre.pc`.
- Calculate the second-best principal component (`pc.order [2]`) between the (`kmax-1`) by `fregre.pc` and calculate the criteria value of the two principal components.
- The process (point 1 and 2) is repeated until `kmax` principal component (`pc.order[kmax]`).
- The process (point 1, 2 and 3) is repeated for each `lambda` value.
- The method selects the principal components (`pc.opt=pc.order[1:k.min]`) and (optionally) the `lambda` parameter with minimum MSC criteria.

If `kmax` is a sequence of integer the procedure is as follows (see example 2):

- The method selects the best principal components with minimum MSC criteria by stepwise regression using `fregre.pc` in each step.
- The process (point 1) is repeated for each `lambda` value.
- The method selects the principal components (`pc.opt=pc.order[1:k.min]`) and (optionally) the `lambda` parameter with minimum MSC criteria.

Finally, is computing functional PC regression between functional explanatory variable  $X(t)$  and scalar response  $Y$  using the best selection of PC `pc.opt` and ridge parameter `rn.opt`.

The criteria selection is done by cross-validation (CV) or Model Selection Criteria (MSC).

- Predictive Cross-Validation:  $PCV(k_n) = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_{(-i,k_n)})^2$ ,  
criteria="CV"
- Model Selection Criteria:  $MSC(k_n) = \log \left[ \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \right] + p_n \frac{k_n}{n}$   
 $p_n = \frac{\log(n)}{n}$ , criteria="SIC" (by default)  
 $p_n = \frac{\log(n)}{n-k_n-2}$ , criteria="SICc"  
 $p_n = 2$ , criteria="AIC"  
 $p_n = \frac{2n}{n-k_n-2}$ , criteria="AICc"  
 $p_n = \frac{2\log(\log(n))}{n}$ , criteria="HQIC"

where `criteria` is an argument that controls the type of validation used in the selection of the smoothing parameter `kmax= kn` and penalized parameter `lambda= λ`.

**Value**

Return:

fregre.pc	Fitted regression object by the best (pc.opt) components.
pc.opt	Index of PC components selected.
MSC.min	Minimum Model Selection Criteria (MSC) value for the (pc.opt) components.
MSC	Minimum Model Selection Criteria (MSC) value for kmax components.

**Note**

criteria=``CV`` is not recommended: time-consuming.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

**See Also**

See also as: [fregre.pc](#) .

**Examples**

```
## Not run:
data(tecator)
x<-tecator$absorp.fdata[1:129]
y<-tecator$y$Fat[1:129]
# no penalization
res.pc1=fregre.pc.cv(x,y,8)
# 2nd derivative penalization
res.pc2=fregre.pc.cv(x,y,8,lambda=TRUE,P=c(0,0,1))
#Ridge regression
res.pc3=fregre.pc.cv(x,y,1:8,lambda=TRUE,P=1)

## End(Not run)
```

fregre.plm

*Semi-functional partially linear model with scalar response.***Description**

Computes functional regression between functional (and non functional) explanatory variables and scalar response using asymmetric kernel estimation.

An extension of the non-parametric functional regression models is the semi-functional partial linear model proposed in Aneiros-Perez and Vieu (2005). This model uses a non-parametric kernel procedure as that described in [fregre.np](#). The output  $y$  is scalar. A functional covariate  $X$  and a multivariate non functional covariate  $Z$  are considered.

$$y = r(X) + \sum_{j=1}^p Z_j \beta_j + \epsilon$$

The unknown smooth real function  $r$  is estimated by means of

$$\hat{r}_h(X) = \sum_{i=1}^n w_{n,h}(X, X_i) (Y_i - Z_i^T \hat{\beta}_h)$$

where  $W_h$  is the weight function:

$w_{n,h}(X, X_i) = \frac{K(d(X, X_i)/h)}{\sum_{j=1}^n K(d(X, X_j)/h)}$  with smoothing parameter  $h$ , an asymmetric kernel  $K$  and a metric or semi-metric  $d$ . In `fregre.plm()` by default  $W_h$  is a functional version of the Nadaraya-Watson-type weights (`type.S=S.NW`) with asymmetric normal kernel (`Ker=AKer.norm`) in  $L_2$  (`metric=metric.lp` with `p=2`). The unknown parameters  $\beta_j$  for the multivariate non functional covariates are estimated by means of  $\hat{\beta}_j = (\tilde{Z}_h^T \tilde{Z}_h)^{-1} \tilde{Z}_h^T \tilde{Z}_h$  where  $\tilde{Z}_h = (I - W_h)Z$  with the smoothing parameter  $h$ . The errors  $\epsilon$  are independent, with zero mean, finite variance  $\sigma^2$  and  $E[\epsilon|Z_1, \dots, Z_p, X(t)] = 0$ .

**Usage**

```
fregre.plm(formula, data, h=NULL, Ker=AKer.norm, metric=metric.lp,
type.CV = GCV.S, type.S=S.NW, par.CV=list(trim=0, draw=FALSE),
par.S=list(w=1), ...)
```

**Arguments**

formula	an object of class <code>formula</code> (or one that can be coerced to that class): a symbolic description of the model to be fitted. The details of model specification are given under <code>Details</code> .
data	List that containing the variables in the model.
Ker	Type of asymmetric kernel used, by default asymmetric normal kernel.

h	Bandwidth, $h > 0$ . Default argument values are provided as the sequence of length 51 from 2.5%-quantile to 25%-quantile of the distance between the functional data, see <a href="#">h.default</a> .
metric	Metric function, by default <a href="#">metric.lp</a> .
type.CV	Type of cross-validation. By default generalized cross-validation <a href="#">GCV.S</a> method.
type.S	Type of smothing matrix S. By default S is calculated by Nadaraya-Watson kernel estimator (S.NW).
par.CV	List of parameters for type.CV: trim, the alpha of the trimming and draw=TRUE.
par.S	List of parameters for type.S: w, the weights.
...	Further arguments passed to or from other methods.

### Details

The first item in the data list is called "*df*" and is a data frame with the response and non functional explanatory variables, as `link{lm}`. If non functional data into the formula then `lm` regression is performed.

Functional variable (`fdata` or `fd` class) is introduced in the second item in the data list. If only functional variable into the formula then [fregre.np.cv](#) is performed.

The function estimates the value of smoothing parameter or the bandwidth  $h$  through Generalized Cross-validation GCV criteria. It computes the distance between curves using the [metric.lp](#), although you can also use other metric function.

Different asymmetric kernels can be used, see [Kernel.asymmetric](#).

### Value

call	The matched call.
fitted.values	Estimated scalar response.
residuals	$y$ minus fitted values.
df	The residual degrees of freedom.
H	Hat matrix.
r2	Coefficient of determination.
sr2	Residual variance.
y	Scalar response.
fdataobj	Functional explanatory data.
XX	Non functional explanatory data.
mdist	Distance matrix between curves.
betah	beta coefficient estimated
data	List that containing the variables in the model.
Ker	Asymmetric kernel used.

h.opt	Value that minimizes CV or GCV method.
h	Smoothing parameter or bandwidth.
data	List that containing the variables in the model.
gcv	GCV values.
formula	formula.

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

### References

- Aneiros-Perez G. and Vieu P. (2005). *Semi-functional partial linear regression*. *Statistics & Probability Letters*, 76:1102-1110.
- Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.
- Hardle, W. *Applied Nonparametric Regression*. Cambridge University Press, 1994.
- Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. *Journal of Statistical Software*, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

### See Also

See Also as: [predict.fregre.plm](#) and [summary.fregre.fd](#)  
 Alternative methods: [fregre.lm](#), [fregre.np](#) and [fregre.np.cv](#)

### Examples

```
## Not run:
data(tecator)
x=tecator$absorp.fdata[1:129]
dataf=tecator$y[1:129,]

f=Fat~Water+x
ldata=list("df"=dataf,"x"=x)
res.plm=fregre.plm(f,ldata)
summary(res.plm)

# with 2nd derivative of functional data
x.fd=fdata.deriv(x,nderiv=2)
f2=Fat~Water+x.fd
ldata2=list("df"=dataf,"x.fd"=x.fd)
res.plm2=fregre.plm(f2,ldata2)
summary(res.plm2)

## End(Not run)
```

fregre.pls

*Functional Penalized PLS regression with scalar response***Description**

Computes functional linear regression between functional explanatory variable  $X(t)$  and scalar response  $Y$  using penalized Partial Least Squares (PLS)

$$Y = \langle \tilde{X}, \beta \rangle + \epsilon = \int_T \tilde{X}(t)\beta(t)dt + \epsilon$$

where  $\langle \cdot, \cdot \rangle$  denotes the inner product on  $L_2$  and  $\epsilon$  are random errors with mean zero, finite variance  $\sigma^2$  and  $E[\tilde{X}(t)\epsilon] = 0$ .

**Usage**

```
fregre.pls(fdataobj, y=NULL, l = NULL, lambda=0, P=c(0,0,1), ...)
```

**Arguments**

fdataobj	<a href="#">fdata</a> class object.
y	Scalar response with length n.
l	Index of components to include in the model.
lambda	Amount of penalization. Default value is 0, i.e. no penalization is used.
P	If P is a vector: P are coefficients to define the penalty matrix object. By default $P=c(0,0,1)$ penalize the second derivative (curvature) or acceleration. If P is a matrix: P is the penalty matrix object.
...	Further arguments passed to or from other methods.

**Details**

Functional (FPLS) algorithm maximizes the covariance between  $X(t)$  and the scalar response  $Y$  via the partial least squares (PLS) components. The functional penalized PLS are calculated in [fdata2pls](#) by alternative formulation of the NIPALS algorithm proposed by Kraemer and Sugiyama (2011).

Let  $\{\tilde{\nu}_k\}_{k=1}^{\infty}$  the functional PLS components and  $\tilde{X}_i(t) = \sum_{k=1}^{\infty} \tilde{\gamma}_{ik} \tilde{\nu}_k$  and  $\beta(t) = \sum_{k=1}^{\infty} \tilde{\beta}_k \tilde{\nu}_k$ . The functional linear model is estimated by:

$$\hat{y} = \langle X, \hat{\beta} \rangle \approx \sum_{k=1}^{k_n} \tilde{\gamma}_k \tilde{\beta}_k$$

The response can be fitted by:

- $\lambda = 0$ , no penalization,

$$\hat{y} = \nu_k^\top (\nu_k^\top \nu_k)^{-1} \nu_k^\top y$$

- Penalized regression,  $\lambda > 0$  and  $P \neq 0$ . For example,  $P = c(0, 0, 1)$  penalizes the second derivative (curvature) by `P=P.penalty(fdataobj["argvals"],P)`,

$$\hat{y} = \nu_k^\top (\nu_k^\top \nu_k + \lambda \nu_k^\top \mathbf{P} \nu_k)^{-1} \nu_k^\top y$$

## Value

Return:

call	The matched call of <code>fregre.pls</code> function.
beta.est	Beta coefficient estimated of class <code>fdata</code> .
coefficients	A named vector of coefficients.
fitted.values	Estimated scalar response.
residuals	y-fitted values.
H	Hat matrix.
df	The residual degrees of freedom.
r2	Coefficient of determination.
GCV	GCV criterion.
sr2	Residual variance.
l	Index of components to include in the model.
lambda	Amount of shrinkage.
fdata.comp	Fitted object in <code>fdata2pls</code> function.
lm	Fitted object in <code>lm</code> function
fdataobj	Functional explanatory data.
y	Scalar response.

## Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

## References

- Preda C. and Saporta G. *PLS regression on a stochastic process*. *Comput. Statist. Data Anal.* 48 (2005): 149-158.
- N. Kraemer, A.-L. Boulsteix, and G. Tutz (2008). *Penalized Partial Least Squares with Applications to B-Spline Transformations and Functional Data*. *Chemometrics and Intelligent Laboratory Systems*, 94, 60 - 69. <http://dx.doi.org/10.1016/j.chemolab.2008.06.009>
- Martens, H., Naes, T. (1989) *Multivariate calibration*. Chichester: Wiley.
- Kraemer, N., Sugiyama M. (2011). *The Degrees of Freedom of Partial Least Squares Regression*. *Journal of the American Statistical Association*. Volume 106, 697-705.
- Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. *Journal of Statistical Software*, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>



**See Also**

See Also as: [P.penalty](#) and [fregre.pls.cv](#).  
Alternative method: [fregre.pc](#).

**Examples**

```
## Not run:
data(tecator)
x<-tecator$absorp.fdata
y<-tecator$y$Fat
res=fregre.pc(x,y,c(1:8))
summary(res)
res2=fregre.pls(x,y,c(1:8),lambda=10)
summary(res2)

## End(Not run)
```

---

fregre.pls.cv	<i>Functional penalized PLS regression with scalar response using selection of number of PLS components</i>
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**Description**

Functional Regression with scalar response using selection of number of penalized principal components PLS through cross-validation. The algorithm selects the PLS components with best estimates the response. The selection is performed by cross-validation (CV) or Model Selection Criteria (MSC). After is computing functional regression using the best selection of PLS components.

**Usage**

```
fregre.pls.cv(fdataobj, y, kmax=8, lambda = 0, P = c(0, 0, 1),
  criteria = "SIC", ...)
```

**Arguments**

fdataobj	<a href="#">fdata</a> class object.
y	Scalar response with length n.
kmax	The number of components to include in the model.
lambda	Vector with the amounts of penalization. Default value is 0, i.e. no penalization is used. If lambda=TRUE the algorithm computes a sequence of lambda values.
P	The vector of coefficients to define the penalty matrix object. For example, if $P=c(0, 0, 1)$ , penalized regression is computed penalizing the second derivative (curvature).
criteria	Type of cross-validation (CV) or Model Selection Criteria (MSC) applied. Possible values are "CV", "AIC", "AICc", "SIC", "SICc".
...	Further arguments passed to <a href="#">fregre.pls</a> .

## Details

The algorithm selects the best principal components `pls.opt` from the first `kmax` PLS and (optionally) the best penalized parameter `lambda.opt` from a sequence of non-negative numbers `lambda`.

- The method selects the best principal components with minimum MSC criteria by stepwise regression using `fregre.pls` in each step.
- The process (point 1) is repeated for each `lambda` value.
- The method selects the principal components (`pls.opt=pls.order[1:k.min]`) and (optionally) the `lambda` parameter with minimum MSC criteria.

Finally, is computing functional PLS regression between functional explanatory variable  $X(t)$  and scalar response  $Y$  using the best selection of PLS `pls.opt` and ridge parameter `rn.opt`.

The criteria selection is done by cross-validation (CV) or Model Selection Criteria (MSC).

- Predictive Cross-Validation:  $PCV(k_n) = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_{(-i,k_n)})^2$ ,  
criteria="CV"
- Model Selection Criteria:  $MSC(k_n) = \log \left[ \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \right] + p_n \frac{k_n}{n}$   
 $p_n = \frac{\log(n)}{n}$ , criteria="SIC" (by default)  
 $p_n = \frac{\log(n)}{n-k_n-2}$ , criteria="SICc"  
 $p_n = 2$ , criteria="AIC"  
 $p_n = \frac{2n}{n-k_n-2}$ , criteria="AICc"

where `criteria` is an argument that controls the type of validation used in the selection of the smoothing parameter `kmax= kn` and penalized parameter `lambda= λ`.

## Value

Return:

<code>fregre.pls</code>	Fitted regression object by the best ( <code>pls.opt</code> ) components.
<code>pls.opt</code>	Index of PLS components selected.
<code>MSC.min</code>	Minimum Model Selection Criteria (MSC) value for the ( <code>pls.opt</code> ) components.
<code>MSC</code>	Minimum Model Selection Criteria (MSC) value for <code>kmax</code> components.

## Note

`criteria='CV'` is not recommended: time-consuming.

## Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

## References

Preda C. and Saporta G. *PLS regression on a stochastic process*. Comput. Statist. Data Anal. 48 (2005): 149-158.

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

## See Also

See also as:[fregre.ppc](#) .

## Examples

```
## Not run:
data(tecator)
x<-tecator$absorp.fdata[1:129]
y<-tecator$y$Fat[1:129]
# no penalization
pls1<- fregre.ppls.cv(x,y,8)
# 2nd derivative penalization
pls2<-fregre.ppls.cv(x,y,8,lambda=0:5,P=c(0,0,1))

## End(Not run)
```

---

fregre.ppc, fregre.ppls

*Functional Penalized PC (or PLS) regression with scalar response*

---

## Description

Computes functional linear regression between functional explanatory variable  $\tilde{X}(t)$  and scalar response  $Y$  using penalized Principal Components Analysis (PPC) or Partial Least Squares (PPLS), where  $\tilde{X}(t) = MX(t)$  with  $M = (I + \lambda P)^{-1}$ .

$$Y = \langle \tilde{X}, \beta \rangle + \epsilon = \int_T \tilde{X}(t)\beta(t)dt + \epsilon$$

where  $\langle \cdot, \cdot \rangle$  denotes the inner product on  $L_2$  and  $\epsilon$  are random errors with mean zero, finite variance  $\sigma^2$  and  $E[\tilde{X}(t)\epsilon] = 0$ .

## Usage

```
fregre.ppc(fdataobj, y, l = NULL, lambda=0, P=c(0,0,1), ...)
fregre.ppls(fdataobj, y=NULL, l = NULL, lambda=0, P=c(0,0,1), ...)
```

**Arguments**

<code>fdataobj</code>	<code>fdata</code> class object.
<code>y</code>	Scalar response with length <code>n</code> .
<code>l</code>	Index of components to include in the model.
<code>lambda</code>	Amount of penalization. Default value is 0, i.e. no penalization is used.
<code>P</code>	If <code>P</code> is a vector: <code>P</code> are coefficients to define the penalty matrix object. By default <code>P=c(0, 0, 1)</code> penalize the second derivative (curvature) or acceleration. If <code>P</code> is a matrix: <code>P</code> is the penalty matrix object.
<code>...</code>	Further arguments passed to or from other methods.

**Details**

The function computes the  $\{\nu_k\}_{k=1}^{\infty}$  orthonormal basis of functional PC (or PLS) to represent the functional data as  $\tilde{X}_i(t) = \sum_{k=1}^{\infty} \gamma_{ik} \nu_k$ , where  $\tilde{X} = MX$  with  $M = (I + \lambda P)^{-1}$ ,  $\gamma_{ik} = \langle \tilde{X}_i(t), \nu_k \rangle$ .

The functional penalized PC are calculated in `fdata2ppc`.

Functional (FPLS) algorithm maximizes the covariance between  $\tilde{X}(t)$  and the scalar response  $Y$  via the partial least squares (PLS) components. The functional penalized PLS are calculated in `fdata2pppls` by alternative formulation of the NIPALS algorithm proposed by Kraemer and Sugiyama (2011).

Let  $\{\tilde{\nu}_k\}_{k=1}^{\infty}$  the functional PLS components and  $\tilde{X}_i(t) = \sum_{k=1}^{\infty} \tilde{\gamma}_{ik} \tilde{\nu}_k$  and  $\beta(t) = \sum_{k=1}^{\infty} \tilde{\beta}_k \tilde{\nu}_k$ . The functional linear model is estimated by:

$$\hat{y} = \langle \tilde{X}, \hat{\beta} \rangle \approx \sum_{k=1}^{k_n} \tilde{\gamma}_k \tilde{\beta}_k$$

**Value**

Return:

<code>call</code>	The matched call of <code>fregre.pls</code> function.
<code>beta.est</code>	Beta coefficient estimated of class <code>fdata</code> .
<code>coefficients</code>	A named vector of coefficients.
<code>fitted.values</code>	Estimated scalar response.
<code>residuals</code>	y-fitted values.
<code>H</code>	Hat matrix.
<code>df</code>	The residual degrees of freedom.
<code>r2</code>	Coefficient of determination.
<code>GCV</code>	GCV criterion.
<code>sr2</code>	Residual variance.
<code>l</code>	Index of components to include in the model.

rn	Amount of shrinkage.
fdata.comp	Fitted object in <code>fdata2pls</code> function.
lm	Fitted object in <code>lm</code> function
fdataobj	Functional explanatory data.
y	Scalar response.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

- Preda C. and Saporta G. *PLS regression on a stochastic process*. *Comput. Statist. Data Anal.* 48 (2005): 149-158.
- Kraemer, N., Sugiyama M. (2011). *The Degrees of Freedom of Partial Least Squares Regression*. *Journal of the American Statistical Association*. Volume 106, 697-705.
- Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. *Journal of Statistical Software*, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

**See Also**

See Also as: [P.penalty](#), [fregre.ppc.cv](#) and [fregre.ppls.cv](#).  
Alternative method: [fregre.pc](#), and [fregre.pls](#).

---

fregre.ppc.cv	<i>Functional penalized PC (or PLS) regression with scalar response using selection of number of PC (or PLS) components</i>
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---

**Description**

Functional Regression with scalar response using selection of number of penalized principal components PPC(or partial least squares components PPLS) through cross-validation. The algorithm selects the PPLS components with best estimates the response. The selection is performed by cross-validation (CV) or Model Selection Criteria (MSC). After is computing functional regression using the best selection of PPC (or PPLS) components.

**Usage**

```
fregre.ppc.cv(fdataobj, y, kmax=8, lambda = 0, P = c(0, 0, 1),
  criteria = "SIC", ...)
```

```
fregre.ppls.cv(fdataobj, y, kmax=8, lambda = 0, P = c(0, 0, 1),
  criteria = "SIC", ...)
```

**Arguments**

fdataobj	fdata class object.
y	Scalar response with length n.
kmax	The number of components to include in the model.
lambda	Vector with the amounts of penalization. Default value is 0, i.e. no penalization is used. If lambda=TRUE the algorithm computes a sequence of lambda values.
P	If P is a vector: P are coefficients to define the penalty matrix object. By default $P=c(0, 0, 1)$ penalize the second derivative (curvature) or acceleration. If P is a matrix: P is the penalty matrix object.
criteria	Type of cross-validation (CV) or Model Selection Criteria (MSC) applied. Possible values are "CV", "AIC", "AICc", "SIC".
...	Further arguments passed to <a href="#">fregre.ppc</a> or <a href="#">fregre.ppls</a>

**Details**

The algorithm is as follows:

- Select the bests components (pc.opt or pls.opt) with minimum MSC criteria by stepwise regression using [fregre.ppc](#) or [fregre.ppls](#) in each step.
- Fit the functional PPLS regression between  $\tilde{X}(t)$  and  $Y$  using the best selection of FPLS components pls.opt.

For more details in estimation process see [fregre.ppc](#) or [fregre.ppls](#).

The criteria selection is done by cross-validation (CV) or Model Selection Criteria (MSC).

- Predictive Cross-Validation:  $PCV(k_n) = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_{(-i, k_n)})^2$ ,  
criteria="CV"
- Model Selection Criteria:  $MSC(k_n) = \log \left[ \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \right] + p_n \frac{k_n}{n}$   
 $p_n = \frac{\log(n)}{n}$ , criteria="SIC", Schwarz information criterion (by default).  
 $p_n = \frac{\log(n)}{n-k_n-2}$ , criteria="SICc", corrected Schwarz information criterion.  
 $p_n = 2$ , criteria="AIC", Akaike information criterion.  
 $p_n = \frac{2n}{n-k_n-2}$ , criteria="AICc", corrected Akaike information criterion  
 $p_n = \frac{2\log(\log(n))}{n}$ , criteria="HQIC", Hannan-Quinn information criterion.
- The generalized minimum description length (gmdl) criteria:

$$gmdl(k_n) = \log \left[ \frac{1}{n-k_n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \right] + K_n \log \left( \frac{(n-k_n) \sum_{i=1}^n \hat{y}_i^2}{\sum_{i=1}^n (y_i - \hat{y}_i)^2} \right) + \log(n)$$

where `criteria` is an argument that controls the type of validation used in the selection of the smoothing parameter  $k_{\max} = k_n$  and penalized parameter  $\lambda = \lambda$ .

`criteria = "CV"` is not recommended: time-consuming.

### Value

Return:

<code>pls.opt</code>	Index of PC or PLS components selected.
<code>MSC.min</code>	Minimum Model Selection Criteria (MSC) value for the ( <code>pc.opt</code> or <code>pls.opt</code> ) components.
<code>MSC</code>	Minimum Model Selection Criteria (MSC) value for <code>kmax</code> components.
<code>fregre.ppls</code>	Fitted regression object by the best ( <code>pc.opt</code> or <code>pls.opt</code> ) components.

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <[manuel.oviedo@usc.es](mailto:manuel.oviedo@usc.es)>

### References

- Schwarz, G. (1978). *Estimating the dimension of a model*. The annals of statistics, 6(2), 461-464.
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- Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

### See Also

See also as: [fregre.ppls](#) and [fregre.ppc](#).

---

GCCV.S

*The generalized correlated cross-validation (GCCV) score.*

---

### Description

The generalized correlated cross-validation (GCV) score.

**Usage**

```
GCCV.S(y,S,criteria="GCCV1",W=NULL,trim=0,draw=FALSE,
       metric=metric.lp,...)
```

**Arguments**

y	Response vector with length n or Matrix of set cases with dimension (n x m), where n is the number of curves and m are the points observed in each curve.
S	Smoothing matrix, see <a href="#">S.NW</a> , <a href="#">S.LLR</a> or <a href="#">S.KNN</a> .
criteria	The penalizing function. By default "Rice" criteria. "GCCV1","GCCV2","GCCV3","GCV") Possible values are "GCCV1", "GCCV2", "GCCV3", "GCV".
W	Matrix of weights.
trim	The alpha of the trimming.
draw	=TRUE, draw the curves, the sample median and trimmed mean.
metric	Metric function, by default <a href="#">metric.lp</a> .
...	Further arguments passed to or from other methods.

**Details**

$$GCCV = \frac{\sum_{i=1}^n y_i - \hat{y}_{i,b}^2}{1 - \frac{tr(C)}{n}}$$

where  $S$  is the smoothing matrix  $S$  and:

A.-If  $C = 2S\Sigma - S\Sigma S$

B.-If  $C = S\Sigma$

C.-If  $C = S\Sigma S'$

with  $\Sigma$  is the  $n \times n$  covariance matrix with  $cor(\epsilon_i, \epsilon_j) = \sigma$

Note: Provided that  $C = I$  and the smoother matrix  $S$  is symmetric and idempotent, as is the case for many linear fitting techniques, the trace term reduces to  $n - tr[S]$ , which is proportional to the familiar denominator in GCV.

**Value**

res Returns GCCV score calculated for input parameters.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <[manuel.oviedo@usc.es](mailto:manuel.oviedo@usc.es)>

**References**

Carmack, P. S., Spence, J. S., and Schucany, W. R. (2012). Generalised correlated cross-validation. *Journal of Nonparametric Statistics*, 24(2):269–282.

Oviedo de la Fuente, M., Febrero-Bande, M., Pilar Munoz, and Dominguez, A. Predicting seasonal influenza transmission using Functional Regression Models with Temporal Dependence. arXiv:1610.08718.

<https://arxiv.org/abs/1610.08718>



**See Also**

See Also as [min.np](#).

Alternative method (independent case): [GCV.S](#)

**Examples**

```
## Not run:
data(tecator)
x=tecator$absorp.fdata
x.d2<-fdata.deriv(x,nderiv=)
tt<-x[["argvals"]]
dataf=as.data.frame(tecator$y)
y=tecator$y$Fat
# plot the response
plot(ts(tecator$y$Fat))

nbasis.x=11;nbasis.b=7
basis1=create.bspline.basis(rangeval=range(tt),nbasis=nbasis.x)
basis2=create.bspline.basis(rangeval=range(tt),nbasis=nbasis.b)
basis.x=list("x.d2"=basis1)
basis.b=list("x.d2"=basis2)
ldata=list("df"=dataf,"x.d2"=x.d2)
# No correlation
res.gls=fregre.gls(Fat~x.d2,data=ldata,
                  basis.x=basis.x,basis.b=basis.b)
# AR1 correlation
res.gls=fregre.gls(Fat~x.d2,data=ldata, correlation=corAR1(),
                  basis.x=basis.x,basis.b=basis.b)
GCCV.S(y,res.gls$H,"GCCV1",W=res.gls$W)
res.gls$gcv

## End(Not run)
```

---

GCV.S

*The generalized cross-validation (GCV) score.*


---

**Description**

The generalized cross-validation (GCV) score.

**Usage**

```
GCV.S(y,S,criteria="GCV",W=NULL,trim=0,
      draw=FALSE,metric=metric.lp,...)
```

**Arguments**

y	Matrix of set cases with dimension (n x m), where n is the number of curves and m are the points observed in each curve.
S	Smoothing matrix, see <a href="#">S.NW</a> , <a href="#">S.LLR</a> or <a href="#">S.KNN</a> .
criteria	The penalizing function. By default "Rice" criteria. Possible values are "GCV", "AIC", "FPE", "Shibata", "Rice".
W	Matrix of weights.
trim	The alpha of the trimming.
draw	=TRUE, draw the curves, the sample median and trimmed mean.
metric	Metric function, by default <a href="#">metric.lp</a> .
...	Further arguments passed to or from other methods.

**Details**

$$GCV(h) = p(h)\Xi(n^{-1}h^{-1})$$

Where  
A.-If trim=0:

$$p(h) = \left\| \sqrt{(W)} (y_i - \hat{y}_i) \right\|$$

B.-If trim>0:

$$p(h) = \frac{1}{l} \sum_{i=1}^l \left( y_i - r_i(x_i) \right)^2 w(x_i)$$

l: index of (1-trim) curves with less error. where  $h$  is the bandwidth parameter,  $w$  the weights and the penalty function  $\Xi$  can be selected from the following criteria:

- Generalized Cross-validation (GCV):

$$\Xi_{GCV}(n^{-1}h^{-1}) = (1 - n^{-1}S_{ii})^{-2}$$

- Akaike's Information Criterion (AIC):

$$\Xi_{AIC}(n^{-1}h^{-1}) = \exp(2n^{-1}S_{ii})$$

- Finite Prediction Error (FPE):

$$\Xi_{FPE}(n^{-1}h^{-1}) = \frac{(1 + n^{-1}S_{ii})}{(1 - n^{-1}S_{ii})}$$

- Shibata's model selector (Shibata):

$$\Xi_{Shibata}(n^{-1}h^{-1}) = (1 + 2n^{-1}S_{ii})$$

- Rice's bandwidth selector (Rice):

$$\Xi_{Rice}(n^{-1}h^{-1}) = (1 - 2n^{-1}S_{ii})^{-1}$$

where  $S_{ii}$  the  $i$ th diagonal element of the smoothing matrix  $S$ , in see [S.NW](#), [S.LLR](#) or [S.KNN](#).

## Value

res Returns GCV score calculated for input parameters.

## Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

## References

Wasserman, L. *All of Nonparametric Statistics*. Springer Texts in Statistics, 2006.

Hardle, W. *Applied Nonparametric Regression*. Cambridge University Press, 1994.

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

## See Also

See Also as [min.np](#).  
Alternative method: [CV.S](#)

## Examples

```
data(phoneme)
mlearn<-phoneme$learn
tt<-1:ncol(mlearn)
S1 <- S.NW(tt,2.5)
S2 <- S.LLR(tt,2.5)
gcv1 <- GCV.S(mlearn, S1)
gcv2 <- GCV.S(mlearn, S2)
gcv3 <- GCV.S(mlearn, S1,criteria="AIC")
gcv4 <- GCV.S(mlearn, S2,criteria="AIC")
gcv1; gcv2; gcv3; gcv4
```

---

gridfdata, rcombdata *Utils for generate functional data*

---

### Description

gridfdata generates n curves as lineal combination of the original curves fdataobj plus a functional trend mu.

rcombdata generates n random linear combinations of the fdataobj curves plus a functional trend mu. The coefficients of the combinations follows a normal distribution with zero mean and standard deviation sdarg.

### Usage

```
gridfdata(coef, fdataobj, mu)
```

```
rcombdata(n = 10, fdataobj, mu,  
          sdarg = rep(1, nrow(fdataobj)), norm = 1)
```

### Arguments

coef	Coefficients of the combination. A matrix with number of columns equal to number of curves in fdataobj
fdataobj	<a href="#">fdata</a> class object.
mu	Functional trend, by default $\mu = \mu(t) = 0$ . An object of class <a href="#">fdata</a> .
n	Number of curves to be generated
sdarg	Standard deviation of the coefficients.
norm	Norm of the coefficients. The norm is adjusted before the transformation for sdarg is performed.

### Value

Return the functional trajectories as a fdata class object.

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

### See Also

See Also as [rproc2fdata](#)

**Examples**

```

tt=seq(0,1,len=51)
fou3=create.fourier.basis(c(0,1),nbasis=3)
fdataobj=fdata(t(eval.basis(tt,fou3)),argvals=tt)

coef=expand.grid(0,seq(-1,1,len=11),seq(-1,1,len=11))
grid=gridfdata(coef,fdataobj)
plot(grid,lty=1)

rcomb=rcombfdata(n=51,fdataobj,mu=fdata(30*tt*(1-tt),tt))
plot(rcomb,lty=1)

```

h.default

*Calculation of the smoothing parameter (h) for a functional data***Description**

Calculation of the smoothing parameter (h) for a functional data using nonparametric kernel estimation.

**Usage**

```

h.default(fdataobj, prob=c(0.025,0.25),len=51, metric = metric.lp,
Ker = "AKer.norm", type.S ="S.NW",...)

```

**Arguments**

fdataobj	fdata class object.
prob	Range of probabilities for the quantiles of the distance matrix.
len	Vector length of smoothing parameter h to return.
metric	If is a function: name of the function to calculate the distance matrix between the curves, by default <code>metric.lp</code> . If is a matrix: distance matrix between the curves.
Ker	Type of asymmetric kernel used, by default asymmetric normal kernel.
type.S	Type of smothing matrix S. Possible values are: Nadaraya-Watson estimator "S.NW" and K nearest neighbors estimator "S.KNN"
...	Arguments to be passed for metric argument.

**Value**

Returns the vector of smoothing parameter or bandwidth h.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**See Also**

See Also as [metric.lp](#), [Kernel](#) and [S.NW](#).  
Function used in [fregre.np](#) and [fregre.np.cv](#) function.

**Examples**

```
## Not run
# data(aemet)
# h1<-h.default(aemet$temp,prob=c(0.025, 0.25),len=2)
# mdist<-metric.lp(aemet$temp)
# h2<-h.default(aemet$temp,len=2,metric=mdist)
# h3<-h.default(aemet$temp,len=2,metric=semimetric.pca,q=2)
# h4<-h.default(aemet$temp,len=2,metric=semimetric.pca,q=4)
# h1;h2;h3;h4
```

---

influence.quan

*Quantile for influence measures*


---

**Description**

Estimate the quantile of measures of influence for each observation.

**Usage**

```
## S3 method for class 'quan'
influence(model,out.influ,mue.boot=500,
smo=0.1,smoX=0.05,alpha=0.95,kmax.fix=FALSE,...)
```

**Arguments**

model	fregre.pc, fregre.basis or fregre.basis.cv object.
out.influ	influence.fdata object
mue.boot	Number of bootstrap samples
smo	Smoothing parameter as a proportion of response variance.
smoX	Smoothing parameter for fdata object as a proportion of variance-covariance matrix of the explanatory functional variable.
alpha	Significance level.
kmax.fix	The maximum number of principal components or number of basis is fixed by model object.
...	Further arguments passed to or from other methods.

**Details**

Compute the quantile of measures of influence estimated in [influence.fdata](#) for functional regression using principal components representation ([fregre.pc](#)) or basis representation ([fregre.basis](#) or [fregre.basis.cv](#)).

A smoothed bootstrap method is used to estimate the quantiles of the influence measures, which allows to point out which observations have the larger influence on estimation and prediction.

**Value**

Return:

quan.cook.for	Distance Cook Prediction Quantile.
quan.cook.est	Distance Cook Estimation Quantile.
quan.cook.peña	Peña Distance Quantile.
mues.est	Sample Cook generated.
mues.pena	Sample Peña generated.
beta.boot	Functional beta estimated by bootstrap method.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Febrero-Bande, M., Galeano, P. and Gonzalez-Manteiga, W. (2010). *Measures of influence for the functional linear model with scalar response*. Journal of Multivariate Analysis 101, 327-339.

**See Also**

See Also as: [influence.fdata](#), [fregre.basis](#), [fregre.pc](#).

**Examples**

```
## Not run:
data(tecator)
x=tecator$absorp.fdata
y=tecator$y$Fat
res=fregre.pc(x,y,1:6)

#time consuming
res.infl=influence.fdata(res)
resquan=influence.quan(res,res.infl,4,0.01,0.05,0.95)
plot(res.infl$betas,type="l",col=2)
lines(res$beta.est,type="l",col=3)
lines(resquan$betas.boot,type="l",col="gray")

res=fregre.basis(x,y)
res.infl=influence.fdata(res)
```

```

resquan=influence.quan(res,res.infl,mue.boot=4,kmax.fix=T)
plot(resquan$betas.boot,type="l",col=4)
lines(res.infl$betas,type="l",col=2)
lines(resquan$betas.boot,type="l",col="gray")

## End(Not run)

```

---

influence.fdata	<i>Functional influence measures</i>
-----------------	--------------------------------------

---

### Description

Once estimated the functional regression model with scalar response, influence.fdata function is used to obtain the functional influence measures.

### Usage

```

## S3 method for class 'fdata'
influence(model,...)

```

### Arguments

model	fregre.pc, fregre.basis or fregre.basis.cv object.
...	Further arguments passed to or from other methods.

### Details

Identify influential observations in the functional linear model in which the predictor is functional and the response is scalar.

Three statistics are introduced for measuring the influence: Distance Cook Prediction DCP, Distance Cook Estimation DCE and Distance Peña DP respectively.

### Value

Return:

DCP	Cook's Distance for Prediction.
DCE	Cook's Distance for Estimation.
DP	Peña's Distance.

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>



## References

Febrero-Bande, M., Galeano, P. and Gonzalez-Manteiga, W. (2010). *Measures of influence for the functional linear model with scalar response*. Journal of Multivariate Analysis 101, 327-339.

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

## See Also

See Also as: [fregre.pc](#), [fregre.basis](#), [influence.quan](#)

## Examples

```
## Not run:
data(tecator)
x=tecator$absorp.fdata[1:129]
y=tecator$y$Fat[1:129]

res1=fregre.pc(x,y,1:5)
# time consuming
res.infl1=influence.fdata(res1)
res2=fregre.basis(x,y)
res.infl2=influence.fdata(res2)

res<-res1
res.infl<-res.infl1
mat=cbind(y,res$fitted.values,res.infl$DCP,res.infl$DCE,res.infl$DP)
colnames(mat)=c("Resp.", "Pred.", "DCP", "DCE", "DP")
pairs(mat)

## End(Not run)
```

---

inprod.fdata

*Inner products of Functional Data Objects o class (fdata)*


---

## Description

Computes a inner products of functional data objects of class fdata.

## Usage

```
inprod.fdata(fdata1,fdata2=NULL, w = 1, ...)
```

**Arguments**

<code>fdata1</code>	Functional data 1 or curve 1. <code>fdata1\$data</code> with dimension $(n1 \times m)$ , where $n1$ is the number of curves and $m$ are the points observed in each curve.
<code>fdata2</code>	Functional data 2 or curve 2. <code>fdata2\$data</code> with dimension $(n2 \times m)$ , where $n2$ is the number of curves and $m$ are the points observed in each curve.
<code>w</code>	Vector of weights with length $m$ . If $w = 1$ approximates the metric $L_p$ by Simpson's rule. By default it uses $w = 1$
<code>...</code>	Further arguments passed to or from other methods.

**Details**

By default it uses weights  $w=1$ .

$$\langle fdata1, fdata2 \rangle = \frac{1}{\int_a^b w(x) dx} \int_a^b fdata1(x) * fdata2(x) w(x) dx$$

The observed points on each curve are equally spaced (by default) or not.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**See Also**

See also [inprod](#) and [norm.fdata](#)

**Examples**

```
x<-seq(0,2*pi,length=1001)
fx1<-sin(x)/sqrt(pi)
fx2<-cos(x)/sqrt(pi)
argv<-seq(0,2*pi,len=1001)
fdat0<-fdata(rep(0,len=1001),argv,range(argv))
fdat1<-fdata(fx1,x,range(x))
inprod.fdata(fdat1,fdat1)
inprod.fdata(fdat1,fdat1)
metric.lp(fdat1)
metric.lp(fdat1,fdat0)
norm.fdata(fdat1)
# The same
integrate(function(x){(abs(sin(x)/sqrt(pi))^2)},0,2*pi)
integrate(function(x){(abs(cos(x)/sqrt(pi))^2)},0,2*pi)
```

---

int.simpson	<i>Simpson integration</i>
-------------	----------------------------

---

**Description**

Computes the integral of `fdataobj$data` with respect to `fdataobj$argvals` using `simpson` or trapezoid rule integration.

**Usage**

```
int.simpson(fdataobj, equi=TRUE, method="TRAPZ")
```

**Arguments**

<code>fdataobj</code>	fdata object.
<code>equi</code>	=TRUE, the observed points on each curve are equally spaced (by default).
<code>method</code>	Method for numerical integration, see details.

**Details**

If `method="CSR"`, composite Simpson's rule integration is used.

If `method="ESR"`, extended Simpson's rule integration is used.

If `method="TRAPZ"`, Trapezoid rule integration is used.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**See Also**

See also [integrate](#).

**Examples**

```
x<-seq(0,2*pi,length=1001)
fx<-fdata(sin(x)/sqrt(pi),x)
fx0<-fdata(rep(0,length(x)),x)
int.simpson(fx0)
int.simpson(fx)
```

---

Kernel	<i>Symmetric Smoothing Kernels.</i>
--------	-------------------------------------

---

**Description**

Represent symmetric smoothing kernels:: normal, cosine, triweight, quartic and uniform.

```

Ker.norm=dnorm(u)
Ker.cos=ifelse(abs(u)<=1,pi/4*(cos(pi*u/2)),0)
Ker.epa=ifelse(abs(u)<=1,3/4*(1-u^2),0)
Ker.tri=ifelse(abs(u)<=1,35/32*(1-u^2)^3,0)
Ker.quar=ifelse(abs(u)<=1,15/16*(1-u^2)^2,0)
Ker.unif=ifelse(abs(u)<=1,1/2,0)

```

### Usage

```

Kernel(u, type.Ker="Ker.norm")
Ker.norm(u)
Ker.cos(u)
Ker.epa(u)
Ker.tri(u)
Ker.quar(u)
Ker.unif(u)

```

### Arguments

type.Ker	Type of Kernel. By default normal kernel.
u	Data.

### Details

Type of kernel:

```

Normal Kernel: Ker.norm
Cosine Kernel: Ker.cos
Epanechnikov Kernel: Ker.epa
Triweight Kernel: Ker.tri
Quartic Kernel: Ker.quar
Uniform Kernel: Ker.unif

```

### Value

res	Returns symmetric kernel.
-----	---------------------------

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

### References

Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.

Hardle, W. *Applied Nonparametric Regression*. Cambridge University Press, 1994.

**Examples**

```

y=qnorm(seq(.1,.9,len=100))
a<-Kernel(u=y)
b<-Kernel(type.Ker="Ker.tri",u=y)
c=Ker.cos(y)

```

---

Kernel.asymmetric	<i>Asymmetric Smoothing Kernel</i>
-------------------	------------------------------------

---

**Description**

Represent Asymmetric Smoothing Kernels: normal, cosine, triweight, quartic and uniform.

```

AKer.norm=ifelse(u>=0,2*dnorm(u),0)
AKer.cos=ifelse(u>=0,pi/2*(cos(pi*u/2)),0)
AKer.epa=ifelse(u>=0 & u<=1,3/2*(1-u^2),0)
AKer.tri=ifelse(u>=0 & u<=1,35/16*(1-u^2)^3,0)
AKer.quar=ifelse(u>=0 & u<=1,15/8*(1-u^2)^2,0)
AKer.unif=ifelse(u>=0 & u<=1,1,0)

```

**Usage**

```

Kernel.asymmetric(u,type.Ker="AKer.norm")
AKer.norm(u)
AKer.cos(u)
AKer.epa(u)
AKer.tri(u)
AKer.quar(u)
AKer.unif(u)

```

**Arguments**

type.Ker	Type of asymmetric metric kernel, by default asymmetric normal kernel.
u	Data.

**Details**

Type of Asymmetric kernel:

```

Asymmetric Normal Kernel: AKer.norm
Asymmetric Cosine Kernel: AKer.cos
Asymmetric Epanechnikov Kernel: AKer.epa
Asymmetric Triweight Kernel: AKer.tri
Asymmetric Quartic Kernel: AKer.quar
Asymmetric Uniform Kernel: AKer.unif

```

**Value**

res Returns asymmetric kernel.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.

Hardle, W. *Applied Nonparametric Regression*. Cambridge University Press, 1994.

**Examples**

```
y=qnorm(seq(.1,.9,len=100))
a<-Kernel.asymmetric(u=y)
b<-Kernel.asymmetric(type.Ker="AKer.tri",u=y)
c=AKer.cos(y)
```

---

Kernel.integrate

*Integrate Smoothing Kernels.*

---

**Description**

Represent integrate kernels: normal, cosine, triweight, quartic and uniform.

**Usage**

```
Kernel.integrate(u,Ker=Ker.norm,a=-1)
IKer.norm(u)
IKer.cos(u)
IKer.epa(u)
IKer.tri(u)
IKer.quar(u)
IKer.unif(u)
```

**Arguments**

u data  
 Ker Type of Kernel. By default normal kernel.  
 a Lower limit of integration.

**Details**

Type of integrate kernel:

Integrate Normal Kernel: IKer.norm  
Integrate Cosine Kernel: IKer.cos  
Integrate Epanechnikov Kernel: IKer.epa  
Integrate Triweight Kernel: IKer.tri  
Integrate Quartic Kernel: IKer.quar  
Integrate Uniform Kernel: IKer.unif

**Value**

res Returns integrate kernel.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.

Hardle, W. *Applied Nonparametric Regression*. Cambridge University Press, 1994.

**See Also**

See Also as: [kernel](#) and [integrate](#).

**Examples**

```
y=qnorm(seq(.1,.9,len=100))
d=IKer.tri(y)
e=IKer.cos(y)
e2=Kernel.integrate(u=y,Ker=Ker.cos)
e-e2
f=IKer.epa(y)
f2=Kernel.integrate(u=y,Ker=Ker.epa)
f-f2

plot(d,type="l",ylab="Integrate Kernel")
lines(e,col=2,type="l")
lines(f,col=4,type="l")
```

kmeans.fd

*K-Means Clustering for functional data***Description**

Perform k-means clustering on functional data.

**Usage**

```
kmeans.fd(fdataobj,ncl=2,metric=metric.lp,dfunc=func.trim.FM,
max.iter=100,par.metric=NULL,par.dfunc=list(trim=0.05),
par.ini=list(method="sample"),draw=TRUE,...)
kmeans.center.ini(fdataobj,ncl=2,metric=metric.lp,
draw=TRUE,method="sample",iter=100,par.metric=NULL,...)
```

**Arguments**

fdataobj	<a href="#">fdata</a> class object.
ncl	See details section.
metric	Metric function, by default <a href="#">metric.lp</a> .
dfunc	Type of depth measure, by default FM depth.
max.iter	Maximum number of iterations for the detection of centers.
draw	=TRUE, draw the curves in the color of the centers.
par.dfunc	List of arguments to pass to the dfunc function .
par.ini	List of arguments to pass to the kmeans.center.ini function .
method	Method for selecting initial centers. If method="Sample" (by default) takes n times a random selection by the ncl centers. The ncl curves with greater distance are the initial centers. If method="Exact" calculated all combinations of ncl centers. The ncl curves with greater distance are the initial centers (this method may be too slow).
iter	Maximum number of random samples for the initial selection of centers.
par.metric	List of arguments to pass to the metric function.
...	Further arguments passed to or from other methods.

**Details**

The method searches the locations around which are grouped data (for a predetermined number of groups).

If ncl=NULL, randomizes the initial centers, ncl=2 using kmeans.center.ini function.  
 If ncl is an integer, indicating the number of groups to classify,  
 are selected ncl initial centers using kmeans.center.ini function.



If `nc1` is a vector of integers, indicating the position of the initial centers with `length(nc1)` equal to number of groups.

If `nc1` is a `fdata` class object, `nc1` are the initial centers curves with `nrow(nc1)` number of groups.

### Value

Return:

<code>cluster</code>	Indexes of groups assigned.
<code>centers</code>	Curves centers.

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

### References

Hartigan, J. A. and Wong, M. A. (1979). *A K-means clustering algorithm*. Applied Statistics 28, 100 \-108.

### See Also

See Also generic [kmeans](#) function.

### Examples

```
## Not run:
data(phoneme)
mlearn<-phoneme$learn[c(1:50,101:150,201:250),]

#Unsupervised classification
out.fd1=kmeans.fd(mlearn,nc1=3,draw=TRUE)
out.fd2=kmeans.fd(mlearn,nc1=3,draw=TRUE,par.ini=list(method="exact"))
# Different Depth function
ind=c(17,77,126)
out.fd3=kmeans.fd(mlearn,nc1=mlearn[ind,],draw=FALSE,
dfunc=func.trim.FM,par.dfunc=list(trim=0.1))
out.fd4=kmeans.fd(mlearn,nc1=mlearn[ind,],draw=FALSE,
dfunc=func.med.FM)
out.fd5=kmeans.fd(mlearn,nc1=3,dfunc=func.trim.RPD,
max.iter=10,par.dfunc=list(dfunc="depth.FM",deriv=c(0,1,1)))
group=c(rep(1,50),rep(2,50),rep(3,50))
table(out.fd5$cluster,group)

## End(Not run)
```

---

LMDC.select	<i>Impact points selection of functional predictor and regression using local maxima distance correlation (LMDC)</i>
-------------	--

---

### Description

LMDC.select function selects impact points of functional predictor using local maxima distance correlation (LMDC) for a scalar response given.

LMDC.regre function fits a multivariate regression method using the selected impact points like covariates for a scalar response.

### Usage

```
LMDC.select(y, covar, data, tol = .06, pvalue = .05,
            plot = FALSE, local.dc = TRUE,
            smo = FALSE, verbose = FALSE)
```

```
LMDC.regre (y, covar, data, newdata, pvalue = .05, method = "lm",
            par.method = NULL, plot = FALSE, verbose = FALSE)
```

### Arguments

y	name of the response variable.
covar	vector with the names of the covariables (or points of impact) with length p.
data	data frame with length n rows and at least p + 1 columns, containing the scalar response and the potential p covariables (or points of impact) in the model.
tol	Tolerance value for distance correlation and impact point.
pvalue	pvalue of bias corrected distance correlation t-test.
plot	logical value, if TRUE plots the distance correlation curve for each covariate in multivariate case and in each discretization points (args) in the functional case.
local.dc	Compute local distance correlation.
smo	logical. If TRUE, the curve of distance correlation computed in the impact points is smoothed using B-spline representation with a suitable number of basis elements.
verbose	print iterative and relevant steps of the procedure.
newdata	An optional data frame in which to look for variables with which to predict.
method	Name of regression method used, see details. This argument is used in do.call function like "what" argument.
par.method	List of parameters used to call the method. This argument is used in do.call function like "args" argument.

## Details

String of characters corresponding to the name of the regression method called.

Model available options:

- "lm": Step-wise lm regression model (uses lm function, stats package). Recommended for linear models, test linearity using `flm.test` function.
- "gam": Step-wise gam regression model (uses gam function, mgcv package). Recommended for non-linear models.

Models that use the indicated function of the required package:

- "svm": Support vector machine (svm function, e1071 package).
- "knn": k-nearest neighbor regression (knn.reg function, FNN package).
- "lars": Least Angle Regression using Lasso (lars function, lars package).
- "glmnet": Lasso and Elastic-Net Regularized Generalized Linear Models (glmnet and cv.glmnet function, glmnet package).
- "rpart": Recursive partitioning for regression a (rpart function, rpart package).
- "flam": Fit the Fused Lasso Additive Model for a Sequence of Tuning Parameters (flam function, flam package).
- "novas": NONparametric VArable Selection (code available in <https://www.math.univ-toulouse.fr/~ferraty/SOFTWARES/NOVAS/novas-routines.R>).
- "cosso": Fit Regularized Nonparametric Regression Models Using COSSO Penalty (cosso function, cosso package).
- "npreg": kernel regression estimate of a one (1) dimensional dependent variable on p-variate explanatory data (npreg function, np package).
- "mars": Multivariate adaptive regression splines (mars function, mda package).
- "nnet": Fit Neural Networks (nnet function, nnet package).
- "lars": Fits Least Angle Regression, Lasso and Infinitesimal Forward Stagewise regression models (lars function, lars package).

## Value

LMDC.select function return a list of two elements:

cor	the value of distance correlation for each covariate.
maxLocal	index or locations of local maxima distance correlations.

LMDC.regre function return a list of following elements:

model	object corresponding to the estimated method using the selected variables
xvar	names of selected variables (impact points).
edf	Effective Degrees of Freedom.
nvar	Number of selected variables (impact points).

**Author(s)**

Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ordóñez, C., Oviedo de la Fuente, M., Roca-Pardinas, J., Rodríguez-Pérez, J. R. (2017). Determining optimum wavelengths for leaf water content estimation from reflectance: A distance correlation approach. *Chemometrics and Intelligent Laboratory Systems*. <https://doi.org/10.1016/j.chemolab.2017.12.001>. In Press.

**See Also**

See Also as: [lm](#), [gam](#), [dcor.xy](#).

**Examples**

```
## Not run:
data(tecator)
absorp=fdata.deriv(tecator$absorp.fdata,2)
ind=1:129
x=absorp[ind,]
y=tecator$y$Fat[ind]
newx=absorp[-ind,]
newy=tecator$y$Fat[-ind]

## Functional PC regression
res.pc=fregre.pc(x,y,1:6)
pred.pc=predict.fregre.fd(res.pc,newx)

# Functional regression with basis representation
res.basis=fregre.basis.cv(x,y)
pred.basis=predict.fregre.fd(res.basis,newx)

# Functional nonparametric regression
res.np=fregre.np.cv(x,y)
pred.np=predict.fregre.fd(res.np,newx)

dat <- data.frame("y"=y,x$data)
newdat <- data.frame("y"=newy,newx$data)

res.gam=fregre.gsam(y~s(x),data=list("df"=dat,"x"=x))
pred.gam=predict.fregre.gsam(res.gam,list("x"=newx))

dc.raw <- LMDC.select("y",data=dat, tol = 0.05, pvalue= 0.05,
                    plot=F, smo=T,verbose=F)
covar <- paste("X",dc.raw$maxLocal,sep="")
# Preselected design/impact points
covar
ftest<-flm.test(dat[,-1],dat[,"y"], B=500, verbose=F,
                plot.it=F,type.basis="pc",est.method="pc",p=4,G=50)
```

```

if (ftest$p.value>0.05) {
  # Linear relationship, step-wise lm is recommended
  out <- LMDC.regre("y",covar,dat,newdat,pvalue=.05,
    method ="lm",plot=F,verbose=F)
} else {
  # Non-Linear relationship, step-wise gam is recommended
  out <- LMDC.regre("y",covar,dat,newdat,pvalue=.05,
    method ="gam",plot=F,verbose=F) }

# Final design/impact points
out$xvar

# Predictions
mean((newy-pred.pc)^2)
mean((newy-pred.basis)^2)
mean((newy-pred.np)^2)
mean((newy-pred.gam)^2)
mean((newy-out$pred)^2)

## End(Not run)

```

---

MCO

---

*Mitochondrial calcium overload (MCO) data set*


---

## Description

The mitochondrial calcium overload (MCO) was measured in two groups (control and treatment) every 10 seconds during an hour in isolated mouse cardiac cells. In fact, due to technical reasons, the original experiment [see Ruiz-Meana et al. (2000)] was performed twice, using both the "intact", original cells and "permeabilized" cells (a condition related to the mitochondrial membrane).

## Usage

```
data(MCO)
```

## Format

Elements of MCO:

..\$intact: fdata class object with "intact cells"curves,

- "data": Matrix of class fdata with 89 intact cells curves (rows) measured every 10 seconds during an hour in isolated mouse cardiac cell.
- "argvals", 360 discretization points from seond 0 to 3590.
- "rangeval": range("argvals").
- "names" list with: main an overall title "Control Intact Treatment", xlab title for x axis "seconds" and ylab title for y axis "Ca".

```

.. $classintact: Factor levels of “intact cells” curves: "1" control group and "2" treatment group.

.. $permea: fdata class object with “permeabilized cells” curves (whose membrane has been re-
removed),


- "data": Matrix of class fdata with 90 permeabilized cells curves (rows) measured every 10 seconds during an hour in isolated mouse cardiac cell.
- "argvals", 360 discretization points from second 0 to 3590.
- "rangeval": range("argvals").
- "names" list with: main an overall title "Control Intact Treatment", xlab title for x axis "seconds" and ylab title for y axis "Ca".


.. $classpermea: Factor levels of “permeabilized cells” curves: "1" control group and "2" treatment group.

```

### Note

The structure of the curves during the initial period (first 180 seconds) of the experiment shows a erratic behavior (not very relevant in the experiment context) during this period.

### References

Ruiz–Meana M, Garcia-Dorado D, Pina P, Inseñe J, Agullo L, Soler–Soler J. Cariporide preserves mitochondrial proton gradient and delays ATP depletion in cardiomyocytes during ischemic conditions. *American Journal Physiology Heart Circulatory Physiology*. 2003 Sep;285(3):H999–1006.

### Examples

```

data(MCO)
names(MCO)
par(mfrow=c(1,2))
plot(MCO$intact,col=MCO$classintact)
plot(MCO$permea,col=MCO$classpermea)

```

---

metric.dist

*Distance Matrix Computation*

---

### Description

This function computes the distances between the rows of a data matrix by using the specified distance measure.

### Usage

```
metric.dist(x, y = NULL, method = "euclidean", p = 2, dscale=1, ...)
```

**Arguments**

x	Data frame 1. The dimension is (n1 x m).
y	Data frame 2. The dimension is (n2 x m).
method	The distance measure to be used. This must be one of "euclidean", "maximum", "manhattan", "canberra", "binary" or "minkowski".
p	The power of the Minkowski distance.
dscale	If scale is a numeric, the distance matrix is divided by the scale value. If scale is a function (as the mean for example) the distance matrix is divided by the corresponding value from the output of the function.
...	Further arguments passed to <code>dist</code> function.

**Details**

This function returns a distance matrix by using `dist` function.  
The matrix dimension is (n1 x n1) if y=NULL, (n1 x n2) otherwise.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**See Also**

See also `dist` for multivariate date case and `metric.lp` for functional data case

**Examples**

```
# data(iris)
# d<-metric.dist(iris[,1:4])
# matplot(d,type="l",col=as.numeric(iris[,5]))
```

---

metric.hausdorff      *Compute the Hausdorff distances between two curves.*

---

**Description**

Hausdorff distance is the greatest of all the distances from a point in one curve to the closest point in the other curve (been closest the euclidean distance).

**Usage**

```
metric.hausdorff(fdata1, fdata2 = fdata1)
```

**Arguments**

fdata1	Curves 1 of fdata class. The dimension of fdata1 object is (n1 x m), where n1 is the number of points observed in t coordinates with length m.
fdata2	Curves 2 of fdata class. The dimension of fdata2 object is (n2 x m), where n2 is the number of points observed in t coordinates with length m.

**Details**

Let  $G(X) = \{(t, X(t)) \in R^2\}$  and  $G(Y) = \{(t, Y(t)) \in R^2\}$  be two graphs of the considered curves  $X$  and  $Y$  respectively, the Hausdorff distance  $d_H(X, Y)$  is defined as,

$$d_H(X, Y) = \max \left\{ \sup_{x \in G(X)} \inf_{y \in G(Y)} d_2(x, y), \sup_{y \in G(Y)} \inf_{x \in G(X)} d_2(x, y) \right\},$$

where  $d_2(x, y)$  is the euclidean distance, see [metric.lp](#).

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**Examples**

```
## Not run:
data(poblenou)
nox<-poblenou$nox[1:6]
# Hausdorff vs maximum distance
out1<-metric.hausdorff(nox)
out2<-metric.lp(nox,lp=0)
out1
out2
par(mfrow=c(1,3))
plot(nox)
plot(hclust(as.dist(out1)))
plot(hclust(as.dist(out2)))

## End(Not run)
```

---

metric.kl

*Kullback–Leibler distance*

---

**Description**

Measures the proximity between two groups of densities (of class fdata) by computing the Kullback–Leibler distance.

**Usage**

```
metric.kl(fdata1, fdata2 = NULL, symm=TRUE,
          base=exp(1), eps=1e-10,...)
```



**Arguments**

fdata1	Functional data 1 (fdata class) with the densities. The dimension of fdata1 object is (n1 x m), where n1 is the number of densities and m is the number of coordinates of the points where the density is observed.
fdata2	Functional data 2 (fdata class) with the densities. The dimension of fdata2 object is (n2 x m).
symm	If TRUE the symmetric K–L distance is computed, see details section.
base	The logarithm base used to compute the distance.
eps	Tolerance value.
...	Further arguments passed to or from other methods.

**Details**

Kullback–Leibler distance between  $f(t)$  and  $g(t)$  is

$$metric.kl(f(t), g(t)) = \int_a^b f(t) \log \left( \frac{f(t)}{g(t)} \right) dt$$

where  $t$  are the  $m$  coordinates of the points where the density is observed (the `argvals` of the `fdata` object).

The Kullback–Leibler distance is asymmetric,

$$metric.kl(f(t), g(t)) \neq metric.kl(g(t), f(t))$$

A symmetry version of K–L distance (by default) can be obtained by

$$0.5 (metric.kl(f(t), g(t)) + metric.kl(g(t), f(t)))$$

If  $(f_i(t) = 0 \ \& \ g_j(t) = 0) \implies metric.kl(f(t), g(t)) = 0$ .

If  $|f_i(t)g_i(t)| \leq \epsilon \implies f_i(t) = f_i(t) + \epsilon$ , where  $\epsilon$  is the tolerance value (by default `eps=1e-10`).

The coordinates of the points where the density is observed (discretization points  $t$ ) can be equally spaced (by default) or not.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Kullback, S., Leibler, R.A. (1951). *On information and sufficiency*. *Annals of Mathematical Statistics*, 22: 79-86

**See Also**

See also [metric.lp](#) and [fdata](#)

**Examples**

```
## Not run:
int.simpson2<-fda.usc:::int.simpson2
n<-201
tt01<-seq(0,1,len=n)
rtt01<-c(0,1)

x1<-dbeta(tt01,20,5)
x2<-dbeta(tt01,21,5)
y1<-dbeta(tt01,5,20)
y2<-dbeta(tt01,5,21)
xy<-fdata(rbind(x1,x2,y1,y2),tt01,rtt01)
plot(xy)
round(metric.kl(xy,xy,eps=1e-5),6)
round(metric.kl(xy,xy,eps=1e-5),6)
round(metric.kl(xy,xy,eps=1e-6),6)
round(metric.kl(xy,xy,symm=FALSE,eps=1e-5),6)
round(metric.kl(xy,symm=FALSE,eps=1e-5),6)

plot(c(fdata(y1[1:101]),fdata(y2[1:101])))
metric.kl(fdata(x1))
metric.kl(fdata(x1),fdata(x2),eps=1e-5,symm=F)
metric.kl(fdata(x1),fdata(x2),eps=1e-6,symm=F)
metric.kl(fdata(y1[1:101]),fdata(y2[1:101]),eps=1e-13,symm=F)
metric.kl(fdata(y1[1:101]),fdata(y2[1:101]),eps=1e-14,symm=F)

## End(Not run)
```

---

metric.lp

*Approximates Lp-metric distances for functional data.*


---

**Description**

Measures the proximity between the functional data and curves approximating Lp-metric. If  $w = 1$  approximates the Lp-metric by Simpson's rule. By default it uses  $lp = 2$  and weights  $w = 1$ .

**Usage**

```
metric.lp(fdata1,fdata2=NULL,lp=2,w=1, dscale=1,...)
```

**Arguments**

**fdata1** Functional data 1 or curve 1. If fdata class, the dimension of fdata1\$data object is  $(n1 \times m)$ , where  $n1$  is the number of curves and  $m$  are the points observed in each curve.

fdata2	Functional data 2 or curve 2. If fdata class, the dimension of fdata2\$data object is (n2 x m), where n2 is the number of curves and m are the points observed in each curve.
lp	Lp norm, by default it uses lp = 2
w	Vector of weights with length m, If w = 1 approximates the metric Lp by Simpson's rule. By default it uses w = 1
dscale	If scale is a numeric, the distance matrix is divided by the scale value. If scale is a function (as the mean for example) the distance matrix is divided by the corresponding value from the output of the function.
...	Further arguments passed to or from other methods.

### Details

By default it uses the L2-norm with lp = 2.

$$Let f(x) = fdata1(x) - fdata2(x)$$

$$\|f\|_p = \left( \frac{1}{\int_a^b w(x)dx} \int_a^b |f(x)|^p w(x)dx \right)^{1/p}$$

The observed points on each curve are equally spaced (by default) or not.

The L $\infty$ -norm is computed with lp = 0.

$$d(fdata1(x), fdata2(x))_\infty = sup |fdata1(x) - fdata2(x)|$$

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

### References

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

### See Also

See also [semimetric.basis](#) and [semimetric.NPFDA](#)

### Examples

```
# INFERENCE PHONDAT
data(phoneme)
mlearn<-phoneme$mlearn[1:100]
mtest<-phoneme$mtest[1:100]
glearn<-phoneme$classlearn[1:100]
gtest<-phoneme$classmtest[1:100]
# Matrix of distances of curves of DATA1
```

```

mdist1<-metric.lp(mlearn)

# Matrix of distances between curves of DATA1 and curves of DATA2
mdist2<-metric.lp(mlearn,mtest,lp=2)
# mdist with L1 norm and weighth=v
v=dnorm(seq(-3,3,len=dim(mlearn)[2]))
mdist3<-metric.lp(mlearn,mtest,lp=1,w=v)
plot(1:100,mdist2[1,],type="l",ylim=c(1,max(mdist3[1,])))
lines(mdist3[1,],type="l",col="2")

# mdist with mlearn with different discretization points.
#mlearn2=mlearn
#mlearn2[["argvals"]]=seq(0,1,len=150)
#mdist5<-metric.lp(mlearn,mlearn2)
#mdist6<-metric.lp(mlearn2,mlearn)
#sum(mdist5-mdist6)
#sum(mdist1-mdist6)

x<-seq(0,2*pi,length=1001)
fx<-fdata(sin(x)/sqrt(pi),x)
fx0<-fdata(rep(0,length(x)),x)
metric.lp(fx,fx0)
# The same
integrate(function(x){(abs(sin(x)/sqrt(pi))^2)},0,2*pi)

```

---

min.basis

*Select the number of basis using GCV method.*


---

## Description

Functional data estimation via basis representation using cross-validation (CV) or generalized cross-validation (GCV) method with a roughness penalty.

## Usage

```

## S3 method for class 'basis'
min(fdataobj, type.CV=GCV.S, W=NULL, lambda=0,
numbasis=floor(seq(ncol(fdataobj)/16, ncol(fdataobj)/2, len=10)),
type.basis="bspline", par.CV=list(trim=0, draw=FALSE),
verbose=FALSE, ...)

```

## Arguments

fdataobj      [fdata](#) class object.

type.CV	Type of cross-validation. By default generalized cross-validation (GCV) method.
W	Matrix of weights.
lambda	A roughness penalty. By default, no penalty $\lambda=0$ .
numbasis	Number of basis to use.
type.basis	Character string which determines type of basis. By default "bspline".
par.CV	List of parameters for type.CV: trim, the alpha of the trimming and draw=TRUE.
verbose	If TRUE information about GCV values and input parameters is printed. Default is FALSE.
...	Further arguments passed to or from other methods. Arguments to be passed by default to <a href="#">create.basis</a> .

### Details

Provides the least GCV for functional data for a list of number of basis `num.basis` and `lambda` values `lambda`. You can define the type of CV to use with the `type.CV`, the default is used GCV. S. Smoothing matrix is performed by [S.basis](#). `W` is the matrix of weights of the discretization points.

### Value

<code>gcv</code>	Returns GCV values calculated for input parameters.
<code>fdataobj</code>	Matrix of set cases with dimension $(n \times m)$ , where $n$ is the number of curves and $m$ are the points observed in each curve.
<code>fdata.est</code>	Estimated <code>fdata</code> class object.
<code>numbasis.opt</code>	<code>numbasis</code> value that minimizes CV or GCV method.
<code>lambda.opt</code>	<code>lambda</code> value that minimizes CV or GCV method.
<code>basis.opt</code>	<code>basis</code> for the minimum CV or GCV method.
<code>S.opt</code>	Smoothing matrix for the minimum CV or GCV method.
<code>gcv.opt</code>	Minimum of CV or GCV method.
<code>lambda</code>	A roughness penalty. By default, no penalty $\lambda=0$ .
<code>numbasis</code>	Number of basis to use.
<code>verbose</code>	If TRUE information about GCV values and input parameters is printed. Default is FALSE.

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <[manuel.oviedo@usc.es](mailto:manuel.oviedo@usc.es)>

### References

- Ramsay, James O., and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.
- Wasserman, L. *All of Nonparametric Statistics*. Springer Texts in Statistics, 2006.
- Hardle, W. *Applied Nonparametric Regression*. Cambridge University Press, 1994.
- Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

**See Also**

See Also as [S.basis](#).

Alternative method: [min.np](#)

**Examples**

```

a1<-seq(0,1,by=.01)
a2=rnorm(length(a1),sd=0.2)
f1<-(sin(2*pi*a1))+rnorm(length(a1),sd=0.2)
nc<-50
np<-length(f1)
tt=1:101
S<-S.NW(tt,2)
mdata<-matrix(NA,ncol=np,nrow=50)
for (i in 1:50) mdata[i,]<- (sin(2*pi*a1))+rnorm(length(a1),sd=0.2)
mdata<-fdata(mdata)
nb<-floor(seq(5,29,len=5))
l<-2^(-5:15)
out<-min.basis(mdata,lambda=1,numbasis=nb,type.basis="fourier")
matplot(t(out$gcv),type="l",main="GCV with fourier basis")

# out1<-min.basis(mdata,type.CV = CV.S,lambda=1,numbasis=nb)
# out2<-min.basis(mdata,lambda=1,numbasis=nb)

# variance calculations
y<-mdata
i<-3
z=qnorm(0.025/np)
fdata.est<-out$fdata.est
var.e<-Var.e(mdata,out$.opt)
var.y<-Var.y(mdata,out$.opt)
var.y2<-Var.y(mdata,out$.opt,var.e)

# estimated fdata and point confidence interval
upper.var.e<-out$fdata.est[["data"]][i,]-z*sqrt(diag(var.e))
lower.var.e<-out$fdata.est[["data"]][i,]+z*sqrt(diag(var.e))
dev.new()
plot(y[i,],lwd=1,ylim=c(min(lower.var.e),max(upper.var.e)))
lines(out$fdata.est[["data"]][i,],col=gray(.1),lwd=1)
lines(out$fdata.est[["data"]][i,]+z*sqrt(diag(var.y)),col=gray(0.7),lwd=2)
lines(out$fdata.est[["data"]][i,]-z*sqrt(diag(var.y)),col=gray(0.7),lwd=2)
lines(upper.var.e,col=gray(.3),lwd=2,lty=2)
lines(lower.var.e,col=gray(.3),lwd=2,lty=2)
legend("top",legend=c("Var.y","Var.error"),col = c(gray(0.7),
gray(0.3)),lty=c(1,2))

```

min.np

*Smoothing of functional data using nonparametric kernel estimation***Description**

Smoothing of functional data using nonparametric kernel estimation with cross-validation (CV) or generalized cross-validation (GCV) methods.

**Usage**

```
## S3 method for class 'np'
min(fdataobj, h=NULL, W=NULL, Ker=Ker.norm,
    type.CV=GCV.S, type.S=S.NW, par.CV=list(trim=0, draw=FALSE),
    verbose=FALSE, ...)
```

**Arguments**

fdataobj	<a href="#">fdata</a> class object.
h	Smoothing parameter or bandwidth.
W	Matrix of weights.
Ker	Type of kernel used, by default normal kernel.
type.CV	Type of cross-validation. By default generalized cross-validation (GCV) method. Possible values are <i>GCV.S</i> and <i>CV.S</i>
type.S	Type of smothing matrix S. By default S is calculated by Nadaraya-Watson kernel estimator ( <i>S.NW</i> ). Possible values are <i>S.NW</i> and <i>S.KNN</i>
par.CV	List of parameters for type.CV: <code>trim</code> , the alpha of the trimming and <code>draw=TRUE</code> .
verbose	If TRUE information about GCV values and input parameters is printed. Default is FALSE.
...	Further arguments passed to or from other methods. Arguments to be passed for kernel method.

**Details**

Calculate the minimum GCV for a vector of values of the smoothing parameter  $h$ . Nonparametric smoothing is performed by the kernel function. The type of kernel to use with the parameter `Ker` and the type of smothing matrix `S` to use with the parameter `type.S` can be selected by the user, see function [Kernel](#). `W` is the matrix of weights of the discretization points.

**Value**

Returns GCV or CV values calculated for input parameters.

gcv	GCV or CV for a vector of values of the smoothing parameter h
fdataobj	<a href="#">fdata</a> class object.
fdata.est	Estimated fdata class object.
h.opt	h value that minimizes CV or GCV method.
S.opt	Smoothing matrix for the minimum CV or GCV method.
gcv.opt	Minimum of CV or GCV method.
h	Smoothing parameter or bandwidth.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.

Wasserman, L. *All of Nonparametric Statistics*. Springer Texts in Statistics, 2006.

Hardle, W. *Applied Nonparametric Regression*. Cambridge University Press, 1994.

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

**See Also**

See Also as [S.NW](#).

Alternative method: [min.basis](#)

**Examples**

```
# Exemple, phoneme DATA
data(phoneme)
mlearn<-phoneme$learn[1:100]

out1<-min.np(mlearn,type.CV=CV.S,type.S=S.NW)
np<-ncol(mlearn)
# variance calculations
y<-mlearn
out<-out1
i<-1
z=qnorm(0.025/np)
fdata.est<-out$fdata.est
tt<-y[["argvals"]]
var.e<-Var.e(y,out$S.opt)
```



```

var.y<-Var.y(y,out$$S.opt)
var.y2<-Var.y(y,out$$S.opt,var.e)

# plot estimated fdata and point confidence interval
upper.var.e<-fdata.est[i,]-z*sqrt(diag(var.e))
lower.var.e<-fdata.est[i,]+z*sqrt(diag(var.e))
dev.new()
plot(y[i,],lwd=1,
ylim=c(min(lower.var.e$data),max(upper.var.e$data)),xlab="t")
lines(fdata.est[i,],col=gray(.1),lwd=1)
lines(fdata.est[i,]+z*sqrt(diag(var.y)),col=gray(0.7),lwd=2)
lines(fdata.est[i,]-z*sqrt(diag(var.y)),col=gray(0.7),lwd=2)
lines(upper.var.e,col=gray(.3),lwd=2,lty=2)
lines(lower.var.e,col=gray(.3),lwd=2,lty=2)
legend("bottom",legend=c("Var.y","Var.error"),
col = c(gray(0.7),gray(0.3)),lty=c(1,2))

```

---

na.omit.fdata

*A wrapper for the na.omit and na.fail function for fdata object*


---

## Description

na.fail returns the object if it does not contain any missing values, and signals an error otherwise. na.omit returns the object with incomplete cases removed. If na.omit.fdata removes cases, the row numbers of the cases form the "na.action" attribute of the result, of class "omit", see generic function [na.omit](#).

## Usage

```

## S3 method for class 'fdata'
na.omit(object,...)
## S3 method for class 'fdata'
na.fail(object,...)

```

## Arguments

object            an fdata object.  
...                further potential arguments passed to methods.

## Value

The value returned from `omit` is a fdata object with incomplete cases removed.

## Author(s)

Manuel Febrero Bande

**Examples**

```
fdataobj<-fdata(MontrealTemp)
fdataobj$data[3,3]<-NA
fdataobj$data[10,]<-NA
fdastaobj2<-na.omit.fdata(fdataobj)
```

norm.fdata

*Approximates Lp-norm for functional data.***Description**

Approximates Lp-norm for functional data (fdata) object using metric or semimetric functions. Norm for functional data using by default Lp-metric.

**Usage**

```
norm.fdata(fdataobj,metric=metric.lp,...)
norm.fd(fdobj)
```

**Arguments**

fdataobj      [fdata](#) class object.  
fdobj          Functional data or curves of [fd](#) class.  
metric         Metric function, by default [metric.lp](#).  
...             Further arguments passed to or from other methods.

**Details**

By default it computes the L2-norm with  $p = 2$  and weights  $w$  with  $\text{length}=(m-1)$ .

Let  $f(x) = \text{fdataobj}(x)$

$$\|f\|_p = \left( \frac{1}{\int_a^b w(x)dx} \int_a^b |f(x)|^p w(x)dx \right)^{1/p}$$

The observed points on each curve are equally spaced (by default) or not.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**See Also**

See also [metric.lp](#) and [norm](#)  
Alternative method: [inprod](#) of [fda](#)-package

**Examples**

```

x<-seq(0,2*pi,length=1001)
fx1<-sin(x)/sqrt(pi)
fx2<-cos(x)/sqrt(pi)
argv<-seq(0,2*pi,len=1001)
fdat0<-fdata(rep(0,len=1001),argv,range(argv))
fdat1<-fdata(fx1,x,range(x))
metric.lp(fdat1)
metric.lp(fdat1,fdat0)
norm.fdata(fdat1)
# The same
integrate(function(x){(abs(sin(x)/sqrt(pi))^2)},0,2*pi)
integrate(function(x){(abs(cos(x)/sqrt(pi))^2)},0,2*pi)

bspl1<- create.bspline.basis(c(0,2*pi),21)
fd.bspl1 <- fd(basisobj=bspl1)
fd.bspl2<-fdata2fd(fdat1,nbasis=21)
norm.fd(fd.bspl1)
norm.fd(fd.bspl2)

```

---

order.fdata

*A wrapper for the [order](#) function*


---

**Description**

The functional data is ordered w.r.t the sample order of the values of vector.

**Usage**

```

## S3 method for class 'fdata'
order(y,fdataobj,na.last = TRUE, decreasing = FALSE)

```

**Arguments**

y	a sequence of numeric, complex, character or logical vectors, all of the same length, or a classed R object.
fdataobj	fdata class object.
na.last	for controlling the treatment of NAs. If TRUE, missing values in the data are put last; if FALSE, they are put first; if "keep" they are kept with rank NA.
decreasing	logical. Should the sort order be increasing or decreasing?.

**Value**

[order.fdata](#) returns the functional data fdataobj w.r.t. a permutation which rearranges its first argument into ascending or descending order.

---

Outliers.fdata                      *Detecting outliers for functional dataset*

---

### Description

Procedure for detecting functional outliers.

### Usage

```
outliers.thres.lrt(fdataobj,nb=200,smo=0.05,trim=0.10,...)
outliers.lrt(fdataobj,nb=200,smo=0.05,trim=0.10,...)
outliers.depth.trim(fdataobj,nb=200,smo=0.05,trim=0.01,quan=0.5,
dfunc=depth.mode,...)
outliers.depth.pond(fdataobj,nb=200,smo=0.05,quan=0.5,
dfunc=depth.mode,...)
## S3 method for class 'outliers.pond'
quantile(x,dfunc=depth.mode,
nb=200,smo=0.05,ns=0.01,...)
## S3 method for class 'outliers.trim'
quantile(x,dfunc=depth.mode,trim=0.25,
nb=200,smo=0.05,ns=0.01,...)
```

### Arguments

<code>fdataobj, x</code>	<code>fdata</code> class object.
<code>nb</code>	The number of bootstrap samples.
<code>smo</code>	The smoothing parameter for the bootstrap samples.
<code>trim</code>	The alpha of the trimming.
<code>quan</code>	Quantile to determine the cutoff from the Bootstrap procedure (by default=0.5)
<code>dfunc</code>	Type of depth measure, by default <code>depth.mode</code> .
<code>ns</code>	Significance level, by default 1%.
<code>...</code>	Further arguments passed to or from other methods.

### Details

Outlier detection in functional data by likelihood ratio test (`outliers.lrt`). The threshold for outlier detection is given by the `outliers.thres.lrt`.

Outlier detection in functional data by depth measures:

- i.-`outliers.depth.pond` function weights the data according to depth.
- ii.-`outliers.depth.trim` function uses trimmed data.

`quantile.outliers.pond` and `quantile.outliers.trim` functions provides the quantiles of the bootstrap samples for functional outlier detection by, respectively, weighed and trimmed procedures. Bootstrap smoothing function (`fdata.bootstrap` with `nb` resamples) is applied to these weighted or

trimmed data. If `smo=0` smoothed bootstrap is not performed. The function returns a vector of size `1xnb` with bootstrap replicas of the quantile.

### Value

<code>outliers</code>	Indexes of functional outlier.
<code>dep.out</code>	Depth value of functional outlier.
<code>dep.out</code>	Iteration in which the functional outlier is detected.
<code>quantile</code>	Threshold for outlier detection.
<code>dep</code>	Depth value of functional data.

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <[manuel.oviedo@usc.es](mailto:manuel.oviedo@usc.es)>

### References

Cuevas A, Febrero M, Fraiman R. 2006. *On the use of bootstrap for estimating functions with functional data*. Computational Statistics and Data Analysis 51: 1063-1074.

Febrero-Bande, M., Galeano, P., and Gonzalez-Manteiga, W. (2008). *Outlier detection in functional data by depth measures with application to identify abnormal NOx levels*. Environmetrics 19, 4, 331-345.

Febrero-Bande, M., Galeano, P. and Gonzalez-Manteiga, W. (2007). *A functional analysis of NOx levels: location and scale estimation and outlier detection*. Computational Statistics 22, 3, 411-427.

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

### See Also

See Also: [fdata.bootstrap](#), [Depth](#).

### Examples

```
## Not run:
data(aemet)
nb=20 # Time consuming
out.trim<-outliers.depth.trim(aemet$temp,dfunc=depth.FM,nb=nb)
plot(aemet$temp,col=1,lty=1)
lines(aemet$temp[out.trim[[1]]],col=2)

## End(Not run)
```

P.penalty

*Penalty matrix for higher order differences***Description**

This function computes the matrix that penalizes the higher order differences.

**Usage**

```
P.penalty(tt,P=c(0,0,1))
```

**Arguments**

**tt** vector of the n discretization points or argvals.  
**P** vector of coefficients with the order of the differences. Default value P=c(0,0,1) penalizes the second order difference.

**Details**

For example, if P=c(0,1,2), the function return the penalty matrix the second order difference of a vector *tt*. That is

$$v^T P_j tt = \sum_{i=3}^n (\Delta tt_i)^2$$

where

$$\Delta tt_i = tt_i - 2tt_{i-1} + tt_{i-2}$$

is the second order difference. More details can be found in Kraemer, Boulesteix, and Tutz (2008).

**Value**

penalty matrix of size sum(n) x sum(n)

**Note**

The discretization points can be equidistant or not.

**Author(s)**

This version is created by Manuel Oviedo de la Fuente modified the original version created by Nicole Kramer in pp1s package.

**References**

N. Kraemer, A.-L. Boulsteix, and G. Tutz (2008). *Penalized Partial Least Squares with Applications to B-Spline Transformations and Functional Data*. Chemometrics and Intelligent Laboratory Systems, 94, 60 - 69. <http://dx.doi.org/10.1016/j.chemolab.2008.06.009>

**See Also**[fdata2ppls](#)**Examples**

```
P.penalty((1:10)/10,P=c(0,0,1))
# a more detailed example can be found under script file
```

PCvM.statistic

*PCvM statistic for the Functional Linear Model with scalar response***Description**

Projected Cramer-von Mises statistic (PCvM) for the Functional Linear Model with scalar response (FLM):  $Y = \langle X, \beta \rangle + \varepsilon$ .

**Usage**

```
PCvM.statistic (X, residuals, p, Adot.vec)
Adot (X, inpr)
```

**Arguments**

<code>X</code>	Functional covariate for the FLM. The object must be either in the class <a href="#">fdata</a> or in the class <a href="#">fd</a> . It is used to compute the matrix of inner products.
<code>residuals</code>	Residuals of the estimated FLM.
<code>p</code>	Number of elements of the functional basis where the functional covariate is represented.
<code>Adot.vec</code>	Output from the <code>Adot</code> function (see Details). Computed if not given.
<code>inpr</code>	Matrix of inner products of <code>X</code> . Computed if not given.

**Details**

In order to optimize the computation of the statistic, the critical parts of these two functions are coded in FORTRAN. The hardest part corresponds to the function `Adot`, which involves the computation of a symmetric matrix of dimension  $n \times n$  where each entry is a sum of  $n$  elements. As this matrix is symmetric, the order of the method can be reduced from  $O(n^3)$  to  $O(\frac{n^3-n^2}{2})$ . The memory requirement can also be reduced to  $O(\frac{n^2-n+2}{2})$ . The value of `Adot` is a vector of length  $\frac{n^2-n+2}{2}$  where the first element is the common diagonal element and the rest are the lower triangle entries of the matrix, sorted by rows (see Examples).

**Value**

For `PCvM.statistic`, the value of the statistic. For `Adot`, a suitable output to be used in the argument `Adot.vec`.

**Note**

No NA's are allowed in the functional covariate.

**Author(s)**

Eduardo Garcia-Portugues. Please, report bugs and suggestions to <egarcia@math.ku.dk>

**References**

Escanciano, J. C. (2006). A consistent diagnostic test for regression models using projections. *Econometric Theory*, 22, 1030-1051. <http://dx.doi.org/10.1017/S0266466606060506>

Garcia-Portugues, E., Gonzalez-Manteiga, W. and Febrero-Bande, M. (2014). A goodness-of-fit test for the functional linear model with scalar response. *Journal of Computational and Graphical Statistics*, 23(3), 761-778. <http://dx.doi.org/10.1080/10618600.2013.812519>

**See Also**

[flm.test](#)

**Examples**

```
# Functional process
X=rproc2fdata(n=10,t=seq(0,1,l=101))

# Adot
Adot.vec=Adot(X)

# Obtain the entire matrix Adot
Ad=diag(rep(Adot.vec[1],dim(X$data)[1]))
Ad[upper.tri(Ad,diag=FALSE)]=Adot.vec[-1]
Ad=t(Ad)
Ad=Ad+t(Ad)-diag(diag(Ad))
Ad

# Statistic
PCvM.statistic(X,residuals=rnorm(10),p=5)
```

---

phoneme

*phoneme data*

---

**Description**

Phoneme curves



**Usage**

```
data(phoneme)
```

**Format**

Elements of phoneme:

..\$learn: learning sample of curves. fdata class object with: i.- "data": Matrix of class fdata with 250 curves (rows) discretized in 150 points or argvals (columns).

, ii.- "argvals", iii.- "rangeval": range("argvals"), iv.- "names" list with: main an overall title "Phoneme learn", xlab title for x axis "frequencies" and ylab title for y axis "log-periodograms".

..\$test: testing sample of curves. fdata class object with: i.- "data": Matrix of class fdata with 250 curves (rows) discretized in 150 points or argvals (columns).

, ii.- "argvals", iii.- "rangeval": range("argvals"), iv.- "names" list with: main an overall title "Phoneme learn", xlab title for x axis "frequencies" and ylab title for y axis "log-periodograms".

..\$classlearn: learning class numbers (as factor). Factor levels: "sh" 1, "iy" 2, "dcl" 3, "aa" 4 and "ao" 5.

..\$classtest: testing class numbers (as factor). Factor levels: "sh" 1, "iy" 2, "dcl" 3, "aa" 4 and "ao" 5.

**Details**

The following instructions have been used file:

<http://www.math.univ-toulouse.fr/staph/npfda/npfda-phondiscRS.txt>  
of Phoneme dataset file.

**Author(s)**

Manuel Febrero-Bande and Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**Source**

<http://www.math.univ-toulouse.fr/staph/npfda/npfda-datasets.html>

**References**

Ferraty, F. and Vieu, P. (2006). *NPFDA in practice*. Free access on line at <http://www.lsp.ups-tlse.fr/staph/npfda/>

**Examples**

```
data(phoneme)
names(phoneme)
names(phoneme$learn)
class(phoneme$learn)
dim(phoneme$learn)
table(phoneme$classlearn)
```

---

plot.fdata	<i>Plot functional data: fdata.</i>
------------	-------------------------------------

---

### Description

Plot object of class fdata.

### Usage

```
## S3 method for class 'fdata'
plot(x, type, main, xlab, ylab,
mfrow = c(1, 1), time = 1,...)
## S3 method for class 'fdata'
lines(x,...)
## S3 method for class 'fdata'
title(x,main=NULL,xlab=NULL,ylab=NULL,rownames=NULL)
## S3 method for class 'bifd'
plot(x,argvals.s,argvals.t,...)
```

### Arguments

**x** fdata class object with:

- "data":  
For fdata class object as curve (1d), "data" is a matrix (by default), data.frame or array of set cases with dimension (n x m), where n is the number of curves and m are the points observed in each curve over the x-axe.  
For fdata2d class object as surface (2d). "data" is a array of set cases with dimension (n x m1 x m2), where n is the number of functional data and m1 and m2 are the points observed over the x-y plane.
- "argvals": vector or list of vectors with the discretizations points values.
- "rangeval": vector or list of vectors with the range of the discretizations points values, by default range(argvals).
- "names": (optional) list with main an overall title, xlab title for x axis and ylab title for y axis.

or a two-argument functional data object, see [bifd](#).

**type** 1-character string giving the type of plot desired.  
The following values are possible for fdata class object: "l" for lines (by default), "p" for points, "o" for overplotted points and lines, "b", "c" for (empty if "c") points joined by lines, "s" and "S" for stair steps and "h" for histogram-like vertical lines. Finally, "n" does not produce any points or lines.  
The following values are possible for fdata2d class object: "image.contour" (by default) to display three-dimensional data and add the contour lines, "image" to display three-dimensional data, "contour" to display a contour plot, "persp" to display a perspective plots of a surface over the x-y plane and "filled.contour" to display a contour plot with the areas between the contours filled in solid color.

main	an overall title for the plot: see <a href="#">title</a> .
xlab	xlab title for x axis, as in plot.
ylab	ylab title for y axis, as in plot.
mfrow	A vector of the form <code>c(nr, nc)</code> . Subsequent figures will be drawn in an <code>nr</code> -by- <code>nc</code> array on the device by rows ( <code>mfrow</code> ).
time	The time interval to suspend plot execution for, in seconds, see <a href="#">Sys.sleep</a> .
rownames	Row names.
argsvals.s	a vector of argument values for the first argument <code>s</code> of the functional data object to be evaluated.
argsvals.t	a vector of argument values for the second argument <code>t</code> of the functional data object to be evaluated.
...	Further arguments passed to <a href="#">matplot</a> function (for <code>fdata</code> class) or <a href="#">image</a> , <a href="#">contour</a> , <a href="#">persp</a> or <a href="#">filled.contour</a> (for <code>fdata2d</code> class).

**Author(s)**

Manuel Febrero Bande and Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**See Also**

See Also as [fdata](#)

**Examples**

```
## Not run:

# example for fdata class of 1 dimension (curve)
a1<-seq(0,1,by=.01)
a2=rnorm(length(a1),sd=0.2)
f1<-(sin(2*pi*a1))+rnorm(length(a1),sd=0.2)
nc<-10
np<-length(f1)
tt=seq(0,1,len=101)
mdata<-matrix(NA,ncol=np,nrow=nc)
for (i in 1:nc) mdata[i,]<- (sin(2*pi*a1))+rnorm(length(a1),sd=0.2)
fdataobj<-fdata(mdata,tt)
res=plot.fdata(fdataobj,type="l",col=gray(1:nrow(mdata)/nrow(mdata)))
lines(func.mean(fdataobj),col=3,lwd=2) #original curve

# example for fdata2d class of 2 dimension (surface)
t1 <- seq(0, 1, length= 51)
t2 <- seq(0, 1, length= 31)
z<-array(NA,dim=c(4,51,31))
for (i in 1:4) z[i,] <- outer(t1, t2, function(a, b) (i*a)*(b)^i)
z.fdata<-fdata(z,list(t1,t2))
plot(z.fdata,time=2)
plot(z.fdata,mfrow=c(2,2),type="persp",theta=30)

## End(Not run)
```

poblenou

*poblenou data***Description**

NOx levels measured every hour by a control station in Poblenou in Barcelona (Spain).

**Usage**

```
data(poblenou)
```

**Format**

The format is:

.. \$nox: fdata class object with:

i.- "data": Matrix with 115 curves (rows) discretized in 24 points or argvals (columns).

ii.- "argvals": 0:23

iii.- "rangeval"=(0,23): range("argvals"),

iv.- "names" list with: main an overall title "NOx data set", xlab title for x axis "Hours" and ylab title for y axis "NOx (mg/m<sup>3</sup>)".

.. \$df: Data Frame with (115x3) dimension.

"date" in the first column.

Second column ("day.week"). Factor levels: "Monday" 1, "Tuesday" 2, "Wednesday" 3, "Thursday" 4, "Friday" 5, "Saturday" 6 and "Sunday" 7.

Third column "day.festive". Factor levels: "non festive day" 0 and "festive day" 1.

**Details**

The dataset starts on 23 February and ends on 26 June, in 2005. We split the whole sample of hourly measures in a dataset of functional trajectories of 24 h observations (each curve represents the evolution of the levels in 1 day).

Twelve curves that contained missing data were eliminated.

**Author(s)**

Febrero-Bande, M and Oviedo de la Fuente, Manuel

**Source**

<http://mediambient.gencat.cat>

**References**

Febrero-Bande, M., Galeano, P., and Gonzalez-Manteiga, W. (2008). *Outlier detection in functional data by depth measures with application to identify abnormal NOx levels*. *Environmetrics* 19, 4, 331-345.

## Examples

```
data(poblenou)
names(poblenou)
names(poblenou$nox)
nox<-poblenou$nox
class(nox)
ind.weekend<-as.integer(poblenou$df[, "day.week"]>5)
plot(nox,col=ind.weekend+1)
```

---

predict.classif	<i>Predicts from a fitted classif object.</i>
-----------------	---

---

## Description

Classifier of functional data by kernel method using functional data object of class `classif`.

## Usage

```
## S3 method for class 'classif'
predict(object, new.fdataobj, type="class", ...)
```

## Arguments

object	Object object estimated by: k nearest neighbors method <code>classif.knn</code> , kernel method <code>classif.kernel</code> .
new.fdataobj	New functional explanatory data of <code>fdata</code> class.
type	Type of prediction ("class or probability of each group membership").
...	Further arguments passed to or from other methods.

## Details

Returns the predicted classes using a previously trained model.

## Value

If `type="class"`, produces a vector of predictions.

If `type="probs"`, a list with the following components is returned:

- `group.pred` the vector of predictions.
- `prob.group` the matrix of predicted probability by factor level.

## Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

## References

- Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.
- Ramsay, James O., and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.

## See Also

See also [classif.np](#) [classif.glm](#), [classif.gsam](#) and [classif.gkam](#) .

## Examples

```
## Not run:
data(phoneme)
mlearn<-phoneme[["learn"]][1:100]
glearn<-phoneme[["classlearn"]][1:100]

# ESTIMATION knn
out1=classif.knn(glearn,mlearn,knn=3)
summary.classif(out1)

# PREDICTION knn
mtest<-phoneme[["test"]][1:100]
gtest<-phoneme[["classtest"]][1:100]
pred1=predict.classif(out1,mtest)
table(pred1,gtest)

# ESTIMATION kernel
h=2^(0:5)
# using metric distances computed in classif.knn
out2=classif.kernel(glearn,mlearn,h=h,metric=out1$mdist)
summary.classif(out2)
# PREDICTION kernel
pred2=predict.classif(out2,mtest)
table(pred2,gtest)

## End(Not run)
```

---

predict.classif.DD      *Predicts from a fitted classif.DD object.*

---

## Description

Classifier of functional (and multivariate) data by DD–classifier.

**Usage**

```
## S3 method for class 'classif.DD'
predict(object,new.fdataobj,type="predictive",...)
```

**Arguments**

object	Object object estimated by <code>classif.DD</code> .
new.fdataobj	By default, new $p$ functional explanatory dataset or new multivariate data of <code>data.frame</code> class
type	!="predictive", for each row of data shows the probability of each group membership.
...	Further arguments passed to or from other methods.

**Details**

Returns the groups or classes predicted using a previously trained model.

**Value**

group.pred	Vector of groups or classes predicted
------------	---------------------------------------

**Author(s)**

Febrero-Bande, M., and Oviedo de la Fuente, M.

**References**

Li, J., P.C., Cuesta-Albertos, J.A. and Liu, R. *DD-Classifier: Nonparametric Classification Procedure Based on DD-plot*. Journal of the American Statistical Association (2012), Vol. 107, 737–753.

**See Also**

See also [classif.DD](#).

**Examples**

```
## Not run:
# DD-classif for multivariate data
data(iris)
iris<-iris[1:100,]
ii<-sample(1:100,80)
group.train<-factor(iris[ii,5])
x.train<-iris[ii,1:4]
out1=classif.DD(group.train,x.train,depth="MhD",classif="lda")
out2=classif.DD(group.train,x.train,depth="MhD",classif="glm")
summary.classif(out1)
summary.classif(out2)
x.test<-iris[-ii,1:4]
```

```

pred1=predict.classif.DD(out1,x.test)
pred2=predict.classif.DD(out2,x.test)
group.test<-iris[-ii,5]
table(pred1,group.test)
table(pred2,group.test)

# DD-classif for Functional data
data(phoneme)
mlearn<-phoneme[["learn"]]
glearn<-phoneme[["classlearn"]]

# ESTIMATION
out1=classif.DD(glearn,mlearn,depth="FM",classif="glm")
summary.classif(out1)
# PREDICTION
mtest<-phoneme[["test"]]
gtest<-phoneme[["classtest"]]
pred1=predict.classif.DD(out1,mtest)
table(pred1,gtest)

## End(Not run)

```

---

predict.fregre.fd

*Predict method for functional linear model (fregre.fd class)*

---

## Description

Computes predictions for regression between functional explanatory variables and scalar response using: basis representation, Principal Components Analysis, Partial least squares or nonparametric kernel estimation.

## Usage

```

## S3 method for class 'fregre.fd'
predict(object,new.fdataobj=NULL,se.fit=FALSE,
        scale = NULL,df=df, interval = "none", level = 0.95,
        weights = 1, pred.var = res.var/weights,...)

```

## Arguments

object	fregre.fd object.
new.fdataobj	New functional explanatory data of fdata class.
se.fit	=TRUE (not default) standard error estimates are returned for each prediction.
scale	Scale parameter for std.err. calculation.
df	Degrees of freedom for scale.
interval	Type of interval calculation.



level	Tolerance/confidence level.
pred.var	the variance(s) for future observations to be assumed for prediction intervals. See <code>link{predict.lm}</code> for more details.
weights	variance weights for prediction. This can be a numeric vector or a one-sided model formula. In the latter case, it is interpreted as an expression evaluated in <code>newdata</code>
...	Further arguments passed to or from other methods.

### Details

Predicts from a fitted `fregre.basis` object, see [fregre.basis](#) or [fregre.basis.cv](#)

Predicts from a fitted `fregre.pc` object, see [fregre.pc](#) or [fregre.pc.cv](#)

Predicts from a fitted `fregre.pls` object, see [fregre.pls](#) or [fregre.pls.cv](#)

Predicts from a fitted `fregre.np` object, see [fregre.np](#) or [fregre.np.cv](#).

### Value

If `se.fit = FALSE`, a vector of predictions of scalar response is returned or a matrix of predictions and bounds with column names `fit`, `lwr`, and `upr` if `interval` is set.

If `se.fit = TRUE` a list with the following components is returned:

<code>fit</code>	A vector of predictions or a matrix of predictions and bounds as above
<code>se.fit</code>	Associated standard error estimates of predictions
<code>residual.scale</code>	Residual standard deviations
<code>df</code>	Degrees of freedom for residual

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <[manuel.oviedo@usc.es](mailto:manuel.oviedo@usc.es)>

### References

Cai TT, Hall P. 2006. *Prediction in functional linear regression*. *Annals of Statistics* 34: 2159-2179.

Cardot H, Ferraty F, Sarda P. 1999. *Functional linear model*. *Statistics and Probability Letters* 45: 11-22.

Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.

Hall P, Hosseini-Nasab M. 2006. *On properties of functional principal components analysis*. *Journal of the Royal Statistical Society B* 68: 109-126.

Hardle, W. *Applied Nonparametric Regression*. Cambridge University Press, 1994.

Ramsay, James O., and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. *Journal of Statistical Software*, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

**See Also**

See Also as: [fregre.basis](#), [fregre.basis.cv](#), [fregre.np](#), [fregre.np.cv](#), [fregre.pc](#), [fregre.pc.cv](#), [fregre.pls](#), [fregre.pls.cv](#) and [summary.fregre.fd](#).

**Examples**

```
## Not run:
data(tecator)
absorp=tecator$absorp.fdata
ind=1:129
x=absorp[ind,]
y=tecator$y$Fat[ind]
newx=absorp[-ind,]
newy=matrix(tecator$y$Fat[-ind],ncol=1)
## Functional PC regression
res.pc=fregre.pc(x,y,1:6)
pred.pc=predict.fregre.fd(res.pc,newx)
# Functional PLS regression
res.pls=fregre.pls(x,y,1:6)
pred.pls=predict.fregre.fd(res.pls,newx)
# Functional nonparametric regression
res.np=fregre.np(x,y,Ker=AKer.tri,metric=semimetric.deriv)
pred.np=predict.fregre.fd(res.np,newx)
# Functional regression with basis representation
res.basis=fregre.basis.cv(x,y)
pred.basis=predict.fregre.fd(res.basis,newx)

dev.new()
plot(pred.pc-newy)
points(pred.pls-newy,col=2,pch=2)
points(pred.np-newy,col=3,pch=3)
points(pred.basis-newy,col=4,pch=4)
sum((pred.pc-newy)^2,na.rm=TRUE)/sum((newy-mean(newy))^2,na.rm=TRUE)
sum((pred.pls-newy)^2,na.rm=TRUE)/sum((newy-mean(newy))^2,na.rm=TRUE)
sum((pred.np-newy)^2,na.rm=TRUE)/sum((newy-mean(newy))^2,na.rm=TRUE)
sum((pred.basis-newy)^2,na.rm=TRUE)/sum((newy-mean(newy))^2,na.rm=TRUE)

## End(Not run)
```

---

predict.fregre.GAM

*Predict method for functional regression model*

---

**Description**

Computes predictions for regression between functional (and non functional) explanatory variables and scalar response.

- `predict.fregre.lm`, Predict method for functional linear model of `fregre.lm` fits object using basis or principal component representation.
- `predict.fregre.plm`, Predict method for semi-functional linear regression model of `fregre.plm` fits object using using asymmetric kernel estimation.
- `predict.fregre.glm`, Predict method for functional generalized linear model of `fregre.glm` fits object using basis or principal component representation.
- `predict.fregre.gsam`, Predict method for functional generalized spectral additive model of `fregre.gsam` fits object using basis or principal component representation.
- `predict.fregre.gkam`, Predict method for functional generalized kernel additive model of `fregre.gkam` fits object using backfitting algorithm.

### Usage

```
## S3 method for class 'fregre.lm'
predict(object,newx=NULL,type="response",se.fit=FALSE,
        scale = NULL,df=df, interval = "none", level = 0.95,weights = 1,
        pred.var = res.var/weights, ...)
## S3 method for class 'fregre.plm'
predict(object,newx=NULL,...)
## S3 method for class 'fregre.glm'
predict(object,newx=NULL,type="response",...)
## S3 method for class 'fregre.gsam'
predict(object, newx = NULL, type = "response",...)
## S3 method for class 'fregre.gkam'
predict(object, newx = NULL, type = "response",...)
```

### Arguments

<code>object</code>	<code>fregre.lm</code> , <code>fregre.plm</code> , <code>fregre.glm</code> , <code>fregre.gsam</code> or <code>fregre.gkam</code> object.
<code>newx</code>	An optional data list in which to look for variables with which to predict. If omitted, the fitted values are used. List of new explanatory data.
<code>type</code>	Type of prediction (response or model term).
<code>se.fit</code>	=TRUE (not default) standard error estimates are returned for each prediction.
<code>scale</code>	Scale parameter for std.err. calculation.
<code>df</code>	Degrees of freedom for scale.
<code>interval</code>	Type of interval calculation.
<code>level</code>	Tolerance/confidence level.
<code>pred.var</code>	the variance(s) for future observations to be assumed for prediction intervals. See <code>link{predict.lm}</code> for more details.
<code>weights</code>	variance weights for prediction. This can be a numeric vector or a one-sided model formula. In the latter case, it is interpreted as an expression evaluated in <code>newdata</code>
<code>...</code>	Further arguments passed to or from other methods.

## Details

These functions use the model fitting function `lm`, `glm` or `gam` properties.

If using functional data derived, is recommended to use a number of bases to represent beta lower than the number of bases used to represent the functional data.

The first item in the data list of `newx` argument is called "*df*" and is a data frame with the response and non functional explanatory variables, as `lm`, `glm` or `gam`. Functional variables (`fdata` and `fd` class) are introduced in the following items in the data list of `newx` argument.

## Value

Return the predicted values and optionally:

<code>predict.lm</code> , <code>predict.glm</code> , <code>predict.gam</code>	produces a vector of predictions or a matrix of predictions and bounds with column names <code>fit</code> , <code>lwr</code> , and <code>upr</code> if interval is set. If <code>se.fit</code> is TRUE, a list with the following components is returned: <code>fit</code> vector or matrix as above.
<code>se.fit</code>	standard error of predicted means.
<code>residual.scale</code>	residual standard deviations.
<code>df</code>	degrees of freedom for residual.

## Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <[manuel.oviedo@usc.es](mailto:manuel.oviedo@usc.es)>

## References

Febrero-Bande, M., Oviedo de la Fuente, M. (2012). *Statistical Computing in Functional Data Analysis: The R Package fda.usc*. Journal of Statistical Software, 51(4), 1-28. <http://www.jstatsoft.org/v51/i04/>

## See Also

See Also as: [fregre.lm](#), [fregre.plm](#), [fregre.glm](#), [fregre.gsam](#) and [fregre.gkam](#).

## Examples

```
## Not run:
data(tecator)
ind<-1:129
x=tecator$absorp.fdata
x.d2<-fdata.deriv(x,nderiv=2)
tt<-x[["argvals"]]
dataf=as.data.frame(tecator$y)
nbasis.x=11;nbasis.b=7
basis1=create.bspline.basis(rangeval=range(tt),nbasis=nbasis.x)
basis2=create.bspline.basis(rangeval=range(tt),nbasis=nbasis.b)
basis.x=list("x.d2"=basis1)
```

```

basis.b=list("x.d2"=basis2)
ldata=list("df"=dataf[ind,],"x.d2"=x.d2[ind])

res=fregre.gsam(Fat~s(Water,k=3)+s(x.d2,k=3),data=ldata,
family=gaussian(),basis.x=basis.x,basis.b=basis.b)
newldata=list("df"=dataf[-ind,],"x.d2"=x.d2[-ind])
pred<-predict.fregre.gsam(res,newldata)
plot(pred,tecator$y$Fat[-ind])

res.glm=fregre.glm(Fat~Water+x.d2,data=ldata,family=gaussian(),
basis.x=basis.x,basis.b=basis.b)
pred.glm<-predict.fregre.glm(res.glm,newldata)
newy<-tecator$y$Fat[-ind]
points(pred.glm,tecator$y$Fat[-ind],col=2)

res.plm=fregre.plm(Fat~Water+x.d2,data=ldata)
pred.plm<-predict.fregre.plm(res.plm,newldata)
points(pred.plm,tecator$y$Fat[-ind],col=3)

# Time-consuming
res.gkam=fregre.gkam(Fat~Water+x.d2,data=ldata)
pred.gkam=predict(res.gkam,newldata)
points(pred.gkam,tecator$y$Fat[-ind],col=4)

((1/length(newy))*sum((drop(newy)-pred)^2))/var(newy)
((1/length(newy))*sum((newy-pred.plm)^2))/var(newy)
((1/length(newy))*sum((newy-pred.glm)^2))/var(newy)
((1/length(newy))*sum((newy-pred.gkam)^2))/var(newy)

## End(Not run)

```

---

predict.fregre.gls      *Predictions from a functional gls object*

---

## Description

The predictions for the functional generalized least squares fitted linear model represented by object are obtained at the covariate values defined in newx.

## Usage

```

## S3 method for class 'fregre.gls'
predict(object, newx = NULL, type = "response",
        se.fit= FALSE, scale = NULL,df , interval = "none",
        level = 0.95, weights = 1, pred.var, n.ahead = 1, ...)
## S3 method for class 'fregre.igls'
predict(object, new.fdataobj = NULL, data,
        se.fit = FALSE, scale = NULL, df = df, interval = "none",
        level = 0.95, weights = 1, pred.var, n.ahead =1L,...)

```

**Arguments**

object	fregre.gls object.
newx,new.fdataobj	An optional data list in which to look for variables with which to predict. If omitted, the fitted values are used. List of new explanatory data.
type	Type of prediction (response or model term).
se.fit	=TRUE (not default) standard error estimates are returned for each prediction.
scale	Scale parameter for std.err. calculation.
df	Degrees of freedom for scale.
interval	Type of interval calculation.
level	Tolerance/confidence level.
weights	variance weights for prediction. This can be a numeric vector or a one-sided model formula. In the latter case, it is interpreted as an expression evaluated in newdata
pred.var	the variance(s) for future observations to be assumed for prediction intervals. See <code>link{predict.lm}</code> for more details.
data	Data frame with the time or spatinal index
n.ahead	number of steps ahead at which to predict.
...	Further arguments passed to or from other methods.

**Value**

a vector with the predicted values.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Oviedo de la Fuente, M., Febrero-Bande, M., Pilar Munoz, and Dominguez, A. Predicting seasonal influenza transmission using Functional Regression Models with Temporal Dependence. arXiv:1610.08718. <https://arxiv.org/abs/1610.08718>

**See Also**

[fregre.gls](#)

**Examples**

```
## Not run:
data(tecator)
ind<-1:129
x <-fdata.deriv(tecator$absorp.fdata,nderiv=1)
dataf=as.data.frame(tecator$y)
dataf$time <- 1:nrow(x)
```

```
ldata=list("df"=dataf[ind,],"x"=x[ind])
newldata=list("df"=dataf[-ind,],"x"=x[-ind])
newy <- tecator$y$Fat[-ind]
ff <- Fat~x
res.gls=fregre.gls(ff,data=ldata, correlation=corAR1())
pred.gls <- predict.fregre.gls(res.gls,newldata)
par.cor <- list("cor.ARMA"=list("index"=c("itime"),"p"=1))
res.igls <- fregre.igls(ff,data=ldata,correlation=par.cor)
pred.igls<-predict.fregre.igls(res.igls,newldata,data=newldata$df["index",drop=F])

## End(Not run)
```

---

predict.functional.response

*Predict method for functional response model*

---

## Description

Computes predictions for regression between functional explanatory variables and functional response.

## Usage

```
## S3 method for class 'fregre.fr'
predict(object,new.fdataobj=NULL,...)
```

## Arguments

object	fregre.fr object.
new.fdataobj	New functional explanatory data of fdata class.
...	Further arguments passed to or from other methods.

## Value

Return the predicted functional data.

## Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

## See Also

See Also as: [fregre.basis.fr](#)

## Examples

```
## Not run:

# CV prediction for CandianWeather data
rtt<-c(0, 365)
basiss <- create.bspline.basis(rtt,7)
basist <- create.bspline.basis(rtt,9)
nam<-dimnames(CanadianWeather$dailyAv)[[2]]

# fdata class (raw data)
tt<-1:365
tempfdata<-fdata(t(CanadianWeather$dailyAv[, ,1]),tt,rtt)
log10precfddata<-fdata(t(CanadianWeather$dailyAv[, ,3]),tt,rtt)
rng<-range(log10precfddata)
for (ind in 1:35){
  res1<- fregre.basis.fr(tempfdata[-ind], log10precfddata[-ind],
    basis.s=basiss,basis.t=basist)
  pred1<-predict.fregre.fr(res1,tempfdata[ind])
  plot( log10precfddata[ind],col=1,ylim=rng,main=nam[ind])
  lines(pred1,lty=2,col=2)
  Sys.sleep(1)
}

# fd class (smooth data)
basis.alpha <- create.constant.basis(rtt)
basisx <- create.bspline.basis(rtt,65)

dayfd<-Data2fd(day.5,CanadianWeather$dailyAv,basisx)
tempfd<-dayfd[,1]
log10precfd<-dayfd[,3]
for (ind in 1:35){
  res2 <- fregre.basis.fr(tempfd[-ind], log10precfd[-ind],
    basis.s=basiss,basis.t=basist)
  pred2<-predict.fregre.fr(res2,tempfd[ind])
  plot(log10precfd[ind],col=1,ylim=range(log10precfd$coef),main=nam[ind])
  lines(pred2,lty=2,col=2)
  Sys.sleep(1)
}

## End(Not run)
```

## Description

Sampling of paths of the Ornstein-Uhlenbeck process.



**Usage**

```
r.ou(n, t = seq(0, 1, len = 201), mu = 0, alpha = 1, sigma = 1,
     x0 = rnorm(n, mean = mu, sd = sigma/sqrt(2 * alpha)))
```

**Arguments**

n	number of functions to sample.
t	time locations for the functional data.
mu	mean of the process.
alpha	strength of the drift.
sigma	diffusion coefficient.
x0	a number or a vector of length n giving the initial value(s) of the Ornstein-Uhlenbeck process. By default, n points are sampled from the stationary distribution.

**Value**

Functional sample, an `fdata` object of length n.

**Author(s)**

Eduardo Garcia-Portugues (<edgarcia@est-econ.uc3m.es>).

**Examples**

```
plot(r.ou(n = 100))
plot(r.ou(n = 100, alpha = 2, sigma = 4, x0 = 1:100))
```

---

 rdir.pc

*Data-driven sampling of random directions guided by sample of functional data*

---

**Description**

Generation of random directions based on the principal components  $\hat{e}_1, \dots, \hat{e}_k$  of a sample of functional data  $X_1, \dots, X_n$ . The random directions are sampled as

$$h = \sum_{j=1}^k h_j \hat{e}_j,$$

with  $h_j \sim \mathcal{N}(0, \sigma_j^2)$ ,  $j = 1, \dots, k$ . Useful for sampling non-orthogonal random directions  $h$  such that they are non-orthogonal for the random sample.

**Usage**

```
rdir.pc(n, X.fdata, ncomp = 0.95, fdata2pc.obj = fda.usc::fdata2pc(X.fdata,
  ncomp = min(length(X.fdata$argvals), nrow(X.fdata))), sd = 0,
  zero.mean = TRUE, norm = FALSE)
```

**Arguments**

n	number of curves to be generated.
X.fdata	an <code>fdata</code> object used to compute the functional principal components.
ncomp	if an integer vector is provided, the index for the principal components to be considered. If a threshold between 0 and 1 is given, the number of components $k$ is determined automatically as the minimum number that explains at least the <code>ncomp</code> proportion of the total variance of <code>X.fdata</code> .
fdata2pc.obj	output of <code>fdata2pc</code> containing as many components as the ones to be selected by <code>ncomp</code> . Otherwise, it is computed internally.
sd	if 0, the standard deviations $\sigma_j$ are estimated by the standard deviations of the scores for $e_j$ . If not, the $\sigma_j$ 's are set to <code>sd</code> .
zero.mean	whether the projections should have zero mean. If not, the mean is set to the mean of <code>X.fdata</code> .
norm	whether the samples should be L2-normalized or not.

**Value**

A `fdata` object with the sampled directions.

**Author(s)**

Eduardo Garcia-Portugues (<edgarcia@est-econ.uc3m.es>) and Manuel Febrero-Bande (<manuel.febrero@usc.es>).

**Examples**

```
# Simulate some data
set.seed(345673)
X.fdata <- r.ou(n = 200, mu = 0, alpha = 1, sigma = 2, t = seq(0, 1, l = 201),
  x0 = rep(0, 200))
pc <- fdata2pc(X.fdata, ncomp = 20)

# Samples
set.seed(34567)
rdir.pc(n = 5, X.fdata = X.fdata, zero.mean = FALSE)$data[, 1:5]
set.seed(34567)
rdir.pc(n = 5, X.fdata = X.fdata, fdata2pc.obj = pc)$data[, 1:5]

## Not run:
# Comparison for the variance type
set.seed(456732)
n.proj <- 100
set.seed(456732)
```

```

samp1 <- rdir.pc(n = n.proj, X.fdata = X.fdata, sd = 1, norm = FALSE, ncomp = 0.99)
set.seed(456732)
samp2 <- rdir.pc(n = n.proj, X.fdata = X.fdata, sd = 0, norm = FALSE, ncomp = 0.99)
set.seed(456732)
samp3 <- rdir.pc(n = n.proj, X.fdata = X.fdata, sd = 1, norm = TRUE, ncomp = 0.99)
set.seed(456732)
samp4 <- rdir.pc(n = n.proj, X.fdata = X.fdata, sd = 0, norm = TRUE, ncomp = 0.99)
par(mfrow = c(1, 2))
plot(X.fdata, col = gray(0.85), lty = 1)
lines(samp1[1:10], col = 2, lty = 1)
lines(samp2[1:10], col = 4, lty = 1)
legend("topleft", legend = c("Data", "Different variances", "Equal variances"),
      col = c(gray(0.85), 2, 4), lwd = 2)
plot(X.fdata, col = gray(0.85), lty = 1)
lines(samp3[1:10], col = 5, lty = 1)
lines(samp4[1:10], col = 6, lty = 1)
legend("topleft", legend = c("Data", "Different variances, normalized",
      "Equal variances, normalized"), col = c(gray(0.85), 5:6), lwd = 2)

# Correlations (stronger with different variances and unnormalized;
# stronger with lower ncomp)
ind <- lower.tri(matrix(nrow = n.proj, ncol = n.proj))
median(abs(cor(sapply(1:n.proj, function(i) inprod.fdata(X.fdata, samp1[i]))))[ind])
median(abs(cor(sapply(1:n.proj, function(i) inprod.fdata(X.fdata, samp2[i]))))[ind])
median(abs(cor(sapply(1:n.proj, function(i) inprod.fdata(X.fdata, samp3[i]))))[ind])
median(abs(cor(sapply(1:n.proj, function(i) inprod.fdata(X.fdata, samp4[i]))))[ind])

# Comparison for the threshold
samp1 <- rdir.pc(n = 100, X.fdata = X.fdata, ncomp = 0.25, fdata2pc.obj = pc)
samp2 <- rdir.pc(n = 100, X.fdata = X.fdata, ncomp = 0.50, fdata2pc.obj = pc)
samp3 <- rdir.pc(n = 100, X.fdata = X.fdata, ncomp = 0.90, fdata2pc.obj = pc)
samp4 <- rdir.pc(n = 100, X.fdata = X.fdata, ncomp = 0.95, fdata2pc.obj = pc)
samp5 <- rdir.pc(n = 100, X.fdata = X.fdata, ncomp = 0.99, fdata2pc.obj = pc)
cols <- rainbow(5, alpha = 0.25)
par(mfrow = c(3, 2))
plot(X.fdata, col = gray(0.75), lty = 1, main = "Data")
plot(samp1, col = cols[1], lty = 1, main = "Threshold = 0.25")
plot(samp2, col = cols[2], lty = 1, main = "Threshold = 0.50")
plot(samp3, col = cols[3], lty = 1, main = "Threshold = 0.90")
plot(samp4, col = cols[4], lty = 1, main = "Threshold = 0.95")
plot(samp5, col = cols[5], lty = 1, main = "Threshold = 0.99")

# Normalizing
samp1 <- rdir.pc(n = 100, X.fdata = X.fdata, ncomp = 0.50, fdata2pc.obj = pc,
  norm = TRUE)
samp2 <- rdir.pc(n = 100, X.fdata = X.fdata, ncomp = 0.90, fdata2pc.obj = pc,
  norm = TRUE)
samp3 <- rdir.pc(n = 100, X.fdata = X.fdata, ncomp = 0.95, fdata2pc.obj = pc,
  norm = TRUE)
samp4 <- rdir.pc(n = 100, X.fdata = X.fdata, ncomp = 0.99, fdata2pc.obj = pc,
  norm = TRUE)
samp5 <- rdir.pc(n = 100, X.fdata = X.fdata, ncomp = 0.999, fdata2pc.obj = pc,
  norm = TRUE)

```

```

cols <- rainbow(5, alpha = 0.25)
par(mfrow = c(3, 2))
plot(X.fdata, col = gray(0.75), lty = 1, main = "Data")
plot(samp1, col = cols[1], lty = 1, main = "Threshold = 0.50")
plot(samp2, col = cols[2], lty = 1, main = "Threshold = 0.90")
plot(samp3, col = cols[3], lty = 1, main = "Threshold = 0.95")
plot(samp4, col = cols[4], lty = 1, main = "Threshold = 0.99")
plot(samp5, col = cols[5], lty = 1, main = "Threshold = 0.999")

## End(Not run)

```

---

rp.flm.statistic	<i>Statistics for testing the functional linear model using random projections</i>
------------------	--

---

### Description

Computes the Cramer-von Mises (CvM) and Kolmogorov-Smirnov (kS) statistics on the projected process

$$T_{n,h}(u) = \frac{1}{n} \sum_{i=1}^n (Y_i - \langle X_i, \hat{\beta} \rangle) 1_{\{\langle X_i, h \rangle \leq u\}},$$

designed to test the goodness-of-fit of a functional linear model with scalar response.

### Usage

```
rp.flm.statistic(proj.X, residuals, proj.X.ord = NULL, F.code = TRUE)
```

### Arguments

proj.X	matrix of size c(n, n.proj) containing, for each column, the projections of the functional data $X_1, \dots, X_n$ into a random direction $h$ . Not required if proj.X.ord is provided.
residuals	the residuals of the fitted functional linear model, $Y_i - \langle X_i, \hat{\beta} \rangle$ . Either a vector of length n (same residuals for all projections) or a matrix of size c(n.proj, n) (each projection has an associated set residuals).
proj.X.ord	matrix containing the row permutations of proj.X which rearranges them increasingly, for each column. So, for example proj.X[proj.X.ord[, 1], 1] equals sort(proj.X[, 1]). If not provided, it is computed internally.
F.code	whether to use faster FORTRAN code or R code.

### Details

NA's are not allowed neither in the functional covariate nor in the scalar response.

**Value**

A list containing:

`statistic` a matrix of size  $c(n.proj, 2)$  with the CvM (first column) and KS (second) statistics, for the `n.proj` different projections.

`proj.X.ord` the computed row permutations of `proj.X`, useful for recycling in subsequent calls to `rp.flm.statistic` with the same projections but different residuals.

**Author(s)**

Eduardo Garcia-Portugues (<edgarcia@est-econ.uc3m.es>) and Manuel Febrero-Bande (<manuel.febrero@usc.es>).

**References**

Cuesta-Albertos, J.A., Garcia-Portugues, E., Febrero-Bande, M. and Gonzalez-Manteiga, W. (2017). Goodness-of-fit tests for the functional linear model based on randomly projected empirical processes. arXiv:1701.08363. <https://arxiv.org/abs/1701.08363>

**Examples**

```
# Simulated example
set.seed(345678)
t <- seq(0, 1, l = 101)
n <- 100
X <- r.ou(n = n, t = t)
beta0 <- fdata(mdata = cos(2 * pi * t) - (t - 0.5)^2, argvals = t,
               rangeval = c(0,1))
Y <- inprod.fdata(X, beta0) + rnorm(n, sd = 0.1)

# Linear model
mod <- fregre.pc(fdataobj = X, y = Y, l = 1:3)

# Projections
proj.X1 <- inprod.fdata(X, r.ou(n = 1, t = t))
proj.X2 <- inprod.fdata(X, r.ou(n = 1, t = t))
proj.X12 <- cbind(proj.X1, proj.X2)

# Statistics
t1 <- rp.flm.statistic(proj.X = proj.X1, residuals = mod$residuals)
t2 <- rp.flm.statistic(proj.X = proj.X2, residuals = mod$residuals)
t12 <- rp.flm.statistic(proj.X = proj.X12, residuals = mod$residuals)
t1$statistic
t2$statistic
t12$statistic

# Recycling proj.X.ord
rp.flm.statistic(proj.X.ord = t1$proj.X.ord, residuals = mod$residuals)$statistic
t1$statistic

# Sort in the columns
cbind(proj.X12[t12$proj.X.ord[, 1], 1], proj.X12[t12$proj.X.ord[, 2], 2]) -
```

```

apply(proj.X12, 2, sort)

# FORTRAN and R code
rp.flm.statistic(proj.X = proj.X1, residuals = mod$residuals)$statistic -
rp.flm.statistic(proj.X = proj.X1, residuals = mod$residuals,
                  F.code = FALSE)$statistic

# Matrix and vector residuals
rp.flm.statistic(proj.X = proj.X12, residuals = mod$residuals)$statistic
rp.flm.statistic(proj.X = proj.X12,
                  residuals = rbind(mod$residuals, mod$residuals * 2))$statistic

```

---

rp.flm.test	<i>Goodness-of-fit test for the functional linear model using random projections</i>
-------------	--

---

### Description

Tests the composite null hypothesis of a Functional Linear Model with scalar response (FLM),

$$H_0 : Y = \langle X, \beta \rangle + \epsilon \quad \text{vs} \quad H_1 : Y \neq \langle X, \beta \rangle + \epsilon.$$

If  $\beta = \beta_0$  is provided, then the simple hypothesis  $H_0 : Y = \langle X, \beta_0 \rangle + \epsilon$  is tested. The way of testing the null hypothesis is via a norm (Cramer-von Mises or Kolmogorov-Smirnov) in the empirical process indexed by the projections.

### Usage

```

rp.flm.test(X.fdata, Y, beta0.fdata = NULL, B = 1000, n.proj = 10,
            est.method = "pc", p = NULL, p.criterion = "SICc", pmax = 20,
            type.basis = "bspline", projs = 0.95, verbose = TRUE,
            same.rwild = FALSE, ...)

```

### Arguments

X.fdata	functional observations in the class <a href="#">fdata</a> .
Y	scalar responses for the FLM. Must be a vector with the same number of elements as functions are in X.fdata.
beta0.fdata	functional parameter for the simple null hypothesis, in the <a href="#">fdata</a> class. The argvals and rangeval arguments of beta0.fdata must be the same of X.fdata. If beta0.fdata=NULL (default), the function will test for the composite null hypothesis.
B	number of bootstrap replicates to calibrate the distribution of the test statistic.
n.proj	vector with the number of projections to consider.
est.method	estimation method for $\beta$ , only used in the composite case. There are three methods:

	"pc" if p is given, then $\beta$ is estimated by <code>fregre.pc</code> . Otherwise, p is chosen using <code>fregre.pc.cv</code> and the <code>p.criterion</code> criterion.
	"pls" if p is given, $\beta$ is estimated by <code>fregre.pls</code> . Otherwise, p is chosen using <code>fregre.pls.cv</code> and the <code>p.criterion</code> criterion.
	"basis" if p is given, $\beta$ is estimated by <code>fregre.basis</code> . Otherwise, p is chosen using <code>fregre.basis.cv</code> and the <code>p.criterion</code> criterion. Both in <code>fregre.basis</code> and <code>fregre.basis.cv</code> , the same basis for <code>basis.x</code> and <code>basis.b</code> is considered.
p	number of elements for the basis representation of <code>beta0.fdata</code> and <code>X.fdata</code> with the <code>est.method</code> (only composite hypothesis). If not supplied, it is estimated from the data.
p.criterion	for <code>est.method</code> equal to "pc" or "pls", either "SIC", "SICc" or one of the criterions described in <code>fregre.pc.cv</code> . For "basis" a value for <code>type.CV</code> in <code>fregre.basis.cv</code> such as GCV.S.
pmax	maximum size of the basis expansion to consider in when using <code>p.criterion</code> .
type.basis	type of basis if <code>est.method</code> = "basis".
projs	a <code>fdata</code> object containing the random directions employed to project <code>X.fdata</code> . If numeric, the convenient value for <code>ncomp</code> in <code>rdir.pc</code> .
verbose	whether to show or not information about the testing progress.
same.rwild	wether to employ the same wild bootstrap residuals for different projections or not.
...	further arguments passed to <code>create.basis</code> (not <code>rangeval</code> that is taken as the <code>rangeval</code> of <code>X.fdata</code> ).

## Details

No NA's are allowed neither in the functional covariate nor in the scalar response.

## Value

An object with class "htest" whose underlying structure is a list containing the following components:

p.values.fdr	a matrix of size $c(n.proj, 2)$ , containing in each row the FDR p-values of the CvM and KS tests up to that projection.
proj.statistics	a matrix of size $c(\max(n.proj), 2)$ with the value of the test statistic on each projection.
boot.proj.statistics	an array of size $c(\max(n.proj), 2, B)$ with the values of the bootstrap test statistics for each projection.
proj.p.values	a matrix of size $c(\max(n.proj), 2)$
method	information about the test performed and the kind of estimation performed.
B	number of bootstrap replicates used.
n.proj	number of projections specified
projs	random directions employed to project <code>X.fdata</code> .

type.basis type of basis for est.method = "basis".

beta.est estimated functional parameter  $\hat{\beta}$  in the composite hypothesis. For the simple hypothesis, beta0.fdata.

p number of basis elements considered for estimation of  $\beta$ .

p.criterion criterion employed for selecting p.

data.name the character string "Y = <X, b> + e"

### Author(s)

Eduardo Garcia-Portugues (<edgarcia@est-econ.uc3m.es>) and Manuel Febrero-Bande (<manuel.febrero@usc.es>).

### References

Cuesta-Albertos, J.A., Garcia-Portugues, E., Febrero-Bande, M. and Gonzalez-Manteiga, W. (2017). Goodness-of-fit tests for the functional linear model based on randomly projected empirical processes. arXiv:1701.08363. <https://arxiv.org/abs/1701.08363>

Garcia-Portugues, E., Gonzalez-Manteiga, W. and Febrero-Bande, M. (2014). A goodness-of-fit test for the functional linear model with scalar response. Journal of Computational and Graphical Statistics, 23(3), 761–778. <http://dx.doi.org/10.1080/10618600.2013.812519>

### Examples

```
#' # Simulated example

set.seed(345678)
t <- seq(0, 1, l = 101)
n <- 100
X <- r.ou(n = n, t = t, alpha = 2, sigma = 0.5)
beta0 <- fdata(mdata = cos(2 * pi * t) - (t - 0.5)^2, argvals = t,
               rangeval = c(0,1))
Y <- inprod.fdata(X, beta0) + rnorm(n, sd = 0.1)

# Test all cases
rp.flm.test(X.fdata = X, Y = Y, est.method = "pc")
## Not run:
rp.flm.test(X.fdata = X, Y = Y, est.method = "pls")
rp.flm.test(X.fdata = X, Y = Y, est.method = "basis",
            p.criterion = fda.usc::GCV.S)
rp.flm.test(X.fdata = X, Y = Y, est.method = "pc", p = 5)
rp.flm.test(X.fdata = X, Y = Y, est.method = "pls", p = 5)
rp.flm.test(X.fdata = X, Y = Y, est.method = "basis", p = 5)
rp.flm.test(X.fdata = X, Y = Y, beta0.fdata = beta0)

# Composite hypothesis: do not reject FLM
rp.test <- rp.flm.test(X.fdata = X, Y = Y, est.method = "pc")
rp.test$p.values.fdr
pcvm.test <- flm.test(X.fdata = X, Y = Y, est.method = "pc", B = 1e3,
                    plot.it = FALSE)

pcvm.test
```



```

# Estimation of beta
par(mfrow = c(1, 3))
plot(X, main = "X")
plot(beta0, main = "beta")
lines(rp.test$beta.est, col = 2)
lines(pcv.test$beta.est, col = 3)
plot(density(Y), main = "Density of Y", xlab = "Y", ylab = "Density")
rug(Y)

# Simple hypothesis: do not reject beta = beta0
rp.flm.test(X.fdata = X, Y = Y, beta0.fdata = beta0)$p.values.fdr
flm.test(X.fdata = X, Y = Y, beta0.fdata = beta0, B = 1e3, plot.it = FALSE)

# Simple hypothesis: reject beta = beta0^2
rp.flm.test(X.fdata = X, Y = Y, beta0.fdata = beta0^2)$p.values.fdr
flm.test(X.fdata = X, Y = Y, beta0.fdata = beta0^2, B = 1e3, plot.it = FALSE)

# Tecator dataset

# Load data
data(tecator)
absorp <- tecator$absorp.fdata
ind <- 1:129 # or ind <- 1:215
x <- absorp[ind, ]
y <- tecator$y$Fat[ind]

# Composite hypothesis
rp.tecat <- rp.flm.test(X.fdata = x, Y = y, est.method = "pc")
pcvm.tecat <- flm.test(X.fdata = x, Y = y, est.method = "pc", B = 1e3,
                      plot.it = FALSE)
rp.tecat$p.values.fdr[c(5, 10), ]
pcvm.tecat

# Simple hypothesis
zero <- fdata(mdata = rep(0, length(x$argvals)), argvals = x$argvals,
              rangeval = x$rangeval)
rp.flm.test(X.fdata = x, Y = y, beta0.fdata = zero)
flm.test(X.fdata = x, Y = y, beta0.fdata = zero, B = 1e3)

# With derivatives
rp.tecat <- rp.flm.test(X.fdata = fdata.deriv(x, 1), Y = y, est.method = "pc")
rp.tecat$p.values.fdr
rp.tecat <- rp.flm.test(X.fdata = fdata.deriv(x, 2), Y = y, est.method = "pc")
rp.tecat$p.values.fdr

# AEMET dataset

# Load data
data(aemet)
wind.speed <- apply(aemet$wind.speed$data, 1, mean)
temp <- aemet$temp

# Remove the 5% of the curves with less depth (i.e. 4 curves)

```

```

par(mfrow = c(1, 1))
res.FM <- depth.FM(temp, draw = TRUE)
qu <- quantile(res.FM$dep, prob = 0.05)
l <- which(res.FM$dep <= qu)
lines(aemet$temp[l], col = 3)

# Data without outliers
wind.speed <- wind.speed[-l]
temp <- temp[-l]

# Composite hypothesis
rp.aemet <- rp.flm.test(X.fdata = temp, Y = wind.speed, est.method = "pc")
pcvm.aemet <- flm.test(X.fdata = temp, Y = wind.speed, B = 1e3,
  est.method = "pc", plot.it = FALSE)
rp.aemet$p.values.fdr
apply(rp.aemet$p.values.fdr, 2, range)
pcvm.aemet

# Simple hypothesis
zero <- fdata(mdata = rep(0, length(temp$argvals)), argvals = temp$argvals,
  rangeval = temp$rangeval)
flm.test(X.fdata = temp, Y = wind.speed, beta0.fdata = zero, B = 1e3,
  plot.it = FALSE)
rp.flm.test(X.fdata = temp, Y = wind.speed, beta0.fdata = zero)

## End(Not run)

```

---

rproc2fdata

*Simulate several random processes.*


---

## Description

Simulate Functional Data from different processes: Ornstein Uhlenbeck, Brownian, Fractional Brownian, Gaussian or Exponential variogram.

## Usage

```

rproc2fdata(n, t=NULL, mu=rep(0, length(t)), sigma=1,
  par.list=list("scale"=1, "theta"=.2*diff(rtt), "H"=0.5),
  norm=FALSE, verbose=FALSE, ...)

```

## Arguments

n	Number of functional curves to be generated.
t	Discretization points.
mu	vector which specifies the trend values at the discretization points, by default $\mu = \mu(t) = 0$ . If mu is a fdata class object, $t = \text{argvals}(\mu)$ .

sigma	<p>A positive-definite symmetric matrix, <math>\Sigma_{s,t}</math>, specifying the covariance matrix among grid points. If sigma is a scalar, creates a random Gaussian process with <math>\Sigma_{s,t} = \text{sigma}I</math> (by default sigma=1).          If sigma is a vector, creates a random Gaussian process with <math>\Sigma_{s,t} = \text{diag}(\text{sigma})</math>.          If sigma is a character: create a random process using the covariance matrix <math>\Sigma_{s,t}</math> indicated in the argument,</p> <ul style="list-style-type: none"> <li>• "OU" or "OrnsteinUhlenbeck", creates a random Ornstein Uhlenbeck process with <math>\Sigma_{s,t} = \frac{\sigma^2}{2\theta} e^{-\theta(s+t)} (e^{2\theta(s+t)} - 1)</math>, by default <math>\theta = 1/(3\text{range}(t))</math>, <math>\sigma^2 = 1</math>.</li> <li>• "brownian" or "wiener", creates a random Wiener process with <math>\Sigma_{s,t} = \sigma^2 \min(s, t)</math>, by default <math>\sigma^2 = 1</math>.</li> <li>• "fbrownian", creates a random fractional brownian process with <math>\Sigma_{s,t} = \sigma^{2H} / 2  s ^{2H} +  t ^{2H} -  s - t ^{2H}</math>, by default <math>\sigma^2 = 1</math> and <math>H = 0.5</math> (brownian process).</li> <li>• "vexponential", creates a random gaussian process with exponential variogram <math>\Sigma_{s,t} = \sigma^2 e^{-\frac{ s-t }{\theta}}</math>, by default <math>\theta = 0.2\text{range}(t)</math>, <math>\sigma^2 = 1</math>.</li> </ul>
par.list	List of parameter to process, by default "scale" $\sigma^2 = 1$ , "theta" $\theta = 0.2\text{range}(t)$ and "H"=0.5.
norm	If TRUE the norm of random projection is 1. Default is FALSE
verbose	If TRUE, information about procedure is printed. Default is FALSE.
...	Further arguments passed to or from other methods.

### Value

Return the functional random processes as a fdata class object.

### Author(s)

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

### Examples

```

par(mfrow=c(3,2))
lent<-30
tt<-seq(0,1,len=lent)
mu<-fdata(rep(0,lent),tt)
plot(rproc2fdata(200,t=tt,sigma="OU",par.list=list("scale"=1)))
plot(rproc2fdata(200,mu=mu,sigma="OU",par.list=list("scale"=1)))
plot(rproc2fdata(200,t=tt,sigma="vexponential"))
plot(rproc2fdata(200,t=tt,sigma=1:lent))
plot(rproc2fdata(200,t=tt,sigma="brownian"))
plot(rproc2fdata(200,t=tt,sigma="wiener"))
#plot(rproc2fdata(200,seq(0,1,len=30),sigma="oo")) # this is an error

```

---

 rwild

 Wild bootstrap residuals
 

---

### Description

The wild bootstrap residuals are computed as  $residuals * V$ , where  $V$  is a sampling from a random variable (see details section).

### Usage

```
rwild(residuals, type="golden")
```

### Arguments

residuals	residuals
type	Type of distribution of V.

### Details

For the construction of wild bootstrap residuals, sampling from a random variable  $V$  such that  $E[V^2] = 1$  and  $E[V] = 0$  is needed. A simple and suitable  $V$  is obtained with a discrete variable of the form:

- “golden”, Sampling from golden section bootstrap values suggested by Mammen (1993).

$$P\left\{V = \frac{1 - \sqrt{5}}{2}\right\} = \frac{5 + \sqrt{5}}{10} \text{ and } P\left\{V = \frac{1 + \sqrt{5}}{2}\right\} = \frac{5 - \sqrt{5}}{10},$$

which leads to the *golden section bootstrap*.

- “Rademacher”, Sampling from Rademacher distribution values  $\{-1, 1\}$  with probabilities  $\{\frac{1}{2}, \frac{1}{2}\}$ , respectively.
- “normal”, Sampling from a standard normal distribution.

### Value

The wild bootstrap residuals computed using a sample of the random variable  $V$ .

### Author(s)

Eduardo Garcia-Portugues, Manuel Febrero-Bande and Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>.

### References

- Mammen, E. (1993). *Bootstrap and wild bootstrap for high dimensional linear models*. Annals of Statistics 21, 255-285.
- Davidson, R. and E. Flachaire (2001). *The wild bootstrap, tamed at last*. working paper IER1000, Queens University.

**See Also**

[flm.test](#), [flm.Ftest](#), [dfv.test](#), [fregre.bootstrap](#)

**Examples**

```
n<-100
# For golden wild bootstrap variable
e.boot0=rwild(rep(1,len=n),"golden")
# Construction of wild bootstrap residuals
e=rnorm(n)
e.boot1=rwild(e,"golden")
e.boot2=rwild(e,"Rademacher")
e.boot3=rwild(e,"normal")
summary(e.boot1)
summary(e.boot2)
summary(e.boot3)
```

---

S.basis

*Smoothing matrix with roughness penalties by basis representation.*


---

**Description**

Provides the smoothing matrix  $S$  with roughness penalties.

**Usage**

```
S.basis(tt,basis,lambda=0,Lfdobj=vec2Lfd(c(0,0)),w=NULL,...)
```

**Arguments**

tt	Discretization points.
basis	Basis to use. See <a href="#">create.basis</a> .
lambda	A roughness penalty. By default, no penalty $\lambda=0$ .
Lfdobj	See <a href="#">eval.penalty</a> .
w	Optional case weights.
...	Further arguments passed to or from other methods. Arguments to be passed by default to <a href="#">create.basis</a>

**Details**

Provides the smoothing matrix  $S$  for the discretization points `tt` and `bbasis` with roughness penalties. If  $\lambda=0$  is not used penalty, else a basis roughness penalty matrix is calculated using [getbasispenalty](#).

**Value**

Return the smoothing matrix S.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ramsay, James O. and Silverman, Bernard W. (2006). *Functional Data Analysis*, 2nd ed., Springer, New York.

Wasserman, L. *All of Nonparametric Statistics*. Springer Texts in Statistics, 2006.

**See Also**

See Also as [S.np](#)

**Examples**

```
np=101
tt=seq(0,1,len=np)

nbasis=11
base1 <- create.bspline.basis(c(0, np), nbasis)
base2 <- create.fourier.basis(c(0, np), nbasis)

S1<-S.basis(tt,basis=base1,lambda=3)
image(S1)
S2<-S.basis(tt,basis=base2,lambda=3)
image(S2)
```

---

S.np

*Smoothing matrix by nonparametric methods.*

---

**Description**

Provides the smoothing matrix S for the discretization points `tt` by:  
Nadaraya-Watson kernel estimator (S.NW) with bandwidth parameter `h`.  
Local Linear Smoothing (S.LLR) with bandwidth parameter `h` .  
K nearest neighbors estimator (S.KNN) with parameter `knn` .

**Usage**

```
S.LLR(tt, h, Ker = Ker.norm,w=NULL,cv=FALSE)
S.NW(tt, h, Ker = Ker.norm,w=NULL,cv=FALSE)
S.KNN(tt,h=NULL,Ker=Ker.unif,w=NULL,cv=FALSE)
```

**Arguments**

tt	Vector of discretization points or distance matrix mdist
h	Smoothing parameter or bandwidth. In S.KNN, number of k-nearest neighbors.
Ker	Type of kernel used, by default normal kernel.
w	Optional case weights.
cv	=TRUE cross-validation is done.

**Value**

If S.LLR return the smoothing matrix by Local Linear Smoothing. If S.NW return the smoothing matrix by Nadaraya-Watson kernel estimator. If S.KNN return the smoothing matrix by k nearest neighbors estimator.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.

Wasserman, L. *All of Nonparametric Statistics*. Springer Texts in Statistics, 2006.

**See Also**

See Also as [S.basis](#)

**Examples**

```
tt=1:101
S=S.LLR(tt,h=5)
S2=S.LLR(tt,h=10,Ker=Ker.tri)
S3=S.NW(tt,h=10,Ker=Ker.tri)
S4=S.KNN(tt,h=5,Ker=Ker.tri)
par(mfrow=c(2,2))
image(S)
image(S2)
image(S3)
image(S4)
```

---

semimetric.basis      *Proximities between functional data*

---

### Description

Approximates semi-metric distances for functional data of class `fdata` or `fd`.

### Usage

```
semimetric.basis(fdata1, fdata2 = fdata1, nderiv=0, type.basis1=NULL,
nbasis1=NULL, type.basis2=type.basis1, nbasis2=NULL, ...)
```

### Arguments

<code>fdata1</code>	Functional data 1 or curve 1.
<code>fdata2</code>	Functional data 2 or curve 2.
<code>nderiv</code>	Order of derivation, used in <code>deriv.fd</code>
<code>type.basis1</code>	Type of Basis for <code>fdata1</code> .
<code>nbasis1</code>	Number of Basis for <code>fdata1</code> .
<code>type.basis2</code>	Type of Basis for <code>fdata2</code> .
<code>nbasis2</code>	Number of Basis for <code>fdata2</code> .
<code>...</code>	Further arguments passed to or from other methods.

### Details

Approximates semi-metric distances for functional data of two `fd` class objects. If functional data are not functional `fd` class, the `semimetric.basis` function creates a basis to represent the functional data, by default is used `create.bspline.basis` and the `fdata` class object is converted to `fd` class using the `Data2fd`.

The function calculates distances between the derivative of order `nderiv` of curves using `deriv.fd` function.

### Value

Returns a proximities matrix between functional data.

### References

Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.

### See Also

See also `metric.lp`, `semimetric.NPFDA` and `deriv.fd`



**Examples**

```

data(phoneme)
DATA1<-phoneme$learn[c(30:50,210:230),]
DATA2<-phoneme$test[231:250,]
a1=semimetric.basis(DATA1,DATA2)
a2=semimetric.basis(DATA1,DATA2,type.basis1="fourier",nbasis1=11,
type.basis2="fourier",nbasis2=11)
fd1 <- fdata2fd(DATA1)
fd2 <- fdata2fd(DATA2)
a3=semimetric.basis(fd1,fd2)
a4=semimetric.basis(fd1,fd2,nderiv=1)

```

---

semimetric.NPFDA

*Proximities between functional data (semi-metrics)*


---

**Description**

Computes semi-metric distances of functional data based on Ferraty F and Vieu, P. (2006).

`semimetric.deriv`: approximates  $L_2$  metric between derivatives of the curves based on their B-spline representation. The derivatives set with the argument `nderiv`.

`semimetric.fourier`: approximates  $L_2$  metric between the curves based on their B-spline representation. The derivatives set with the argument `nderiv`.

`semimetric.hshift`: computes distance between curves taking into account an horizontal shift effect.

`semimetric.mplsr`: computes distance between curves based on the partial least squares method.

`semimetric.pca`: computes distance between curves based on the functional principal components analysis method.

**Usage**

```

semimetric.hshift(fdata1, fdata2, t=1:ncol(DATA1),...)
semimetric.mplsr(fdata1, fdata2=fdata1, q=2, class1,...)
semimetric.pca(fdata1, fdata2=fdata1, q=1,...)
semimetric.deriv(fdata1, fdata2=fdata1, nderiv=1,
nknot=ifelse(floor(ncol(DATA1)/3)>floor((ncol(DATA1)-nderiv-4)/2),
floor((ncol(DATA1)-nderiv-4)/2),floor(ncol(DATA1)/3)),...)
semimetric.fourier(fdata1, fdata2=fdata1, nderiv=0,
nbasis=ifelse(floor(ncol(DATA1)/3)>floor((ncol(DATA1)-nderiv-4)/2),
floor((ncol(DATA1) - nderiv - 4)/2), floor(ncol(DATA1)/3)),
period=NULL,...)

```

**Arguments**

`fdata1` Functional data 1 or curve 1. DATA1 with dimension  $(n1 \times m)$ , where  $n1$  is the number of curves and  $m$  are the points observed in each curve.

fdata2	Functional data 2 or curve 2. DATA1 with dimension (n2 x m), where n2 is the number of curves and m are the points observed in each curve.
q	If semimetric.pca: the retained number of principal components. If semimetric.mpls: the retained number of factors.
nknot	semimetric.deriv argument: number of interior knots (needed for defining the B-spline basis).
nderiv	Order of derivation, used in semimetric.deriv and semimetric.fourier
nbasis	semimetric.fourier: size of the basis.
period	semimetric.fourier: allows to select the period for the fourier expansion.
t	semimetric.hshift: vector which defines t (one can choose 1, 2, ..., nbt where nbt is the number of points of the discretization)
class1	semimetric.mpls: vector containing a categorical response which corresponds to class number for units stored in DATA1.
...	Further arguments passed to or from other methods.

### Details

In the next semi-metric functions the functional data  $X$  is approximated by  $k_n$  elements of the Fourier, B-spline, PC or PLS basis using,  $\hat{X}_i = \sum_{k=1}^{k_n} \nu_{k,i} \xi_k$ , where  $\nu_k$  are the coefficient of the expansion on the basis function  $\{\xi_k\}_{k=1}^{\infty}$ .

The distances between the q-order derivatives of two curves  $X_1$  and  $X_2$  is,

$$d_2^{(q)}(X_1, X_2)_{k_n} = \sqrt{\frac{1}{T} \int_T \left( X_1^{(q)}(t) - X_2^{(q)}(t) \right)^2 dt}$$

where  $X_i^{(q)}(t)$  denot the  $q$  derivative of  $X_i$ .

semimetric.deriv and semimetric.fourier function use a B-spline and Fourier approximation respectively for each curve and the derivatives are directly computed by differentiating several times their analytic form, by default  $q=1$  and  $q=0$  respectively. semimetric.pca and semimetric.mpls function compute proximities between curves based on the functional principal components analysis (FPCA) and the functional partial least square analysis (FPLS), respectively. The FPC and FPLS reduce the functional data in a reduced dimensional space ( $q$  components). semimetric.mpls function requires a scalar response.

$$d_2^{(q)}(X_1, X_2)_{k_n} \approx \sqrt{\sum_{k=1}^{k_n} (\nu_{k,1} - \nu_{k,2})^2 \|\xi_k^{(q)}\|^2}$$

semimetric.hshift computes proximities between curves taking into account an horizontal shift effect.

$$d_{hshift}(X_1, X_2) = \min_{h \in [-mh, mh]} d_2(X_1(t), X_2(t+h))$$

where  $mh$  is the maximum horizontal shifted allowed.

**Value**

Returns a proximities matrix between two functional datasets.

**Source**

<http://www.math.univ-toulouse.fr/staph/npfda/>

**References**

Ferraty, F. and Vieu, P. (2006). *Nonparametric functional data analysis*. Springer Series in Statistics, New York.

Ferraty, F. and Vieu, P. (2006). *NPFDA in practice*. Free access on line at <http://www.lsp.ups-tlse.fr/staph/npfda/>

**See Also**

See also [metric.lp](#) and [semimetric.basis](#)

**Examples**

```
# INFERENCE PHONDAT
data(phoneme)
ind=1:100 # 2 groups
mlearn<-phoneme$learn[ind,]
mtest<-phoneme$test[ind,]
n=nrow(mlearn[["data"]])
np=ncol(mlearn[["data"]])
mdist1=semimetric.pca(mlearn,mtest)
mdist2=semimetric.pca(mlearn,mtest,q=2)
mdist3=semimetric.deriv(mlearn,mtest,nderiv=0)
mdist4=semimetric.fourier(mlearn,mtest,nderiv=2,nbasis=21)
#uses hshift function
#mdist5=semimetric.hshift(mlearn,mtest) #takes a lot
glearn<-phoneme$classlearn[ind]
#uses mplsr function
mdist6=semimetric.mplsr(mlearn,mtest,5,glearn)
mdist0=metric.lp(mlearn,mtest)
b=as.dist(mdist6)
c2=hclust(b)
plot(c2)
memb <- cutree(c2, k = 2)
table(memb,phoneme$classlearn[ind])
```

---

subset.fdata	<i>Subsetting</i>
--------------	-------------------

---

**Description**

Return subsets of fdata which meet conditions.

**Usage**

```
## S3 method for class 'fdata'
subset(x, subset, select, drop = TRUE,...)
```

**Arguments**

x	object to be subsetted (fdata class).
subset	logical expression indicating elements or rows to keep.
select	logical expression indicating points or columns to keep.
drop	passed on to [ indexing operator.
...	further arguments to be passed to or from other methods.

**Value**

An object similar to x contain just the selected elements.

**See Also**

See [subset](#) and [fdata](#).

---

summary.classif	<i>Summarizes information from kernel classification methods.</i>
-----------------	---

---

**Description**

Summary function for [classif.knn](#) or [classif.kernel](#).

**Usage**

```
## S3 method for class 'classif'
summary(object,...)
## S3 method for class 'classif'
print(x,digits = max(3, getOption("digits") - 3),...)
```

**Arguments**

object	Estimated by kernel classification.
x	Estimated by kernel classification.
digits	a non-null value for digits specifies the minimum number of significant digits to be printed in values. The default, NULL, uses <code>getOption(digits)</code> .
...	Further arguments passed to or from other methods.

**Details**

object from one of the following functions:

`classif.knn`  
`classif.kernel`

**Value**

Shows:

- Probability of correct classification by group `prob.classification`.
- Confusion matrix between the theoretical groups and estimated groups.
- Highest probability of correct classification `max.prob`.

If the object is returned from the function: `classif.knn`

- Vector of probability of correct classification by number of neighbors `knn`.
- Optimal number of neighbors: `knn.opt`.

If the object is returned from the function: `classif.kernel`

- Vector of probability of correct classification by bandwidth `h`.
- Functional measure of closeness (optimal distance, `h.opt`).

object                      Estimated by kernel classification.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**See Also**

See Also as: [classif.knn](#), [classif.kernel](#)  
and [summary.classif](#)

**Examples**

```
data(phoneme)
mlearn<-phoneme[["learn"]]
glearn<-phoneme[["classlearn"]]
## Not run, time consuming
# out=classif.knn(glearn,mlearn,knn=c(3,5,7))
# summary.classif(out)
# out2=classif.kernel(glearn,mlearn,h=2^(0:5))
#summary.classif(out2)
```

---

summary.fdata.comp      *Correlation for functional data by Principal Component Analysis*

---

**Description**

Compute correlation principal components of functional data and scalar response  $y$ .

**Usage**

```
## S3 method for class 'fdata.comp'
summary(object,y=NULL,biplot=TRUE,corplot=FALSE,...)
```

**Arguments**

object	fdata.comp class object calculated by: <code>fdata2pc</code> , <code>fdata2pls</code> , <code>fregre.pc</code> or <code>fregre.pls</code> .
y	(optional) The argument is only necessary if <code>corplot=TRUE</code> .
biplot	=TRUE draw the biplot and PC (or PLS) components.
corplot	=TRUE draw correlations between $y$ and PC (or PLS) components.
...	Further arguments passed to or from other methods.

**Value**

If `corplot=TRUE`, are displaying the biplot between the PC (or PLS) components.

If `corplot=TRUE`, are displaying the correlations between the PC (or PLS) components and response  $y$ .

If `ask=TRUE`, draw each graph in a window, waiting to confirm the change of page with a click of the mouse or pressing ENTER. If `ask=FALSE` draw graphs in one window.

**Author(s)**

Manuel Febrero-Bande and Manuel Oviedo de la Fuente  
<manuel.oviedo@usc.es>

**References**

Becker, R. A., Chambers, J. M. and Wilks, A. R. (1988). *The New S Language*. Wadsworth & Brooks/Cole.

Venables, W. N. and B. D. Ripley (2002). *Modern Applied Statistics with S*. Springer-Verlag.

**See Also**

See Also as [fdata2pc](#), [fdata2pls](#) and [cor](#)

**Examples**

```
## Not run
# n= 200;tt= seq(0,1,len=101)
# x0<-rproc2fdata(n,tt,sigma="wiener")
# x1<-rproc2fdata(n,tt,sigma=0.1)
# x<-x0*3+x1
# beta = tt*sin(2*pi*tt)^2
# fbeta = fdata(beta,tt)
# y<-inprod.fdata(x,fbeta)+rnorm(n,sd=0.1)
# pc1=fdata2pc(x)
# summary(pc1,y)
# pls1=fdata2pls(x,y)
# summary(pls1,cor=TRUE)
```

---

summary.fregre.fd	<i>Summarizes information from fregre.fd objects.</i>
-------------------	---

---

**Description**

Summary function for [fregre.pc](#), [fregre.basis](#), [fregre.pls](#), [fregre.np](#) and [fregre.plm](#) functions.

**Usage**

```
## S3 method for class 'fregre.fd'
summary(object,times.influ=3,times.sigma=3,draw=TRUE,...)
## S3 method for class 'fregre.fd'
print(x, digits = max(3, getOption("digits") - 3),...)
```

**Arguments**

object,x	Estimated by functional regression, fregre.fd object.
times.influ	Limit for detect possible influence curves.
times.sigma	Limit for detect possible outliers or atypical curves.
draw	=TRUE draw estimation and residuals graphics.

digits a non-null value for digits specifies the minimum number of significant digits to be printed in values. The default, NULL, uses `getOption(digits)`.

... Further arguments passed to or from other methods.

### Details

Shows:

- Call.
- R squared.
- Residual variance.
- Index of possible atypical curves or possible outliers.
- Index of possible influence curves.

If the `fregre.fd` object comes from the `fregre.pc` then shows:

- Variability of explicative variables explained by Principal Components.
- Variability for each principal components -PC-.

If `draw=TRUE` plot:

- y vs y fitted values.
- Residuals vs fitted values.
- Standardized residuals vs fitted values.
- Leverage.
- Residual boxplot.
- Quantile-Quantile Plot (qqnorm).

If `ask=FALSE` draw graphs in one window, by default. If `ask=TRUE`, draw each graph in a window, waiting to confirm.

### Value

Influence Vector of influence measures.

`i.influence` Index of possible influence curves.

`i.atypical` Index of possible atypical curves or possible outliers.

### Author(s)

Manuel Febrero-Bande and Manuel Oviedo de la Fuente  
 <manuel.oviedo@usc.es>



**See Also**

Summary function for [fregre.pc](#), [fregre.basis](#), [fregre.pls](#), [fregre.np](#) and [fregre.plm](#).

**Examples**

```
# Ex 1. Simulated data
n= 200;tt= seq(0,1,len=101)
x0<-rproc2fdata(n,tt,sigma="wiener")
x1<-rproc2fdata(n,tt,sigma=0.1)
x<-x0*3+x1
beta = tt*sin(2*pi*tt)^2
fbeta = fdata(beta,tt)
y<-inprod.fdata(x,fbeta)+rnorm(n,sd=0.1)

# Functional regression
res=fregre.pc(x,y,l=c(1:5))
summary(res,3,ask=TRUE)

# res2=fregre.pls(x,y,l=c(1:4))
# summary(res2)

# res3=fregre.pls(x,y)
# summary(res3)
```

---

summary.fregre.gkam     *Summarizes information from fregre.gkam objects.*

---

**Description**

Summary function for [fregre.gkam](#) function.

**Usage**

```
## S3 method for class 'fregre.gkam'
summary(object,draw=TRUE,selec = NULL, times.influ = 3,...)
## S3 method for class 'fregre.gkam'
print(x, digits = max(3, getOption("digits") - 3),...)
```

**Arguments**

object,x	Estimated by functional regression, fregre.fd object.
draw	=TRUE draw estimation and residuals graphics.
selec	Allows the plot for a single model term to be selected for printing. e.g. if you just want the plot for the second smooth term set selec=2. .
times.influ	Limit for detect possible influence curves.

`digits` a non-null value for `digits` specifies the minimum number of significant digits to be printed in values. The default, `NULL`, uses `getOption(digits)`.

`...` Further arguments passed to or from other methods.

### Details

- Family used.
- Number or iteration of algorithm and if it has converged.
- Residual and null deviance.
- Number of data.

Produces a list of summary information for a fitted `fregre.np` object for each functional covariate.

- Call.
- R squared.
- Residual variance.
- Index of possible atypical curves or possible outliers.
- Index of possible influence curves.

If `draw=TRUE` plot:

- y vs y fitted values.
- Residuals vs fitted values.
- Residual boxplot.
- Quantile-Quantile Plot (qqnorm).
- Plot for a each single model term.

If `ask=FALSE` draw graphs in one window, by default. If `ask=TRUE`, draw each graph in a window, waiting to confirm.

### Value

`object` Object.

### Author(s)

Manuel Febrero-Bande and Manuel Oviedo de la Fuente  
<manuel.oviedo@usc.es>

### See Also

Summary function for [fregre.gkam](#).

**Examples**

```
## Time consuming
# data(tecator)
# ind<-1:129
# ab=tecator$absorp.fdata[ind]
# ab2=fdata.deriv(ab,2)
# yfat=as.integer(cut(tecator$y[ind,"Fat"],c(0,15,100)))-1
# xlist=list("df"=data.frame(yfat),"ab2"=ab2,"ab"=ab)
# f<-yfat~ab+ab2
# res=fregre.gkam(f,data=xlist,family="binomial("logit"),control=list(maxit=2))
# summary(res)
# res
```

---

tecator

*tecator data*


---

**Description**

Water, Fat and Protein content of meat samples

**Usage**

```
data(tecator)
```

**Format**

The format is:

.. \$absorp.fdata: absorbance data. fdata class object with:

- "data": Matrix of class fdata with 215 curves (rows) discretized in 100 points or argvals (columns).
- "argvals": 100 discretization points from 850 to 1050mm
- "rangeval"=(850,1050): range("argvals")
- "names" list with: main an overall title "Tecator data set", xlab title for x axis "Wavelength (mm)" and ylab title for y axis "Absorbances".

.. \$y: the percentages of Fat, Water and Protein. The three contents are determined by analytic chemistry.

**Details**

absorp.fdata absorbance data for 215 samples. The first 129 were originally used as a training set endpoints the percentages of Fat, Water and Protein.  
for more details see tecator package

**Author(s)**

Manuel Febrero-Bande and Manuel Oviedo de la Fuente  
<manuel.oviedo@usc.es>

**Examples**

```
data(tecator)
names(tecator)
names(tecator$absorp.fdata)
names(tecator$y)
names(tecator$y)
class(tecator$absorp.fdata)
class(tecator$y)
dim(tecator$absorp.fdata)
dim(tecator$y)
```

---

Utilities

*A wrapper for the split and unlist function for fdata object*


---

**Description**

split.fdata divides the data in the fdata object x into the groups defined by f.  
Given a list structure x, unlist simplifies it to produce a fdata class object which contains all the atomic components which occur in x.

**Usage**

```
## S3 method for class 'fdata'
split(x,f,drop=FALSE,...)
## S3 method for class 'fdata'
unlist(x, recursive = TRUE, use.names = TRUE)
```

**Arguments**

x	an fdata object containing values to be divided into groups or an list of fdata objects containing values to be combine by rows in a to be flatten one fdata object
f	a factor in the sense that as.factor(f) defines the grouping, or a list of such factors in which case their interaction is used for the grouping.

drop	logical indicating if levels that do not occur should be dropped (if f is a factor or a list).
...	further potential arguments passed to methods.
recursive	logical. Should unlisting be applied to list components of x?
use.names	logical. Should names be preserved?

**Value**

The value returned from `split` is a list of `fdata` objects containing the values for the groups. The components of the list are named by the levels of `f` (after converting to a factor, or if already a factor and `drop = TRUE`, dropping unused levels).

The value returned from `unlist` is a `fdata` object containing the `fdata` components of the list.

**Author(s)**

Manuel Febrero Bande and Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**Examples**

```
fdataobj<-fdata(MontrealTemp)
fac<-factor(c(rep(1,len=17),rep(2,len=17)))
a1<-split.fdata(fdataobj,fac)
dim(a1[[1]]);dim(a1[[2]])
a2<-unlist.fdata(a1)
a2==fdataobj
```

---

 Var.y

*Sampling Variance estimates*


---

**Description**

Sampling variance or error variance estimates for regression estimates.

**Usage**

```
Var.y(y,S,Var.e=NULL)
Var.e(y,S)
```

**Arguments**

y	<code>fdata</code> class object.
S	Smoothing matrix calculated by <code>S.basis</code> or <code>S.NW</code> functions.
Var.e	Error Variance Estimates. If <code>Var.e=NULL</code> , <code>Var.e</code> is calculated.

**Value**

`Var.y`: returns the sampling variance of the functional data. `Var.e`: returns the sampling error variance of the functional data.

**Author(s)**

Manuel Febrero-Bande, Manuel Oviedo de la Fuente <manuel.oviedo@usc.es>

**References**

Ramsay, James O., and Silverman, Bernard W. (2006), *Functional Data Analysis*, 2nd ed., Springer, New York.

**See Also**

See Also as [Var.e](#)

**Examples**

```
a1<-seq(0,1,by=.01)
a2=rnorm(length(a1),sd=0.2)
f1<-(sin(2*pi*a1))+rnorm(length(a1),sd=0.2)
nc<-50
np<-length(f1)
tt=1:101
mdata<-matrix(NA,ncol=np,nrow=nc)
for (i in 1:nc) mdata[i,]<- (sin(2*pi*a1))+rnorm(length(a1),sd=0.2)
mdata<-fdata(mdata,tt)
S=S.NW(tt,h=0.15)
var.e<-Var.e(mdata,S)
var.y<-Var.y(mdata,S)
var.y2<-Var.y(mdata,S,var.e) #the same
```

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