

# Package ‘funData’

February 21, 2018

**Type** Package

**Title** An S4 Class for Functional Data

**Version** 1.2

**Date** 2018-02-21

**Author** Clara Happ [aut, cre]

**Maintainer** Clara Happ <clara.happ@stat.uni-muenchen.de>

**Description** S4 classes for univariate and multivariate functional data with utility functions.

**License** GPL-2

**Depends** methods

**Imports** abind, fields, foreach, graphics, grDevices, stats

**Suggests** covr, ggplot2, gridExtra, reshape2, zoo, testthat

**RoxygenNote** 6.0.1

**NeedsCompilation** no

**Repository** CRAN

**Date/Publication** 2018-02-21 18:15:46 UTC

## R topics documented:

addError . . . . .	2
approxNA . . . . .	3
Arith.funData . . . . .	4
as.funData . . . . .	7
as.irregFunData . . . . .	7
as.multiFunData . . . . .	8
dimSupp . . . . .	8
eFun . . . . .	9
eVal . . . . .	11
extractObs . . . . .	12
flipFuns . . . . .	13
funData-class . . . . .	15

getArgvals . . . . .	18
ggplot . . . . .	20
ggplot.funData . . . . .	21
ggplot.irregFunData . . . . .	23
ggplot.multiFunData . . . . .	24
integrate . . . . .	26
irregFunData-class . . . . .	27
Math.funData . . . . .	29
meanFunction . . . . .	30
multiFunData-class . . . . .	31
nObs . . . . .	34
nObsPoints . . . . .	35
norm . . . . .	36
plot.funData . . . . .	37
plot.irregFunData . . . . .	39
plot.multiFunData . . . . .	41
scalarProduct . . . . .	42
simFunData . . . . .	44
simMultiFunData . . . . .	46
sparsify . . . . .	49
tensorProduct . . . . .	50

<b>Index</b>	<b>52</b>
--------------	-----------

---

addError	<i>Add Gaussian white noise to functional data objects</i>
----------	--

---

## Description

This function generates an artificial noisy version of a functional data object of class `funData` (univariate) or `multiFunData` (multivariate) by adding iid. realizations of Gaussian random variables  $\varepsilon \sim \mathcal{N}(0, \sigma^2)$  to the observations. The standard deviation  $\sigma$  can be supplied by the user.

## Usage

```
addError(funDataObject, sd)
```

## Arguments

<code>funDataObject</code>	A functional data object of class <code>funData</code> or <code>multiFunData</code> .
<code>sd</code>	The standard deviation $\sigma$ of the Gaussian white noise that is added to the data. Defaults to 1. See Description.

## Value

An object of the same class as `funDataObject`, which is a noisy version of the original data.

**See Also**

[funData](#), [multiFunData](#), [simFunData](#), [simMultiFunData](#).

**Examples**

```
oldPar <- par(no.readonly = TRUE)
set.seed(1)

# Univariate functional data
plain <- simFunData(argvals = seq(0,1,0.01), M = 10, eFunType = "Fourier",
                   eValType = "linear", N = 1)$simData
noisy <- addError(plain, sd = 0.5)
veryNoisy <- addError(plain, sd = 2)

plot(plain, main = "Add error", ylim = range(veryNoisy@X))
plot(noisy, type = "p", pch = 20, add = TRUE)
plot(veryNoisy, type = "p", pch = 4, add = TRUE)
legend("topright", c("Plain", "Noisy", "Very Noisy"), lty = c(1, NA, NA), pch = c(NA, 20 ,4))

# Multivariate functional data
plain <- simMultiFunData(type = "split", argvals = list(seq(0,1,0.01), seq(-.5,.5,0.02)), M = 10,
                       eFunType = "Fourier", eValType = "linear", N = 1)$simData
noisy <- addError(plain, sd = 0.5)
veryNoisy <- addError(plain, sd = 2)

par(mfrow = c(1,2))
plot(plain[[1]], main = "Add error (multivariate)", ylim = range(veryNoisy[[1]]@X))
plot(noisy[[1]], type = "p", pch = 20, add = TRUE)
plot(veryNoisy[[1]], type = "p", pch = 4, add = TRUE)

plot(plain[[2]], main = "Add error (multivariate)", ylim = range(veryNoisy[[2]]@X))
plot(noisy[[2]], type = "p", pch = 20, add = TRUE)
plot(veryNoisy[[2]], type = "p", pch = 4, add = TRUE)
legend("topright", c("Plain", "Noisy", "Very Noisy"), lty = c(1, NA, NA), pch = c(NA, 20 ,4))

par(oldPar)
```

---

approxNA

*Approximate missing values for funData objects*

---

**Description**

This function approximates missing values for funData objects based on the [na.approx](#) interpolation method from the package **zoo**.

**Usage**

```
approxNA(object)
```

**Arguments**

object            An object of class funData with missing values (coded by NA).

**Value**

A funData object where missing values have been imputed.

**Warning**

This function requires the package **zoo** to be installed, otherwise it will throw a warning.

**Examples**

```
# Simulate some data
f <- simFunData(N = 10, M = 8, eVal = "linear", eFun = "Poly", argvals = seq(0, 1, 0.01))$simData

# Sparsify, i.e. generate artificial missings in the data
fSparse <- sparsify(f, minObs = 10, maxObs = 50)

# plot
oldpar <- par(no.readonly = TRUE)
par(mfrow = c(1,3))
plot(f, main = "Original Data")
plot(fSparse, main = "Sparse Data")
plot(approxNA(fSparse), main = "Reconstructed Data")
# faster with plot(fSparse, plotNA = TRUE, main = "Reconstructed Data")

par(oldpar)
```

---

Arith.funData

*Arithmetics for functional data objects*


---

**Description**

These functions allow basic arithmetics for functional data and numerics based on [Arith](#). The operations are made pointwise for each observation.

**Usage**

```
## S4 method for signature 'funData,funData'
Arith(e1, e2)

## S4 method for signature 'funData,numeric'
Arith(e1, e2)

## S4 method for signature 'numeric,funData'
Arith(e1, e2)
```

```

## S4 method for signature 'multiFunData,multiFunData'
Arith(e1, e2)

## S4 method for signature 'multiFunData,numeric'
Arith(e1, e2)

## S4 method for signature 'numeric,multiFunData'
Arith(e1, e2)

## S4 method for signature 'irregFunData,numeric'
Arith(e1, e2)

## S4 method for signature 'numeric,irregFunData'
Arith(e1, e2)

## S4 method for signature 'irregFunData,irregFunData'
Arith(e1, e2)

## S4 method for signature 'irregFunData,funData'
Arith(e1, e2)

## S4 method for signature 'funData,irregFunData'
Arith(e1, e2)

```

### Arguments

`e1`, `e2`      Objects of class `funData`, `irregFunData`, `multiFunData` or `numeric`. If two functional data objects are used, they must be of the same class, have the same domain and the same number of observations. For exceptions, see [Details](#).

### Details

If two objects of a functional data class (`funData`, `irregFunData` or `multiFunData`) are used, they normally must be of the same class, have the same domain and the same number of observations. Exceptions are accepted if

- one object has only one observation. In this case, the arithmetic operations are done pairwise for this single function and all functions of the other object (e.g. when subtracting the mean function from a `funData` object). This single function must be defined on the same domain as the other functions (or, in case of `irregFunData`, on the union of all observation grids).
- one of the two objects is of class `irregFunData`. Then, the other object can be of class `funData`, too, if it is defined on the union of all observation grids. The result is an `irregFunData` object which is defined on the same observation grid as the original `irregFunData` object.

### Value

An object of the same functional data class as `e1` or `e2`, respectively.

**Warning**

Note that not all combinations of operations and classes make sense, e.g.  $e1 \wedge e2$  is sensible if  $e1$  is of class `funData`, `irregFunData` or `multiFunData` and  $e2$  is numeric. The reverse is not true.

**See Also**

[funData](#), [irregFunData](#), [multiFunData](#), [Arith](#)

**Examples**

```
oldpar <- par(no.readonly = TRUE)
par(mfrow = c(3,2), mar = rep(2.1,4))

argvals <- seq(0, 2*pi, 0.01)
object1 <- funData(argvals, outer(seq(0.75, 1.25, by = 0.05), sin(argvals)))
object2 <- funData(argvals, outer(seq(0.75, 1.25, by = 0.05), cos(argvals)))

plot(object1, main = "Object1")
plot(object2, main = "Object2")

# Only functional data objects
plot(object1 + object2, main = "Sum")
plot(object1 - object2, main = "Difference")

# Mixed
plot(4 * object1 + 5, main = "4 * Object1 + 5") # Note y-axis!
plot(object1^2 + object2^2, main = "Pythagoras")

### Irregular
ind <- replicate(11, sort(sample(1:length(argvals), sample(5:10, 1))))
i1 <- irregFunData(
  argvals = lapply(1:11, function(i, ind, x){x[ind[[i]]}], ind = ind, x = object1@argvals[[1]]),
  X = lapply(1:11, function(i, ind, y){y[i, ind[[i]]}], ind = ind, y = object1@X)
i2 <- irregFunData(
  argvals = lapply(1:11, function(i, ind, x){x[ind[[i]]}], ind = ind, x = object2@argvals[[1]]),
  X = lapply(1:11, function(i, ind, y){y[i, ind[[i]]}], ind = ind, y = object2@X)

plot(i1, main = "Object 1 (irregular)")
plot(i2, main = "Object 2 (irregular)")

# Irregular and regular functional data objects
plot(i1 + i2, main = "Sum")
plot(i1 - object2, main = "Difference")

# Mixed
plot(4 * i1 + 5, main = "4 * i1 + 5") # Note y-axis!
plot(i1^2 + i2^2, main = "Pythagoras")
par(oldpar)
```

---

as.funData	<i>Coerce an irregFunData object to class funData</i>
------------	---

---

**Description**

This function coerces an object of class `irregFunData` to a `funData` object with missing values, which is defined on the union of all observation points.

**Usage**

```
as.funData(object)

## S4 method for signature 'irregFunData'
as.funData(object)
```

**Arguments**

object	The <code>irregFunData</code> object that is to be converted to a <code>funData</code> object with missing values.
--------	--

**See Also**

[funData](#), [irregFunData](#)

---

as.irregFunData	<i>Coerce a funData object to class irregFunData</i>
-----------------	--

---

**Description**

This function coerces an object of class `funData` to a `irregFunData` object.

**Usage**

```
as.irregFunData(object)

## S4 method for signature 'funData'
as.irregFunData(object)
```

**Arguments**

object	The <code>funData</code> object that is to be converted to a <code>irregFunData</code> object.
--------	--

**See Also**

[funData](#), [irregFunData](#)

---

as.multiFunData	<i>Coerce a funData object to class multiFunData</i>
-----------------	--

---

**Description**

Coerce a funData object to class multiFunData with one element.

**Usage**

```
as.multiFunData(object)

## S4 method for signature 'funData'
as.multiFunData(object)
```

**Arguments**

object	The funData object that is to be converted to a multiFunData object of length 1.
--------	--

**See Also**

[funData](#), [multiFunData](#)

**Examples**

```
# create funData object with 5 observations
x <- seq(0,1,0.01)
f1 <- funData(argvals = x, X = 1:5 %0% x)
f1
class(f1)

# coerce to multiFunData object (of length 1)
m1 <- as.multiFunData(f1)
m1
class(m1)
```

---

dimSupp	<i>Support dimension of functional data</i>
---------	---

---

**Description**

This function returns the support dimension of an object of class funData, irregFunData or multiFunData.

**Usage**

```
dimSupp(object)
```



**Arguments**

object            An object of class funData, irregFunData or multiFunData.

**Value**

If object is univariate (i.e. of class funData or irregFunData), the function returns the dimension of the support of object. If object is multivariate (i.e. of class multiFunData), the function returns a vector, giving the support dimension of each element.

**See Also**

[funData](#), [irregFunData](#), [multiFunData](#)

**Examples**

```
# Univariate (one-dimensional)
object1 <- funData(argvals = 1:5, X = rbind(1:5, 6:10))
dimSupp(object1)

# Univariate (two-dimensional)
object2 <- funData(argvals = list(1:10, 1:5), X = array(rnorm(100), dim = c(2,10,5)))
dimSupp(object2)

# Univariate (irregular)
irregObject <- irregFunData(argvals = list(1:5, 2:4), X = list(2:6, 3:5))
dimSupp(irregObject)

# Multivariate
multiObject <- multiFunData(object1, object2)
dimSupp(multiObject)
```

---

eFun

*Generate orthonormal eigenfunctions*

---

**Description**

This function calculates  $M$  (orthonormal) basis functions on a given interval, that can be interpreted as the first  $M$  eigenfunctions of an appropriate data generating process of functional data.

**Usage**

```
eFun(argvals, M, ignoreDeg = NULL, type)
```

## Arguments

argvals	A vector of numerics, defining a (fine) grid on the interval for which the basis functions are computed.
M	An integer, specifying the number of functions that are calculated.
ignoreDeg	A vector of numerics, specifying the degrees to be ignored for type "PolyHigh". Defaults to NULL. See Details.
type	A character string, specifying the type of functions that are calculated. See Details.

## Details

The function implements three families of orthonormal basis functions plus variations of them. The parameter `type`, that specifies the functions to be calculated, can have the following values:

- "Poly": Calculate orthonormal Legendre polynomials of degree 0,...,M-1.
- "PolyHigh": Calculate  $M$  orthonormal Legendre Polynomials of higher degree. The vector of indices `ignoreDeg` specifies the functions to be ignored. If `ignoreDeg` is not specified, the function returns an error.
- "Fourier": Calculate the first  $M$  Fourier basis functions.
- "FourierLin": Calculate the first  $M - 1$  Fourier basis functions plus the linear function, orthonormalized to the previous functions via Gram-Schmidts method. This type is currently implemented for functions on the unit interval  $[0, 1]$  only. If the function is called with other `argvals`, an error is thrown.
- "Wiener": Calculate the first  $M$  orthonormal eigenfunctions of the Wiener process.

## Value

A univariate functional data object of class `funData` containing the basis functions on the given interval.

## See Also

[funData](#), [simFunData](#), [simMultiFunData](#)

## Examples

```
oldPar <- par(no.readonly = TRUE)

argvals <- seq(0,1,0.01)

par(mfrow = c(3,2))
plot(eFun(argvals, M = 4, type = "Poly"), main = "Poly", ylim = c(-3,3))
plot(eFun(argvals, M = 4, ignoreDeg = 1:2, type = "PolyHigh"), main = "PolyHigh", ylim = c(-3,3))
plot(eFun(argvals, M = 4, type = "Fourier"), main = "Fourier", ylim = c(-3,3))
plot(eFun(argvals, M = 4, type = "FourierLin"), main = "FourierLin", ylim = c(-3,3))
plot(eFun(argvals, M = 4, type = "Wiener"), main = "Wiener", ylim = c(-3,3))
par(oldPar)
```

---

eVal *Generate a sequence of simulated eigenvalues*

---

### Description

This function generates  $M$  decreasing eigenvalues.

### Usage

```
eVal(M, type)
```

### Arguments

**M** An integer, the number of eigenvalues to be generated.  
**type** A character string specifying the type of eigenvalues that should be calculated. See Details.

### Details

The function implements three types of eigenvalues:

- "linear": The eigenvalues start at 1 and decrease linearly towards 0:

$$\nu_m = \frac{M + 1 - m}{m}.$$

- "exponential": The eigenvalues start at 1 and decrease exponentially towards 0:

$$\nu_m = \exp\left(-\frac{m-1}{2}\right).$$

- "wiener": The eigenvalues correspond to the eigenvalues of the Wiener process:

$$\nu_m = \frac{1}{(\pi/2 \cdot (2m-1))^2}.$$

### Value

A vector containing the  $M$  decreasing eigenvalues.

### Examples

```
oldpar <- par(no.readonly = TRUE)

# simulate M = 10 eigenvalues
M <- 10
eLin <- eVal(M = M, type = "linear")
eExp <- eVal(M = M, type = "exponential")
eWien <- eVal(M = M, type = "wiener")
```

```

par(mfrow = c(1,1))
plot(1:M, eLin, pch = 20, xlab = "m", ylab = expression(nu[m]), ylim = c(0,1))
points(1:M, eExp, pch = 20, col = 3)
points(1:M, eWien, pch = 20, col = 4)
legend("topright", legend = c("linear", "exponential", "wiener"), pch = 20, col = c(1,3,4))

par(oldpar)

```

---

extractObs

*Extract observations of functional data*


---

### Description

This function extracts one or more observations and/or observations on a part of the domain from a `funData`, `irregFunData` or `multiFunData` object.

### Usage

```
extractObs(object, obs = 1:nObs(object), argvals = getArgvals(object))
```

### Arguments

<code>object</code>	An object of class <code>funData</code> , <code>irregFunData</code> or <code>multiFunData</code> .
<code>obs</code>	A numeric vector, giving the indices of the observations to extract (default: all observations).
<code>argvals</code>	The part of the domain to be extracted (default: the whole domain <code>object@argvals</code> ). Must be a list or a numeric vector (only for one-dimensional domains, see also the definition of <a href="#">funData</a> , <a href="#">multiFunData</a> ).

### Details

In case of an `irregFunData` object, some functions may not have observation points in the given part of the domain. In this case, the functions are removed from the extracted dataset and a warning is thrown.

### Value

An object of class `funData`, `irregFunData` or `multiFunData` containing the desired observations.

### Warning

The function is currently implemented only for functional data with up to three-dimensional domains.

### See Also

[funData](#), [irregFunData](#), [multiFunData](#)

**Examples**

```

# Univariate - one-dimensional domain
object1 <- funData(argvals = 1:5, X = rbind(1:5, 6:10))
extractObs(object1, obs = 1)
extractObs(object1, argvals = 1:3)
extractObs(object1, argvals = list(1:3)) # the same as the statement before

# Univariate - two-dimensional domains
object2 <- funData(argvals = list(1:5, 1:6), X = array(1:60, dim = c(2, 5, 6)))
extractObs(object2, obs = 1)
extractObs(object2, argvals = list(1:3, c(2,4,6))) # argvals must be supplied as list

# Univariate - irregular
irregObject <- irregFunData(argvals = list(1:5, 2:4), X = list(2:6, 3:5))
extractObs(irregObject, obs = 2)
extractObs(irregObject, argvals = 1:3)
extractObs(irregObject, argvals = c(1,5)) # throws a warning, as second function has no observations

# Multivariate
multiObject <- multiFunData(object1, object2)
extractObs(multiObject, obs = 2)
extractObs(multiObject, argvals = list(1:3, list(1:3, c(2,4,6))))

```

---

flipFuns

*Flip functional data objects*


---

**Description**

This function flips an object `newObject` of class `funData`, `irregFunData` or `multiFunData` with respect to a reference object `refObject` of the same class (or of class `funData`, if `newObject` is irregular). This is particularly useful when dealing with functional principal components, as they are only defined up to a sign change. For details, see below.

**Usage**

```
flipFuns(refObject, newObject, ...)
```

**Arguments**

<code>refObject</code>	An object of class <code>funData</code> , <code>irregFunData</code> or <code>multiFunData</code> that serves as reference. It must have the same number of observations as <code>newObject</code> or have only one observation. In this case, all observations in <code>newObject</code> are flipped with respect to this single observation.
<code>newObject</code>	An object of class <code>funData</code> , <code>irregFunData</code> or <code>multiFunData</code> that is to be flipped with respect to <code>refObject</code> .
<code>...</code>	Further parameters passed to <a href="#">norm</a> .

## Details

Functional principal component analysis is an important tool in functional data analysis. Just as eigenvectors, eigenfunctions (or functional principal components) are only defined up to a sign change. This may lead to difficulties in simulation studies or when bootstrapping pointwise confidence bands, as in these cases one wants the estimates to have the same "orientation" as the true function (in simulation settings) or the non-bootstrapped estimate (when calculating bootstrap confidence bands). This function allows to flip (i.e. multiply by  $-1$ ) all observations in `newObject` that have a different orientation than their counterparts in `refData`.

Technically, the function compares the distance between `newObject` and `refObject`

$$\|f_{\text{new}} - f_{\text{ref}}\|$$

and the distance between `newObject` and  $-1 * \text{refObject}$

$$\|f_{\text{new}} + f_{\text{ref}}\|.$$

If `newObject` is closer to  $-1 * \text{refObject}$ , it is flipped, i.e. multiplied by  $-1$ .

## Value

An object of the same class as `newData` with flipped observations.

## Warning

The function is currently implemented only for functional data with one- and two-dimensional domains.

## See Also

[funData](#), [irregFunData](#), [multiFunData](#), [Arith.funData](#)

## Examples

```
### Univariate
argvals <- seq(0,2*pi,0.01)
refData <- funData(argvals, rbind(sin(argvals))) # one observation as reference
newData <- funData(argvals, outer(sample(c(-1,1), 11, replace = TRUE) * seq(0.75, 1.25, by = 0.05),
                                sin(argvals)))

oldpar <- par(no.readonly = TRUE)
par(mfrow = c(1,2))

plot(newData, col = "grey", main = "Original data")
plot(refData, col = "red", lwd = 2, add = TRUE)

plot(flipFuns(refData, newData), col = "grey", main = "Flipped data")
plot(refData, col = "red", lwd = 2, add = TRUE)

### Univariate (irregular)
ind <- replicate(11, sort(sample(1:length(argvals), sample(5:10,1)))) # sample observation points
```

```

argsvalsIrreg <- lapply(ind, function(i){argsvals[i]})
argsvalsIrregAll <- unique(sort(unlist(argsvalsIrreg)))
# one observation as reference (fully observed)
refDataFull <- funData(argsvals, rbind(sin(argsvals)))
# one observation as reference (irregularly observed)
refDataIrreg <- irregFunData(argsvals = list(argsvalsIrregAll), X = list(sin(argsvalsIrregAll)))
newData <- irregFunData(argsvals = argsvalsIrreg, X = mapply(function(x, a, s){s * a * sin(x)},
  x = argsvalsIrreg, a = seq(0.75, 1.25, by = 0.05), s = sample(c(-1,1), 11, replace = TRUE)))

plot(newData, col = "grey", main = "Original data (regular reference)")
plot(refDataFull, col = "red", lwd = 2, add = TRUE)

plot(flipFuns(refDataFull, newData), col = "grey", main = "Flipped data")
plot(refDataFull, col = "red", lwd = 2, add = TRUE)

plot(newData, col = "grey", main = "Original data (irregular reference)")
plot(refDataIrreg, col = "red", lwd = 2, add = TRUE)

plot(flipFuns(refDataIrreg, newData), col = "grey", main = "Flipped data")
plot(refDataIrreg, col = "red", lwd = 2, add = TRUE)

### Multivariate
refData <- multiFunData(funData(argsvals, rbind(sin(argsvals))), # one observation as reference
  funData(argsvals, rbind(cos(argsvals))))
sig <- sample(c(-1,1), 11, replace = TRUE)
newData <- multiFunData(funData(argsvals, outer(sig * seq(0.75, 1.25, by = 0.05), sin(argsvals))),
  funData(argsvals, outer(sig * seq(0.75, 1.25, by = 0.05), cos(argsvals))))

par(mfrow = c(2,2))

plot(newData[[1]], col = topo.colors(11), main = "Original data")
plot(refData[[1]], col = "red", lwd = 2, add = TRUE)

plot(newData[[2]], col = topo.colors(11), main = "Original data")
plot(refData[[2]], col = "red", lwd = 2, add = TRUE)

plot(flipFuns(refData, newData)[[1]], col = topo.colors(11), main = "Flipped data")
plot(refData[[1]], col = "red", lwd = 2, add = TRUE)

plot(flipFuns(refData, newData)[[2]], col = topo.colors(11), main = "Flipped data")
plot(refData[[2]], col = "red", lwd = 2, add = TRUE)

par(oldpar)

```

---

funData-class

*A class for (univariate) functional data*


---

### Description

The funData class represents functional data on  $d$ -dimensional domains. The two slots represent the domain (x-values) and the values of the different observations (y-values).

**Usage**

```
## S4 method for signature 'list,array'
funData(argvals, X)

## S4 method for signature 'numeric,array'
funData(argvals, X)

## S4 method for signature 'funData'
show(object)

## S4 method for signature 'funData'
names(x)

## S4 replacement method for signature 'funData'
names(x) <- value

## S4 method for signature 'funData'
str(object, ...)

## S4 method for signature 'funData'
summary(object, ...)
```

**Arguments**

argvals	A list of numeric vectors or a single numeric vector, giving the sampling points in the domains. See Details.
X	An array of dimension $N \times M$ (for one-dimensional domains, or $N \times M_1 \times \dots \times M_d$ for higher-dimensional domains), giving the observed values for $N$ individuals. Missing values can be included via NA. See Details.
object	A funData object.
x	The funData object.
value	The names to be given to the funData curves.
...	Other parameters passed to summary.

**Details**

Functional data can be seen as realizations of a random process

$$X : \mathcal{T} \rightarrow \text{IR}$$

on a  $d$ -dimensional domain  $\mathcal{T}$ . The data is usually sampled on a fine grid  $T \subset \mathcal{T}$ , which is represented in the argvals slot of a funData object. All observations are assumed to be sampled over the same grid  $T$ , but can contain missing values (see below). If  $\mathcal{T}$  is one-dimensional, argvals can be supplied either as a numeric vector, containing the x-values or as a list, containing such a vector. If  $\mathcal{T}$  is higher-dimensional, argvals must always be supplied as a list, containing numeric vectors of the x-values in dimensions  $1, \dots, d$ .

The observed values are represented in the X slot of a funData object, which is an array of dimension  $N \times M$  (for one-dimensional domains, or  $N \times M_1 \times \dots \times M_d$  for higher-dimensional



domains). Here  $N$  equals the number of observations and  $M$  denotes the number of sampling points (for higher dimensional domains  $M_i$  denotes the number of sampling points in dimension  $i, i = 1, \dots, d$ ). Missing values in the observations are allowed and must be marked by NA. If missing values occur due to irregular observation points, the data can be stored alternatively as an object of class `irregFunData`.

Generic functions for the `funData` class include a print method, [plotting](#) and [basic arithmetics](#). Further methods for `funData`:

- `dimSupp, nObs`: Informations about the support dimensions and the number of observations,
- `getArgvals, extractObs`: Getting/Setting slot values (instead of accessing them directly via `funData@argvals, funData@X`) and extracting single observations or data on a subset of the domain,
- `integrate, norm`: Integrate all observations over their domain or calculating the  $L^2$  norm.

A `funData` object can be coerced to a `multiFunData` object using `as.multiFunData(funDataObject)`.

### Methods (by generic)

- `funData`: Constructor for functional data objects with `argvals` given as list.
- `funData`: Constructor for functional data objects with `argvals` given as vector of numerics (only valid for one-dimensional domains).
- `show`: Print basic information about the `funData` object in the console. The default console output for `funData` objects.
- `names`: Get the names of the `funData` object.
- `names<-`: Set the names of the `funData` object.
- `str`: A `str` method for `funData` objects, giving a compact overview of the structure.
- `summary`: A `summary` method for `funData` objects.

### Slots

`argvals` The domain  $\mathcal{T}$  of the data. See [Details](#).

`X` The functional data samples. See [Details](#).

### See Also

[irregFunData](#), [multiFunData](#)

### Examples

```
### Creating a one-dimensional funData object with 2 observations
# Basic
f1 <- new("funData", argvals = list(1:5), X = rbind(1:5,6:10))
# Using the constructor with first argument supplied as array
f2 <- funData(argvals = list(1:5), X = rbind(1:5, 6:10))
# Using the constructor with first argument supplied as numeric vector
f3 <- funData(argvals = 1:5, X = rbind(1:5, 6:10))
# Test if all the same
all.equal(f1,f2)
```

```

all.equal(f1,f3)
# Display funData object in the console
f3

# A more realistic object
argvals <- seq(0,2*pi,0.01)
object <- funData(argvals, outer(seq(0.75, 1.25, by = 0.05), sin(argvals)))
# Display / summary give basic information
object
summary(object)
# Use the plot function to get an impression of the data
plot(object)

### Higher-dimensional funData objects with 2 observations
# Basic
g1 <- new("funData", argvals = list(1:5, 1:3),
          X = array(1:30, dim = c(2,5,3)))
# Using the constructor
g2 <- funData(argvals = list(1:5, 1:3),
              X = array(1:30, dim = c(2,5,3)))
# Test if the same
all.equal(g1,g2)
# Display funData object in the console
g2
# Summarize information
summary(g2)

```

---

getArgvals

*Extract and set slots from functional data objects*


---

### Description

These functions can be used to extract and set the slots of `funData`, `irregFunData` and `multiFunData` objects.

### Usage

```

getArgvals(object)

getX(object)

setArgvals(object, newArgvals)

setX(object, newX)

```

### Arguments

`object`            An object of class `funData`, `irregFunData` or `multiFunData`.

newArgvals      See Details.  
 newX            See Details.

## Details

Objects of class `funData` or `irregFunData` have two slots, `argvals` (for the x-values) and `X` (for the y-values for each observation). Using the `getArgvals` and `getX` methods for the classes `funData` and `irregFunData` is equivalent to accessing the slots directly via `object@argvals` and `object@X`. Analogously, the `setArgvals` and `setX` functions are equivalent to setting `object@argvals` to `newArgvals` or `object@X` to `newX`, respectively. The new values must hence have the same structure as the original ones. As an exception, for an object of class `funData` the number of new observations in `newX` may differ from the current (e.g. when adding new observations). In this case, the function throws a warning.

Objects of class `multiFunData` are lists of several `funData` objects. The functions `getArgvals` and `getX` for `multiFunData` objects therefore return a list of the same length as `object`, where each list element corresponds to the `argvals` or `X` slot of the univariate element. The `setArgvals` and `setX` functions for `multiFunData` objects must be lists of the same length as `object`, where each list element corresponds to the new `argvals` or new `X` slot for the univariate elements.

For all classes, the set functions do not change the object, unless their result is assigned to `object` (see Examples).

## Value

See Details.

## See Also

[funData](#), [irregFunData](#), [multiFunData](#)

## Examples

```
### Univariate
object <- funData(argvals = 1:5, X = rbind(1:5, 6:10))
object

# get-methods
getArgvals(object)
getX(object)

# set-methods
setArgvals(object, 0:4)
object # no change
object <- setArgvals(object, 0:4) # reassign the result to object
object # now, argvals is changed
## Not run: object <- setArgvals(object, 1:4) # wrong length
object <- setX(object, rbind(0:4, 5:9))
newObject <- setX(object, rbind(0:4, 5:9, 10:14)) # warning: now 3 observations (was 2 before)
## Not run: object <- setX(object, rbind(1:4, 5:8)) # wrong length

### Univariate (irregular)
```

```
irregObject <- irregFunData(argvals = list(1:5, 2:4), X = list(2:6, 3:5))
irregObject

# get-methods
getArgvals(irregObject)
getX(irregObject)

newIrregObject <- setArgvals(irregObject, list(0:4, 1:3))
newIrregObject <- setX(irregObject, list(12:16, 13:15))

### Multivariate
multiObject <- multiFunData(object, funData(argvals = 1:3, X = rbind(3:5, 6:8)))
multiObject

# get-methods
getArgvals(multiObject)
getX(multiObject)

# set-methods (for special cases see univariate version)
multiObject <- setArgvals(multiObject, list(5:1, 3:1))
multiObject <- setX(multiObject, list(rbind(5:1, 10:6), rbind(5:3, 8:6)))
```

---

ggplot

*ggplot Graphics for Functional Data Objects*

---

## Description

The `funData` allows to plot `funData`, `multiFunData` and `irregFunData` objects via the **ggplot2** package. For Details, see the help pages of the class-specific plot functions (see below).

## Usage

```
ggplot(data, ...)
```

## Arguments

<code>data</code>	A <code>funData</code> , <code>multiFunData</code> or <code>irregFunData</code> object.
<code>...</code>	Further parameters passed to the class-specific methods.

## Value

A `ggplot` object

## See Also

[ggplot.funData](#) for `funData` objects, [ggplot.multiFunData](#) for `multiFunData` objects and [ggplot.irregFunData](#) for `irregFunData` objects.

---

ggplot.funData      *Visualize functional data objects using ggplot*

---

### Description

This function allows to plot funData objects based on the **ggplot2** package. The function provides a wrapper that rearranges the data in a funData object on a one- or two-dimensional domain and provides a basic **ggplot** object, which can be customized using all functionalities of the **ggplot2** package.

### Usage

```
ggplot.funData(data, obs = 1:nObs(data), geom = "line", plotNA = FALSE,  
  add = FALSE, ...)
```

```
## S4 method for signature 'funData'  
ggplot(data, ...)
```

### Arguments

data	A funData object on a one- or two-dimensional domain.
obs	A vector of numerics giving the observations to plot. Defaults to all observations in data. For two-dimensional functions (images) obs must have length 1.
geom	A character string describing the geometric object to use. Defaults to "line". See <b>ggplot2</b> for details.
plotNA	Logical. If TRUE, missing values are interpolated using the <a href="#">approxNA</a> function (only for one-dimensional functions). Defaults to FALSE. See Details.
add	Logical. If TRUE, add to current plot (only for one-dimensional functions). Defaults to FALSE.
...	Further parameters passed to <a href="#">geom_line</a> (for one dimensional domains, e.g. alpha, color, fill, linetype, size) or to <a href="#">geom_raster</a> (for two-dimensional domains, e.g. hjust, vjust, interpolate).

### Details

If some observations contain missing values (coded via NA), the functions can be interpolated using the option `plotNA = TRUE`. This option relies on the [na.approx](#) function in package [zoo](#) and is currently implemented for one-dimensional functions only in the function [approxNA](#).

### Value

A **ggplot** object that can be customized using all functionalities of the **ggplot2** package.

### See Also

[funData](#), [ggplot](#), [plot.funData](#)

**Examples**

```

# Install package ggplot2 before running the examples

# One-dimensional
argvals <- seq(0,2*pi,0.01)
object <- funData(argvals,
                  outer(seq(0.75, 1.25, length.out = 11), sin(argvals)))

g <- ggplot(object) # returns ggplot object
g # plot the object

# add the mean function in red
g + ggplot(meanFunction(object), add = TRUE, col = 2)

# Two-dimensional
X <- array(0, dim = c(2, length(argvals), length(argvals)))
X[1,,] <- outer(argvals, argvals, function(x,y){sin((x-pi)^2 + (y-pi)^2)})
X[2,,] <- outer(argvals, argvals, function(x,y){sin(2*x*pi) * cos(2*y*pi)})
object2D <- funData(list(argvals, argvals), X)

ggplot(object2D, obs = 1)
ggplot(object2D, obs = 2)
## Not run: plot(object2D) # must specify obs!

### More examples ###

par(mfrow = c(1,1))

# using plotNA (needs packages zoo and gridExtra)

objectMissing <- funData(1:5, rbind(c(1, NA, 5, 4, 3), c(10, 9, NA, NA, 6)))
g1 <- ggplot(objectMissing) # the default
g2 <- ggplot(objectMissing, plotNA = TRUE) # requires zoo

gridExtra::grid.arrange(g1 + ggplot2::ggtitle("plotNA = FALSE (default)"),
                        g2 + ggplot2::ggtitle("plotNA = TRUE")) # requires gridExtra

# Customizing plots (see ggplot2 documentation for more details)
# parameters passed to geom_line are passed via the ... argument
gFancy <- ggplot(object, color = "red", linetype = 2)
gFancy

# new layers can be added directly to the ggplot object
gFancy + ggplot2::theme_bw() # add new layers to the ggplot object
gFancy + ggplot2::ggtitle("Fancy Plot with Title and Axis Legends") +
  ggplot2::xlab("The x-Axis") + ggplot2::ylab("The y-Axis")

ggplot(object2D, obs = 1) + ggplot2::ggtitle("Customized 2D plot") + ggplot2::theme_minimal() +
  ggplot2::scale_fill_gradient(high = "green", low = "blue", name = "Legend here")

```

---

ggplot.irregFunData     *Visualize irregular functional data objects using ggplot*

---

### Description

This function allows to plot `irregFunData` objects on their domain based on the **ggplot2** package. The function provides a wrapper that rearranges the data in a `irregFunData` object returns a basic `ggplot` object, which can be customized using all functionalities of the **ggplot2** package.

### Usage

```
ggplot.irregFunData(data, obs = 1:nObs(data), geom = "line", add = FALSE,
  ...)
```

```
## S4 method for signature 'irregFunData'
ggplot(data, ...)
```

### Arguments

<code>data</code>	A <code>irregFunData</code> object.
<code>obs</code>	A vector of numerics giving the observations to plot. Defaults to all observations in data. For two-dimensional functions (images) <code>obs</code> must have length 1.
<code>geom</code>	A character string describing the geometric object to use. Defaults to "line". See <b>ggplot2</b> for details.
<code>add</code>	Logical. If TRUE, add to current plot (only for one-dimensional functions). Defaults to FALSE.
<code>...</code>	Further parameters passed to <code>stat_identity</code> , e.g. <code>alpha</code> , <code>color</code> , <code>fill</code> , <code>linetype</code> , <code>size</code> ).

### Value

A `ggplot` object that can be customized using all functionalities of the **ggplot2** package.

### See Also

[irregFunData](#), [ggplot](#), [plot.irregFunData](#)

### Examples

```
# Install packages ggplot2 and gridExtra before running the examples

# Generate data
argvals <- seq(0, 2*pi, 0.01)
ind <- replicate(5, sort(sample(1:length(argvals), sample(5:10, 1))))
object <- irregFunData(argvals = lapply(ind, function(i){argvals[i]}),
```

```

X = lapply(ind, function(i){sample(1:10,1) / 10 * argvals[i]^2}))

# Plot the data
ggplot(object)

# Parameters passed to geom_line are passed via the ... argument
ggplot(object, color = "red", linetype = 3)

# Plot the data and add green dots for the 2nd function
ggplot(object) + ggplot(object, obs = 2, geom = "point", color = "green", add = TRUE)

# New layers can be added directly to the ggplot object using functions from the ggplot2 package
g <- ggplot(object)
g + ggplot2::theme_bw() + ggplot2::ggtitle("Plot with minimal theme and axis labels") +
  ggplot2::xlab("The x-Axis") + ggplot2::ylab("The y-Axis")

```

---

ggplot.multiFunData    *Visualize multivariate functional data objects using ggplot*

---

## Description

This function allows to plot multiFunData objects based on the **ggplot2** package. The function applies the [ggplot.funData](#) function to each element and returns either a combined plot with all elements plotted in one row or a list containing the different subplots as [ggplot](#) objects. The individual objects can be customized using all functionalities of the **ggplot2** package.

## Usage

```
ggplot.multiFunData(data, obs = 1:nObs(data), dim = 1:length(data),
  plotGrid = FALSE, ...)
```

```
## S4 method for signature 'multiFunData'
ggplot(data, ...)
```

## Arguments

data	A multiFunData object that is to be plotted.
obs	A vector of numerics giving the observations to plot. Defaults to all observations in data. For two-dimensional functions (images) obs must have length 1.
dim	The dimensions to plot. Defaults to length(data), i.e. all functions in data are plotted.
plotGrid	Logical. If TRUE, the data is plotted using <a href="#">grid.arrange</a> and the list of <a href="#">ggplot</a> objects is returned invisibly. If FALSE, only the list of objects is returned. Defaults to FALSE.
...	Further parameters passed to the univariate <a href="#">ggplot</a> functions for funData objects.



**Value**

A list of `ggplot` objects that are also printed directly as a grid if `plotGrid = TRUE`.

**Warning**

Currently, the function does not accept different parameters for the univariate elements.

**See Also**

[multiFunData](#), [ggplot](#), [plot.multiFunData](#)

**Examples**

```
# Install packages ggplot2 and gridExtra before running the examples

# One-dimensional elements
argvals <- seq(0, 2*pi, 0.01)
f1 <- funData(argvals, outer(seq(0.75, 1.25, length.out = 11), sin(argvals)))
f2 <- funData(argvals, outer(seq(0.75, 1.25, length.out = 11), cos(argvals)))

m1 <- multiFunData(f1, f2)

g <- ggplot(m1) # default
g[[1]] # plot first element
g[[2]] # plot second element
gridExtra::grid.arrange(grobs = g, nrow = 1) # requires gridExtra package

ggplot(m1, plotGrid = TRUE) # the same directly with plotGrid = TRUE

# Mixed-dimensional elements
X <- array(0, dim = c(11, length(argvals), length(argvals)))
X[1,,] <- outer(argvals, argvals, function(x,y){sin((x-pi)^2 + (y-pi)^2)})
f2 <- funData(list(argvals, argvals), X)

m2 <- multiFunData(f1, f2)

ggplot(m2, obs = 1, plotGrid = TRUE)

# Customizing plots (see ggplot2 documentation for more details)
g2 <- ggplot(m2, obs = 1)
g2[[1]] <- g2[[1]] + ggplot2::ggtitle("First element") + ggplot2::theme_bw()
g2[[2]] <- g2[[2]] + ggplot2::ggtitle("Second element") +
  ggplot2::scale_fill_gradient(high = "green", low = "blue")
gridExtra::grid.arrange(grobs = g2, nrow = 1) # requires gridExtra package
```

---

integrate	<i>Integrate functional data</i>
-----------	----------------------------------

---

### Description

Integrate all observations of a `funData`, `irregFunData` or `multiFunData` object over their domain.

### Usage

```
integrate(object, ...)
```

### Arguments

<code>object</code>	An object of class <code>funData</code> , <code>irregFunData</code> or <code>multiFunData</code> .
<code>...</code>	Further parameters (see <a href="#">Details</a> ).

### Details

Further parameters passed to this function may include:

- `method`: Character string. The integration rule to be used, passed to the internal function `.intWeights`. Defaults to "trapezoidal" (alternative: "midpoint").
- `fullDom`: Logical. If object is of class `irregFunData`, setting `fullDom = TRUE` extrapolates all functions linearly to the full domain before calculating the integrals. Defaults to `FALSE`. For details on the extrapolation, see [extrapolateIrreg](#).

### Value

A vector of numerics, containing the integral values for each observation.

### Warning

The function is currently implemented only for functional data with up to three-dimensional domains.

### See Also

[funData](#), [irregFunData](#), [multiFunData](#)

### Examples

```
# Univariate
object <- funData(argvals = 1:5, X = rbind(1:5, 6:10))
integrate(object)

# Univariate (irregular)
irregObject <- irregFunData(argvals = list(1:5, 2:4), X = list(2:6, 3:5))
integrate(irregObject) # fullDom = FALSE
```

```

integrate(irregObject, fullDom = TRUE)

# Multivariate
multiObject <- multiFunData(object, funData(argvals = 1:3, X = rbind(3:5, 6:8)))
integrate(multiObject)

```

---

irregFunData-class     *A class for irregularly sampled functional data*

---

### Description

The irregFunData class represents functional data that is sampled irregularly on one-dimensional domains. The two slots represent the observation points (x-values) and the observed function values (y-values).

### Usage

```

## S4 method for signature 'list,list'
irregFunData(argvals, X)

## S4 method for signature 'irregFunData'
show(object)

## S4 method for signature 'irregFunData'
names(x)

## S4 replacement method for signature 'irregFunData'
names(x) <- value

## S4 method for signature 'irregFunData'
str(object, ...)

## S4 method for signature 'irregFunData'
summary(object, ...)

```

### Arguments

argvals	A list of numerics, corresponding to the observation points for each realization $X_i$ (see Details).
X	A list of numerics, corresponding to the observed functions $X_i$ (see Details).
object	An irregFunData object.
x	The irregFunData object.
value	The names to be given to the irregFunData curves.
...	Other parameters passed to summary.

## Details

Irregular functional data are realizations of a random process

$$X : \mathcal{T} \rightarrow \mathbb{R},$$

where each realization  $X_i$  of  $X$  is given on an individual grid  $T_i \subset \mathcal{T}$  of observation points. As for the `funData` class, each object of the `irregFunData` class has two slots; the `argvals` slot represents the observation points and the `X` slot represents the observed data. In contrast to the regularly sampled data, both slots are defined as lists of vectors, where each entry corresponds to one observed function:

- `argvals[[i]]` contains the vector of observation points  $T_i$  for the  $i$ -th function,
- `X[[i]]` contains the corresponding observed data  $X_i(t_{ij}), t_{ij} \in T_i$ .

Generic functions for the `irregFunData` class include a print method, `plotting` and `basic arithmetics`. Further methods for `irregFunData`:

- `dimSupp, nObs`: Informations about the support dimensions and the number of observations,
- `getArgvals, extractObs`: Getting/setting slot values (instead of accessing them directly via `irregObject@argvals, irregObject@X`) and extracting single observations or data on a subset of the domain,
- `integrate, norm`: Integrate all observations over their domain or calculating the  $L^2$  norm.

An `irregFunData` object can be coerced to a `funData` object using `as.funData(irregObject)`. The regular functional data object is defined on the union of all observation grids of the irregular object. The value of the new object is marked as missing (NA) for observation points that are in the union, but not in the original observation grid.

## Methods (by generic)

- `irregFunData`: Constructor for irregular functional data objects.
- `show`: Print basic information about the `irregFunData` object in the console. The default console output for `irregFunData` objects.
- `names`: Get the names of the `irregFunData` object.
- `names<-`: Set the names of the `irregFunData` object.
- `str`: A `str` method for `irregFunData` objects, giving a compact overview of the structure.
- `summary`: A summary method for `irregFunData` objects.

## Slots

`argvals` A list of numerics, representing the observation grid  $T_i$  for each realization  $X_i$  of  $X$ .

`X` A list of numerics, representing the values of each observation  $X_i$  of  $X$  on the corresponding observation points  $T_i$ .

## Warning

Currently, the class is implemented only for functional data on one-dimensional domains  $\mathcal{T} \subset \mathbb{R}$ .

**See Also**

[funData](#), [multiFunData](#)

**Examples**

```
# Construct an irregular functional data object
i1 <- irregFunData(argvals = list(1:5, 2:4), X = list(2:6, 3:5))
# Display in the console
i1
# Summarize
summary(i1)

# A more realistic object
argvals <- seq(0,2*pi, 0.01)
ind <- replicate(11, sort(sample(1:length(argvals), sample(5:10,1)))) # sample observation points
argvalsIrreg <- lapply(ind, function(i){argvals[i]})
i2 <- irregFunData(argvals = argvalsIrreg, X = mapply(function(x, a){a * sin(x)},
  x = argvalsIrreg, a = seq(0.75, 1.25, by = 0.05)))
# Display/summary gives basic information
i2
summary(i2)
# Use the plot function to get an impression of the data
plot(i2)
```

---

 Math.funData

*Mathematical operations for functional data objects*


---

**Description**

These functions allow to apply mathematical operations (such as *exp()*, *log()*, *sin()*, *cos()* or *abs()*) to functional data objects based on [Math](#). The operations are made pointwise for each observation.

**Usage**

```
## S4 method for signature 'funData'
Math(x)

## S4 method for signature 'multiFunData'
Math(x)

## S4 method for signature 'irregFunData'
Math(x)
```

**Arguments**

x                    An object of class `funData`, `irregFunData` or `multiFunData`.

**Value**

An object of the same functional data class as `x`.

**See Also**

[funData](#), [irregFunData](#), [multiFunData](#), [Math](#)

**Examples**

```
oldpar <- par(no.readonly = TRUE)
par(mfrow = c(1,2))

# simulate a funData object on 0..1 with 10 observations
argvals <- seq(0, 1, 0.01)
f <- simFunData(argvals = argvals, N = 10,
                M = 5, eFunType = "Fourier", eValType = "linear")$simData

### FunData
plot(f, main = "Original data")
plot(abs(f), main = "Absolute values")

### Irregular
# create an irregFunData object by sparsifying f
i <- as.irregFunData(sparsify(f, minObs = 5, maxObs = 10))

plot(i, main = "Sparse data")
plot(cumsum(i), main = "'cumsum' of sparse data")

### Multivariate
m <- multiFunData(f, -1*f)
plot(m, main = "Multivariate Data")
plot(exp(m), main = "Exponential")

par(oldpar)
```

---

meanFunction

*Mean for functional data*

---

**Description**

This function calculates the pointwise mean function for objects of class `funData`, `irregFunData` or `multiFunData`.

**Usage**

```
meanFunction(object, na.rm = FALSE)
```

**Arguments**

object	An object of class funData, irregFunData or multiFunData.
na.rm	Logical. If TRUE, NA values are removed before computing the mean. Defaults to FALSE.

**Value**

An object of the same class as object with one observation that corresponds to the pointwise mean function of the functions in object.

**Warning**

If object is of class irregFunData, the option na.rm = TRUE is not implemented and throws an error. If na.rm = FALSE, the functions must be observed on the same domain.

**See Also**

[funData](#), [irregFunData](#), [multiFunData](#), [Arith.funData](#)

**Examples**

```
### Univariate (one-dimensional support)
x <- seq(0, 2*pi, 0.01)
f1 <- funData(x, outer(seq(0.75, 1.25, 0.05), sin(x)))

plot(f1)
plot(meanFunction(f1), col = 1, lwd = 2, add = TRUE)

### Univariate (two-dimensional support)
f2 <- funData(list(1:5, 1:3), array(rep(1:5,each = 11, times = 3), dim = c(11,5,3)))
all.equal(extractObs(f2,1), meanFunction(f2)) # f2 has 11 identical observations

### Multivariate
m1 <- multiFunData(f1,f2)
all.equal(extractObs(m1, obs = 6), meanFunction(m1)) # observation 6 equals the pointwise mean

### Irregular
i1 <- irregFunData(argvals = list(1:3,1:3,1:3), X = list(1:3,2:4,3:5))
all.equal(meanFunction(i1), extractObs(i1, obs = 2))
# don't run: functions are not defined on the same domain
## Not run: meanFunction(irregFunData(argvals = list(1:3,1:5), X = list(1:3,1:5)))
```

## Description

The `multiFunData` class represents multivariate functional data on (potentially) different domains, i.e. a multivariate functional data object is a vector of (univariate) functional data objects, just as a vector in  $\mathbb{R}^n$  is a vector of  $n$  scalars. In this implementation, a `multiFunData` object is represented as a list of univariate `funData` objects, see [Details](#).

## Usage

```
## S4 method for signature 'ANY'
multiFunData(...)

## S4 method for signature 'multiFunData'
names(x)

## S4 replacement method for signature 'multiFunData'
names(x) <- value

## S4 method for signature 'multiFunData'
str(object, ...)

## S4 method for signature 'multiFunData'
summary(object, ...)
```

## Arguments

<code>...</code>	A list of <code>funData</code> objects or several <code>funData</code> objects passed as one argument, each. See <a href="#">Details</a> .
<code>x</code>	The <code>multiFunData</code> object.
<code>value</code>	The names to be given to the <code>multiFunData</code> curves.
<code>object</code>	A <code>multiFunData</code> object.

## Details

A `multiFunData` object is represented as a list of univariate `funData` objects, each having a `argvals` and `X` slot, representing the `x`-values and the observed `y`-values (see the [funData](#) class). When constructing a `multiFunData` object, the elements can be supplied as a list of `funData` objects or can be passed directly as arguments to the constructor function.

Most functions implemented for the `funData` class are also implemented for `multiFunData` objects. In most cases, they simply apply the corresponding univariate method to each element of the multivariate object and return it as a vector (if the result of the univariate function is scalar, such as [dimSupp](#)) or as a `multiFunData` object (if the result of the univariate function is a `funData` object, such as [extractObs](#)).

The norm of a multivariate functional data  $f = (f_1, \dots, f_p)$  is defined as

$$\|f\| := \left( \sum_{j=1}^p \|f_j\|^2 \right)^{1/2}.$$



A funData object can be coerced to a multiFunData object with one element using `as.multiFunData(funDataObject)`.

### Methods (by generic)

- `multiFunData`: Constructor for multivariate functional data objects.
- `names`: Get the names of the multiFunData object.
- `names<-`: Set the names of the multiFunData object.
- `str`: A str method for multiFunData objects, giving a compact overview of the structure.
- `summary`: A summary method for multiFunData objects.

### See Also

[funData](#)

### Examples

```
### Creating a multifunData object with 2 observations on the same domain
# Univariate elements
x <- 1:5
f1 <- funData(x, rbind(x, x+1))
f2 <- funData(x, rbind(x^2, sin(x)))
# Basic
m1 <- new("multiFunData", list(f1,f2))
# Using the constructor, passing the elements as list
m2 <- multiFunData(list(f1,f2))
# Using the constructor, passing the elements directly
m3 <- multiFunData(f1,f2)
# Test if all the same
all.equal(m1,m2)
all.equal(m1,m3)
# Display multiFunData object in the console
m3
# Summarize
summary(m3)

### Creating a multifunData object with 2 observations on different domains (both 1D)
# A new element
y <- 1:3
g1 <- funData(y, rbind(3*y, y+4))
# Create the multiFunData object
m4 <- multiFunData(f1,g1)
# Display multiFunData object in the console
m4

### Creating a multifunData object with 2 observations on different domains (1D and 2D)
# A new element
y <- 1:3; z <- 1:4
g2 <- funData(list(y,z), array(rnorm(24), dim = c(2,3,4)))
# Create the multiFunData object
m5 <- multiFunData(f1,g2)
# Display multiFunData object in the console
```

```

m5

### A more realistic object
# element 1
x <- seq(0,2*pi, 0.01)
f1 <- funData(x, outer(seq(0.75, 1.25, length.out = 6), sin(x)))
# element 2
y <- seq(-1,1, 0.01); z <- seq(-0.5, 0.5, 0.01)
X2 <- array(NA, c(6, length(y), length(z)))
for(i in 1:6) X2[i,,] <- outer(y, z, function(x,y){sin(i*pi*y)*cos(i*pi*z)})
f2 <- funData(list(y,z), X2)
# MultiFunData Object
m6 <- multiFunData(f1,f2)
# Display multiFunData object in the console for basic information
m6
# Summarize
summary(m6)
# Use the plot function to get an impression of the data
## Not run: plot(m6) # m6 has 2D element, must specify one observation for plotting
plot(m6, obs = 1, main = c("1st element (obs 1)", "2nd element (obs 1)"))
plot(m6, obs = 6, main = c("1st element (obs 6)", "2nd element (obs 6)"))

```

---

nObs

*Get the number of observations*


---

## Description

This functions returns the number of observations in a funData, irregFunData or multiFunData object.

## Usage

```
nObs(object)
```

## Arguments

object            An object of class funData, irregFunData or multiFunData.

## Value

The number of observations in object.

## See Also

[funData](#), [irregFunData](#), [multiFunData](#)

## Examples

```
# Univariate
object <- funData(argvals = 1:5, X = rbind(1:5, 6:10))
nObs(object)

# Univariate (irregular)
irregObject <- irregFunData(argvals = list(1:5, 2:4), X = list(2:6, 3:5))
nObs(irregObject)

# Multivariate
multiObject <- multiFunData(object, funData(argvals = 1:3, X = rbind(3:5, 6:8)))
nObs(multiObject)
```

---

nObsPoints	<i>Get the number of observation points</i>
------------	---

---

## Description

This function returns the number of observation points in an object of class `funData`, `multiFunData` or `irregFunData`.

## Usage

```
nObsPoints(object)
```

## Arguments

`object` An object of class `funData`, `multiFunData` or `irregFunData`.

## Details

Depending on the class of `object`, the function returns different values:

- If `object` is of class `funData`, the function returns a vector of length `dimSupp(object)`, giving the number of observations in each dimension.
- If `object` is of class `multiFunData`, the function returns a list of the same length as `object`, where the `j`-th entry is a vector, corresponding to the observations point of `object[[j]]`.
- If `object` is of class `irregFunData`, the function returns an array of length `nObs(object)`, where the `j`-th entry corresponds to the number of observations in the `j`-th observed function.

## Value

The number of observation points in `object`. See Details.

## Warning

Do not confound with `nObs`, which returns the number of observations (i.e. the number of observed functions) in an object of a functional data class.

**See Also**

[irregFunData](#), [extractObs](#)

**Examples**

```
# Univariate (one-dimensional)
object1 <- funData(argvals = 1:5, X = rbind(1:5, 6:10))
nObsPoints(object1)

# Univariate (two-dimensional)
object2 <- funData(argvals = list(1:5, 1:6), X = array(1:60, dim = c(2, 5, 6)))
nObsPoints(object2)

# Multivariate
multiObject <- multiFunData(object1, object2)
nObsPoints(multiObject)

# Univariate (irregular)
irregObject <- irregFunData(argvals = list(1:5, 2:4), X = list(2:6, 3:5))
nObsPoints(irregObject)
```

---

norm

*Calculate the norm of functional data*


---

**Description**

This function calculates the norm for each observation of a `funData`, `irregFunData` or `multiFunData` object.

**Arguments**

`object`            An object of class `funData`, `irregFunData` or `multiFunData`.  
`...`                Further parameters (see `Details`).

**Details**

For `funData` objects, the standard  $L^2$  norm is calculated:

$$\|f\| = \left( \int_{\mathcal{T}} f(t)^2 dt \right)^{1/2}.$$

For `irregFunData` objects, each observed function is integrated only on the observed grid points (unless `fullDom = TRUE`).

The (weighted) norm of a multivariate functional data object  $f = (f_1, \dots, f_p)$  is defined as

$$\| \|f\| \| := \left( \sum_{j=1}^p w_j \|f_j\|^2 \right)^{1/2}.$$

Further parameters passed to this function may include:

- squared: Logical. If TRUE (default), the function calculates the squared norm, otherwise the result is not squared.
- obs: A numeric vector, giving the indices of the observations, for which the norm is to be calculated. Defaults to all observations.
- method: A character string, giving the integration method to be used. See [integrate](#) for details.
- weight: An optional vector of weights for the scalar product; particularly useful for multivariate functional data, where each entry can be weighted in the scalar product / norm. Defaults to 1 for each element.
- fullDom: Logical. If object is of class `irregFunData` and `fullDom = TRUE`, all functions are extrapolated to the same domain. Defaults to FALSE. See [integrate](#) for details.

### Value

A numeric vector representing the norm of each observation.

### Warning

The function is currently implemented only for functional data with one- and two-dimensional domains.

### See Also

[funData](#), [irregFunData](#), [multiFunData](#), [integrate](#)

### Examples

```
# Univariate
object <- funData(argvals = 1:5, X = rbind(1:5, 6:10))
norm(object)

# Univariate (irregular)
irregObject <- irregFunData(argvals = list(1:5, 2:4), X = list(2:6, 3:5))
norm(irregObject) # no extrapolation
norm(irregObject, fullDom = TRUE) # extrapolation (of second function)

# Multivariate
multiObject <- multiFunData(object, funData(argvals = 1:3, X = rbind(3:5, 6:8)))
norm(multiObject)
norm(multiObject, weight = c(2,1)) # with weight vector, giving more weight to the first element
```

---

plot.funData

*Plotting univariate functional data*

---

### Description

This function plots observations of univariate functional data on their domain.

**Usage**

```
plot.funData(x, y, obs = 1:nObs(x), type = "l", lty = 1, lwd = 1,
  col = NULL, xlab = "argvals", ylab = "", legend = TRUE,
  plotNA = FALSE, add = FALSE, ...)

## S4 method for signature 'funData,missing'
plot(x, y, ...)
```

**Arguments**

x	An object of class funData.
y	Missing.
obs	A vector of numerics giving the observations to plot. Defaults to all observations in x. For two-dimensional functions (images) obs must have length 1.
type	The type of plot. Defaults to "l" (line plot). See <a href="#">plot</a> for details.
lty	The line type. Defaults to 1 (solid line). See <a href="#">par</a> for details.
lwd	The line width. Defaults to 1. See <a href="#">par</a> for details.
col	The color of the functions. If not supplied (NULL, default value), one-dimensional functions are plotted in the <a href="#">rainbow</a> palette and two-dimensional functions are plotted using <a href="#">tim.colors</a> from package <a href="#">fields-package</a> .
xlab, ylab	The titles for x- and y-axis. Defaults to "argvals" for the x-axis and no title for the y-axis. See <a href="#">plot</a> for details.
legend	Logical. If TRUE, a color legend is plotted for two-dimensional functions (images). Defaults to TRUE.
plotNA	Logical. If TRUE, missing values are interpolated using the <a href="#">approxNA</a> function (only for one-dimensional functions). Defaults to FALSE.
add	Logical. If TRUE, add to current plot (only for one-dimensional functions). Defaults to FALSE.
...	Additional arguments to <a href="#">matplot</a> (one-dimensional functions) or <a href="#">image.plot/image</a> (two-dimensional functions).

**Details**

If some observations contain missing values (coded via NA), the functions can be interpolated using the option `plotNA = TRUE`. This option relies on the [na.approx](#) function in package [zoo](#) and is currently implemented for one-dimensional functions only in the function [approxNA](#).

**Warning**

The function is currently implemented only for functional data with one- and two-dimensional domains.

**See Also**

[funData](#), [matplot](#), [image.plot](#), [image](#)

**Examples**

```

oldpar <- par(no.readonly = TRUE)

# One-dimensional
argvals <- seq(0,2*pi,0.01)
object <- funData(argvals,
                  outer(seq(0.75, 1.25, length.out = 11), sin(argvals)))

plot(object, main = "One-dimensional functional data")

# Two-dimensional
X <- array(0, dim = c(2, length(argvals), length(argvals)))
X[1,,] <- outer(argvals, argvals, function(x,y){sin((x-pi)^2 + (y-pi)^2)})
X[2,,] <- outer(argvals, argvals, function(x,y){sin(2*x*pi) * cos(2*y*pi)})
object2D <- funData(list(argvals, argvals), X)

plot(object2D, main = "Two-dimensional functional data (obs 1)", obs = 1)
plot(object2D, main = "Two-dimensional functional data (obs 2)", obs = 2)
## Not run: plot(object2D, main = "Two-dimensional functional data") # must specify obs!

### More examples ###
par(mfrow = c(1,1))

# using plotNA
if(requireNamespace("zoo", quietly = TRUE))
{
  objectMissing <- funData(1:5, rbind(c(1, NA, 5, 4, 3), c(10, 9, NA, NA, 6)))
  par(mfrow = c(1,2))
  plot(objectMissing, type = "b", pch = 20, main = "plotNA = FALSE") # the default
  plot(objectMissing, type = "b", pch = 20, plotNA = TRUE, main = "plotNA = TRUE") # requires zoo
}

# Changing colors
plot(object, main = "1D functional data in grey", col = "grey")
plot(object, main = "1D functional data in heat.colors", col = heat.colors(nObs(object)))

plot(object2D, main = "2D functional data in topo.colors", obs = 1, col = topo.colors(64))
par(oldpar)

```

---

plot.irregFunData

*Plotting irregular functional data*


---

**Description**

This function plots observations of irregular functional data on their domain.

**Usage**

```
plot.irregFunData(x, y, obs = 1:nObs(x), type = "b", pch = 20,
  col = grDevices::rainbow(length(obs)), xlab = "argvals", ylab = "",
  xlim = range(x@argvals[obs]), ylim = range(x@X[obs]), log = "",
  add = FALSE, ...)

## S4 method for signature 'irregFunData,missing'
plot(x, y, ...)
```

**Arguments**

x	An object of class <code>irregFunData</code> .
y	Missing.
obs	A vector of numerics giving the observations to plot. Defaults to all observations in x.
type	The type of plot. Defaults to "b" (line and point plot). See <a href="#">plot</a> for details.
pch	The point type. Defaults to 20 (solid small circles). See <a href="#">par</a> for details.
col	The color of the functions. Defaults to the <a href="#">rainbow</a> palette.
xlab, ylab	The titles for x- and y-axis. Defaults to "argvals" for the x-axis and no title for the y-axis. See <a href="#">plot</a> for details.
xlim, ylim	The limits for x- and y-axis. Defaults to the total range of the data that is to plot. See <a href="#">plot</a> for details.
log	A character string, specifying the axis that is to be logarithmic. Can be "" (non-logarithmic axis, the default), "x", "y", "xy" or "yx". See <a href="#">plot.default</a> for details. This parameter is ignored, if add = TRUE.
add	Logical. If TRUE, add to current plot (only for one-dimensional functions). Defaults to FALSE.
...	Additional arguments to <a href="#">plot</a> .

**See Also**

[plot.funData](#), [irregFunData](#), [plot](#)

**Examples**

```
oldpar <- par(no.readonly = TRUE)

# Generate data
argvals <- seq(0, 2*pi, 0.01)
ind <- replicate(5, sort(sample(1:length(argvals), sample(5:10, 1))))
object <- irregFunData(argvals = lapply(ind, function(i){argvals[i]}),
  X = lapply(ind, function(i){sample(1:10, 1) / 10 * argvals[i]^2}))

plot(object, main = "Irregular functional data")

par(oldpar)
```



---

plot.multiFunData      *Plotting multivariate functional data*

---

### Description

This function plots observations of multivariate functional data on their domain. The graphic device is split in a number of subplots (specified by `dim`) via `mfrow` (`par`) and the univariate elements are plotted using `plot`.

### Usage

```
plot.multiFunData(x, y, obs = 1:nObs(x), dim = 1:length(x),
  par.plot = NULL, main = names(x), xlab = "argvals", ylab = "",
  log = "", ylim = NULL, ...)

## S4 method for signature 'multiFunData,missing'
plot(x, y, ...)
```

### Arguments

<code>x</code>	An object of class <code>multiFunData</code> .
<code>y</code>	Missing.
<code>obs</code>	A vector of numerics giving the observations to plot. Defaults to all observations in <code>x</code> . For two-dimensional functions (images) <code>obs</code> must have length 1.
<code>dim</code>	The dimensions to plot. Defaults to <code>length(x)</code> , i.e. all functions in <code>x</code> are plotted.
<code>par.plot</code>	Graphic parameters to be passed to the plotting regions. The option <code>mfrow</code> is ignored. Defaults to <code>NULL</code> . See <code>par</code> for details.
<code>main</code>	A string vector, giving the title of the plot. Can have the same length as <code>dim</code> (different titles for each dimension) or length 1 (one title for all dimensions). Defaults to <code>names(x)</code> .
<code>xlab, ylab</code>	The titles for x- and y-axis. Defaults to <code>"argvals"</code> for the x-axis and no title for the y-axis for all elements. Can be supplied as a vector of the same length as <code>dim</code> (one x-/y-lab for each element) or a single string that is applied for all elements. See <code>plot</code> for details.
<code>log</code>	A character string, specifying the axis that is to be logarithmic. Can be <code>""</code> (non-logarithmic axis), <code>"x"</code> , <code>"y"</code> , <code>"xy"</code> or <code>"yx"</code> . Defaults to <code>""</code> for all plots. Can be supplied as a vector of the same length as <code>dim</code> (one log-specification for each element) or a single string that is applied for all elements. See <code>plot.default</code> for details.
<code>ylim</code>	Specifies the limits of the y-Axis. Can be either <code>NULL</code> (the default, limits are chosen automatically), a vector of length 2 (giving the minimum and maximum range for all elements at the same time) or a list of the same length as <code>dim</code> (specifying the limits for each element separately).
<code>...</code>	Additional arguments to <code>plot</code> .

**Warning**

The function is currently implemented only for functional data with one- and two-dimensional domains.

**See Also**

[funData](#), [multiFunData](#), [plot.funData](#)

**Examples**

```
oldpar <- par(no.readonly = TRUE)
argvals <- seq(0, 2*pi, 0.1)

# One-dimensional elements
f1 <- funData(argvals, outer(seq(0.75, 1.25, length.out = 11), sin(argvals)))
f2 <- funData(argvals, outer(seq(0.75, 1.25, length.out = 11), cos(argvals)))

m1 <- multiFunData(f1, f2)
plot(m1, main = c("1st element", "2nd element")) # different titles
plot(m1, main = "Multivariate Functional Data") # one title for all

# Mixed-dimensional elements
X <- array(0, dim = c(11, length(argvals), length(argvals)))
X[1,,] <- outer(argvals, argvals, function(x,y){sin((x-pi)^2 + (y-pi)^2)})
g <- funData(list(argvals, argvals), X)

m2 <- multiFunData(f1, g)
# different titles and labels
plot(m2, main = c("1st element", "2nd element"), obs = 1,
      xlab = c("xlab1", "xlab2"),
      ylab = "one ylab for all")
# one title for all
plot(m2, main = "Multivariate Functional Data", obs = 1)

## Not run: plot(m2, main = c("1st element", "2nd element")) # must specify obs!

par(oldpar)
```

---

scalarProduct

*Calculate the scalar product for functional data objects*

---

**Description**

This function calculates the scalar product between two objects of the class [funData](#), [irregFunData](#) and [multiFunData](#). For univariate functions  $f, g$  on a domain  $\mathcal{T}$ , the scalar product is defined as

$$\int_{\mathcal{T}} f(t)g(t)dt$$

and for multivariate functions  $f, g$  on domains  $\mathcal{T}_1, \dots, \mathcal{T}_p$ , it is defined as

$$\sum_{j=1}^p \int_{\mathcal{T}_j} f^{(j)}(t)g^{(j)}(t)dt.$$

As seen in the formula, the objects must be defined on the same domain. The scalar product is calculated pairwise for all observations, thus the objects must also have the same number of observations or one object may have only one observation (for which the scalar product is calculated with all observations of the other object). Objects of the classes `funData` and `irregFunData` can be combined, see `integrate` for details.

### Usage

```
scalarProduct(object1, object2, ...)
```

### Arguments

`object1`, `object2`

Two objects of class `funData`, `irregFunData` or `multiFunData`, for that the scalar product is to be calculated.

...

Additional parameters passed to `integrate`. For `multiFunData` objects, one can also pass a weight argument. See Details.

### Details

For `multiFunData` one can pass optional vector weight for calculating a weighted scalar product. This vector must have the same number of elements as the `multiFunData` objects and have to be non-negative with at least one weight that is different from 0. Defaults to 1 for each element. See also `norm`.

### Value

A vector of length `nObs(object1)` (or `nObs(object2)`, if `object1` has only one observation), containing the pairwise scalar product for each observation.

### See Also

`integrate`, `norm`,

### Examples

```
# create two funData objectw with 5 observations on [0,1]
f <- simFunData(N = 5, M = 7, eValType = "linear",
               eFunType = "Fourier", argvals = seq(0,1,0.01))$simData
g <- simFunData(N = 5, M = 4, eValType = "linear",
               eFunType = "Poly", argvals = seq(0,1,0.01))$simData

# calculate the scalar product
scalarProduct(f,g)
```

```

# the scalar product of an object with itself equals the squared norm
all.equal(scalarProduct(f,f), norm(f, squared = TRUE))

# This works of course also for multiFunData objects...
m <- multiFunData(f,g)
all.equal(scalarProduct(m,m), norm(m, squared = TRUE))

# ...and for irregFunData objects
i <- as.irregFunData(sparsify(f, minObs = 5, maxObs = 10))
all.equal(scalarProduct(i,i), norm(i, squared = TRUE))

# Scalar product between funData and irregFunData objects
scalarProduct(i,f)

# Weighted scalar product for multiFunData objects
scalarProduct(m,m, weight = c(1,2))

```

---

simFunData

*Simulate univariate functional data*


---

### Description

This functions simulates (univariate) functional data  $f_1, \dots, f_N$  based on a truncated Karhunen-Loeve representation:

$$f_i(t) = \sum_{m=1}^M \xi_{i,m} \phi_m(t).$$

on one- or higher-dimensional domains. The eigenfunctions (basis functions)  $\phi_m(t)$  are generated using [eFun](#), the scores  $\xi_{i,m}$  are simulated independently from a normal distribution with zero mean and decreasing variance based on the [eVal](#) function. For higher-dimensional domains, the eigenfunctions are constructed as tensors of marginal orthonormal function systems.

### Usage

```
simFunData(argvals, M, eFunType, ignoreDeg = NULL, eValType, N)
```

### Arguments

argvals	A numeric vector, containing the observation points (a fine grid on a real interval) of the functional data that is to be simulated or a list of the marginal observation points.
M	An integer, giving the number of univariate basis functions to use. For higher-dimensional data, M is a vector with the marginal number of eigenfunctions. See <a href="#">Details</a> .
eFunType	A character string specifying the type of univariate orthonormal basis functions to use. For data on higher-dimensional domains, eFunType can be a vector, specifying the marginal type of eigenfunctions to use in the tensor product. See <a href="#">eFun</a> for details.

ignoreDeg	A vector of integers, specifying the degrees to ignore when generating the univariate orthonormal bases. Defaults to NULL. For higher-dimensional data, ignoreDeg can be supplied as list with vectors for each marginal. See <a href="#">eFun</a> for details.
eValType	A character string, specifying the type of eigenvalues/variances used for the generation of the simulated functions based on the truncated Karhunen-Loeve representation. See <a href="#">eVal</a> for details.
N	An integer, specifying the number of multivariate functions to be generated.

**Value**

simData	A <a href="#">funData</a> object with N observations, representing the simulated functional data.
trueFuns	A <a href="#">funData</a> object with M observations, representing the true eigenfunction basis used for simulating the data.
trueVals	A vector of numerics, representing the true eigenvalues used for simulating the data.

**See Also**

[funData](#), [eFun](#), [eVal](#), [addError](#), [sparsify](#)

**Examples**

```
oldPar <- par(no.readonly = TRUE)

# Use Legendre polynomials as eigenfunctions and a linear eigenvalue decrease
test <- simFunData(seq(0,1,0.01), M = 10, eFunType = "Poly", eValType = "linear", N = 10)

plot(test$trueFuns, main = "True Eigenfunctions")
plot(test$simData, main = "Simulated Data")

# The use of ignoreDeg for eFunType = "PolyHigh"
test <- simFunData(seq(0,1,0.01), M = 4, eFunType = "Poly", eValType = "linear", N = 10)
test_noConst <- simFunData(seq(0,1,0.01), M = 4, eFunType = "PolyHigh",
                           ignoreDeg = 1, eValType = "linear", N = 10)
test_noLinear <- simFunData(seq(0,1,0.01), M = 4, eFunType = "PolyHigh",
                           ignoreDeg = 2, eValType = "linear", N = 10)
test_noBoth <- simFunData(seq(0,1,0.01), M = 4, eFunType = "PolyHigh",
                          ignoreDeg = 1:2, eValType = "linear", N = 10)

par(mfrow = c(2,2))
plot(test$trueFuns, main = "Standard polynomial basis (M = 4)")
plot(test_noConst$trueFuns, main = "No constant basis function")
plot(test_noLinear$trueFuns, main = "No linear basis function")
plot(test_noBoth$trueFuns, main = "Neither linear nor constant basis function")

# Higher-dimensional domains
simImages <- simFunData(argvals = list(seq(0,1,0.01), seq(-pi/2, pi/2, 0.02)),
                       M = c(5,4), eFunType = c("Wiener","Fourier"), eValType = "linear", N = 4)
for(i in 1:4)
```

```
plot(simImages$simData, obs = i, main = paste("Observation", i))

par(oldPar)
```

---

simMultiFunData

*Simulate multivariate functional data*


---

### Description

This function provides a unified simulation structure for multivariate functional data  $f_1, \dots, f_N$  on one- or two-dimensional domains, based on a truncated multivariate Karhunen-Loeve representation:

$$f_i(t) = \sum_{m=1}^M \rho_{i,m} \psi_m(t).$$

The multivariate eigenfunctions (basis functions)  $\psi_m$  are constructed from univariate orthonormal bases. There are two different concepts for the construction, that can be chosen by the parameter `type`: A split orthonormal basis (`split`, only one-dimensional domains) and weighted univariate orthonormal bases (`weighted`, one- and two-dimensional domains). The scores  $\rho_{i,m}$  in the Karhunen-Loeve representation are simulated independently from a normal distribution with zero mean and decreasing variance. See Details.

### Usage

```
simMultiFunData(type, argvals, M, eFunType, ignoreDeg = NULL, eValType, N)
```

### Arguments

<code>type</code>	A character string, specifying the construction method for the multivariate eigenfunctions (either "split" or "weighted"). See Details.
<code>argvals</code>	A list, containing the observation points for each element of the multivariate functional data that is to be simulated. The length of <code>argvals</code> determines the number of elements in the resulting simulated multivariate functional data. See Details.
<code>M</code>	An integer ( <code>type = "split"</code> ) or a list of integers ( <code>type = "weighted"</code> ), giving the number of univariate basis functions to use. See Details.
<code>eFunType</code>	A character string ( <code>type = "split"</code> ) or a list of character strings ( <code>type = "weighted"</code> ), specifying the type of univariate orthonormal basis functions to use. See Details.
<code>ignoreDeg</code>	A vector of integers ( <code>type = "split"</code> ) or a list of integer vectors ( <code>type = "weighted"</code> ), specifying the degrees to ignore when generating the univariate orthonormal bases. Defaults to <code>NULL</code> . See Details.
<code>eValType</code>	A character string, specifying the type of eigenvalues/variances used for the simulation of the multivariate functions based on the truncated Karhunen-Loeve representation. See <code>eVal</code> for details.
<code>N</code>	An integer, specifying the number of multivariate functions to be generated.

## Details

The parameter type defines how the eigenfunction basis for the multivariate Karhunen-Loeve representation is constructed:

- type = "split": The basis functions of an underlying 'big' orthonormal basis are split in M parts, translated and possibly reflected. This yields an orthonormal basis of multivariate functions with M elements. This option is implemented only for one-dimensional domains.
- type = "weighted": The multivariate eigenfunction basis consists of weighted univariate orthonormal bases. This yields an orthonormal basis of multivariate functions with M elements. For data on two-dimensional domains (images), the univariate basis is constructed as a tensor product of univariate bases in each direction (x- and y-direction).

Depending on type, the other parameters have to be specified as follows:

**Split 'big' orthonormal basis:** The parameters M (integer), eFunType (character string) and ignoreDeg (integer vector or NULL) are passed to the function eFun to generate a univariate orthonormal basis on a 'big' interval. Subsequently, the basis functions are split and translated, such that the  $j$ -th part of the split function is defined on the interval corresponding to `argvals[[j]]`. The elements of the multivariate basis functions are given by these split parts of the original basis functions multiplied by a random sign  $\sigma_j \in \{-1, 1\}$ ,  $j = 1, \dots, p$ .

**Weighted orthonormal bases:** The parameters `argvals`, M, eFunType and ignoreDeg are all lists of a similar structure. They are passed element-wise to the function eFun to generate orthonormal basis functions for each element of the multivariate functional data to be simulated. In case of bivariate elements (images), the corresponding basis functions are constructed as tensor products of orthonormal basis functions in each direction (x- and y-direction).

If the  $j$ -th element of the simulated data should be defined on a one-dimensional domain, then

- `argvals[[j]]` is a list, containing one vector of observation points.
- `M[[j]]` is an integer, specifying the number of basis functions to use for this entry.
- `eFunType[[j]]` is a character string, specifying the type of orthonormal basis functions to use for this entry (see eFun for possible options).
- `ignoreDeg[[j]]` is a vector of integers, specifying the degrees to ignore when constructing the orthonormal basis functions. The default value is NULL.

If the  $j$ -th element of the simulated data should be defined on a two-dimensional domain, then

- `argvals[[j]]` is a list, containing two vectors of observation points, one for each direction (observation points in x-direction and in y-direction).
- `M[[j]]` is a vector of two integers, giving the number of basis functions for each direction (x- and y-direction).
- `eFunType[[j]]` is a vector of two character strings, giving the type of orthonormal basis functions for each direction (x- and y-direction, see eFun for possible options). The corresponding basis functions are constructed as tensor products of orthonormal basis functions in each direction.
- `ignoreDeg[[j]]` is a list, containing two integer vectors that specify the degrees to ignore when constructing the orthonormal basis functions in each direction. The default value is NULL.

The total number of basis functions (i.e. the product of `M[[j]]` for all  $j$ ) must be equal!

**Value**

simData	A <code>multiFunData</code> object with N observations, representing the simulated multivariate functional data.
trueFuns	A <code>multiFunData</code> object with M observations, representing the multivariate eigenfunction basis used for simulating the data.
trueVals	A vector of numerics, representing the eigenvalues used for simulating the data.

**References**

C. Happ, S. Greven (2015): Multivariate Functional Principal Component Analysis for Data Observed on Different (Dimensional) Domains. Preprint on arXiv: <http://arxiv.org/abs/1509.02029>

**See Also**

`multiFunData`, `eFun`, `eVal`, `simFunData`, `addError`, `sparsify`.

**Examples**

```
oldPar <- par(no.readonly = TRUE)

# split
split <- simMultiFunData(type = "split", argvals = list(seq(0,1,0.01), seq(-0.5,0.5,0.02)),
  M = 5, eFunType = "Poly", eValType = "linear", N = 7)

par(mfrow = c(1,2))
plot(split>trueFuns, main = "Split: True Eigenfunctions", ylim = c(-2,2))
plot(split$simData, main = "Split: Simulated Data")

# weighted (one-dimensional domains)
weighted1D <- simMultiFunData(type = "weighted",
  argvals = list(list(seq(0,1,0.01)), list(seq(-0.5,0.5,0.02))),
  M = c(5,5), eFunType = c("Poly", "Fourier"), eValType = "linear", N = 7)

plot(weighted1D>trueFuns, main = "Weighted (1D): True Eigenfunctions", ylim = c(-2,2))
plot(weighted1D$simData, main = "Weighted (1D): Simulated Data")

# weighted (one- and two-dimensional domains)
weighted <- simMultiFunData(type = "weighted",
  argvals = list(list(seq(0,1,0.01), seq(0,10,0.1)), list(seq(-0.5,0.5,0.01))),
  M = list(c(5,4), 20), eFunType = list(c("Poly", "Fourier"), "Wiener"),
  eValType = "linear", N = 7)

plot(weighted>trueFuns, main = "Weighted: True Eigenfunctions (m = 2)", obs = 2)
plot(weighted>trueFuns, main = "Weighted: True Eigenfunctions (m = 15)", obs = 15)
plot(weighted$simData, main = "Weighted: Simulated Data (1st observation)", obs = 1)
plot(weighted$simData, main = "Weighted: Simulated Data (2nd observation)", obs = 2)

par(oldPar)
```



---

sparsify

*Generate a sparse version of functional data objects*


---

### Description

This function generates an artificially sparsified version of a functional data object of class `funData` (univariate) or `multiFunData` (multivariate). The minimal and maximal number of observation points for all observations can be supplied by the user.

### Usage

```
sparsify(funDataObject, minObs, maxObs)
```

### Arguments

`funDataObject` A functional data object of class `funData` or `multiFunData`.

`minObs`, `maxObs` The minimal/maximal number of observation points. Must be a scalar for univariate functional data (`funData` class) or a vector of the same length as `funDataObject` for multivariate functional data (`multiFunData` class), giving the minimal/maximal number of observations for each element. See Details.

### Details

The technique for artificially sparsifying the data is as described in Yao et al. (2005): For each element  $x_i^{(j)}$  of an observed (multivariate) functional data object  $x_i$ , a random number  $R_i^{(j)} \in \{\text{minObs}, \dots, \text{maxObs}\}$  of observation points is generated. The points are sampled uniformly from the full grid  $\{t_{j,1}, \dots, t_{j,S_j}\} \subset \mathcal{T}_j$ , resulting in observations

$$x_{i,r}^{(j)} = x_i^{(j)}(t_{j,r}), \quad r = 1, \dots, R_i^{(j)}, \quad j = 1, \dots, p.$$

### Value

An object of the same class as `funDataObject`, which is a sparse version of the original data.

### Warning

This function is currently implemented for 1D data only.

### References

Yao, F., H.-G. Mueller and J.-L. Wang (2005): Functional Data Analysis for Sparse Longitudinal Data. *Journal of the American Statistical Association*, 100 (470), 577–590.

### See Also

`funData`, `multiFunData`, `simFunData`, `simMultiFunData`, `addError`.

**Examples**

```

oldPar <- par(no.readonly = TRUE)
par(mfrow = c(1,1))
set.seed(1)

# univariate functional data
full <- simFunData(argvals = seq(0,1, 0.01), M = 10, eFunType = "Fourier",
                  eValType = "linear", N = 3)$simData
sparse <- sparsify(full, minObs = 4, maxObs = 10)

plot(full, main = "Sparsify")
plot(sparse, type = "p", pch = 20, add = TRUE)
legend("topright", c("Full", "Sparse"), lty = c(1, NA), pch = c(NA, 20))

# Multivariate
full <- simMultiFunData(type = "split", argvals = list(seq(0,1, 0.01), seq(-.5,.5, 0.02)),
                      M = 10, eFunType = "Fourier", eValType = "linear", N = 3)$simData
sparse <- sparsify(full, minObs = c(4, 30), maxObs = c(10, 40))

par(mfrow = c(1,2))
plot(full[[1]], main = "Sparsify (multivariate)", sub = "minObs = 4, maxObs = 10")
plot(sparse[[1]], type = "p", pch = 20, add = TRUE)

plot(full[[2]], main = "Sparsify (multivariate)", sub = "minObs = 30, maxObs = 40")
plot(sparse[[2]], type = "p", pch = 20, add = TRUE)
legend("bottomright", c("Full", "Sparse"), lty = c(1, NA), pch = c(NA, 20))

par(oldPar)

```

---

tensorProduct

*Tensor product for univariate functions on one-dimensional domains*

---

**Description**

This function calculates tensor product functions for up to three objects of class `funData` defined on one-dimensional domains.

**Usage**

```
tensorProduct(...)
```

**Arguments**

... Two or three objects of class `funData`, that must be defined on a one-dimensional domain, each.

**Value**

An object of class `funData` that corresponds to the tensor product of the input functions.

**Warning**

The function is only implemented for up to three functions on one-dimensional domains.

**See Also**

[funData](#)

**Examples**

```
### Tensor product of two functional data objects
x <- seq(0, 2*pi, 0.1)
f1 <- funData(x, outer(seq(0.75, 1.25, 0.1), sin(x)))
y <- seq(-pi, pi, 0.1)
f2 <- funData(y, outer(seq(0.25, 0.75, 0.1), sin(y)))

plot(f1, main = "f1")
plot(f2, main = "f2")

tP <- tensorProduct(f1, f2)
dimSupp(tP)
plot(tP, obs = 1)

### Tensor product of three functional data objects
z <- seq(-1, 1, 0.05)
f3 <- funData(z, outer(seq(0.75, 1.25, 0.1), z^2))

plot(f1, main = "f1")
plot(f2, main = "f2")
plot(f3, main = "f3")

tP2 <- tensorProduct(f1, f2, f3)
dimSupp(tP2)
```

# Index

- addError, [2](#), [45](#), [48](#), [49](#)
- approxNA, [3](#), [21](#), [38](#)
- Arith, [4](#), [6](#)
- Arith, funData, funData-method  
(Arith.funData), [4](#)
- Arith, funData, irregFunData-method  
(Arith.funData), [4](#)
- Arith, funData, numeric-method  
(Arith.funData), [4](#)
- Arith, irregFunData, funData-method  
(Arith.funData), [4](#)
- Arith, irregFunData, irregFunData-method  
(Arith.funData), [4](#)
- Arith, irregFunData, numeric-method  
(Arith.funData), [4](#)
- Arith, multiFunData, multiFunData-method  
(Arith.funData), [4](#)
- Arith, multiFunData, numeric-method  
(Arith.funData), [4](#)
- Arith, numeric, funData-method  
(Arith.funData), [4](#)
- Arith, numeric, irregFunData-method  
(Arith.funData), [4](#)
- Arith, numeric, multiFunData-method  
(Arith.funData), [4](#)
- Arith.funData, [4](#), [14](#), [31](#)
- as.funData, [7](#)
- as.funData, irregFunData-method  
(as.funData), [7](#)
- as.irregFunData, [7](#)
- as.irregFunData, funData-method  
(as.irregFunData), [7](#)
- as.multiFunData, [8](#)
- as.multiFunData, funData-method  
(as.multiFunData), [8](#)
- basic arithmetics, [17](#), [28](#)
- dimSupp, [8](#), [17](#), [28](#), [32](#)
- eFun, [9](#), [44](#), [45](#), [47](#), [48](#)
- eVal, [11](#), [44–46](#), [48](#)
- extractObs, [12](#), [17](#), [28](#), [32](#), [36](#)
- extrapolateIrreg, [26](#)
- flipFuns, [13](#)
- funData, [2](#), [3](#), [6–10](#), [12](#), [14](#), [19](#), [21](#), [26](#), [28–34](#),  
[37](#), [38](#), [42](#), [43](#), [45](#), [49](#), [51](#)
- funData (funData-class), [15](#)
- funData, list, array-method  
(funData-class), [15](#)
- funData, numeric, array-method  
(funData-class), [15](#)
- funData-class, [15](#)
- geom\_line, [21](#)
- geom\_raster, [21](#)
- getArgvals, [17](#), [18](#), [28](#)
- getX (getArgvals), [18](#)
- ggplot, [20](#), [20](#), [21](#), [23–25](#)
- ggplot, funData-method (ggplot.funData),  
[21](#)
- ggplot, irregFunData-method  
(ggplot.irregFunData), [23](#)
- ggplot, multiFunData-method  
(ggplot.multiFunData), [24](#)
- ggplot.funData, [20](#), [21](#), [24](#)
- ggplot.irregFunData, [20](#), [23](#)
- ggplot.multiFunData, [20](#), [24](#)
- grid.arrange, [24](#)
- image, [38](#)
- image.plot, [38](#)
- integrate, [17](#), [26](#), [28](#), [37](#), [43](#)
- irregFunData, [6](#), [7](#), [9](#), [12](#), [14](#), [17](#), [19](#), [23](#), [26](#),  
[30](#), [31](#), [34](#), [36](#), [37](#), [40](#), [42](#), [43](#)
- irregFunData (irregFunData-class), [27](#)
- irregFunData, list, list-method  
(irregFunData-class), [27](#)
- irregFunData-class, [27](#)
- Math, [29](#), [30](#)

- Math, funData-method (Math.funData), 29
- Math, irregFunData-method (Math.funData), 29
- Math, multiFunData-method (Math.funData), 29
- Math.funData, 29
- matplot, 38
- meanFunction, 30
- multiFunData, 2, 3, 6, 8, 9, 12, 14, 17, 19, 25, 26, 29–31, 34, 37, 42, 43, 48, 49
- multiFunData (multiFunData-class), 31
- multiFunData, ANY-method (multiFunData-class), 31
- multiFunData-class, 31
  
- na.approx, 3, 21, 38
- names, funData-method (funData-class), 15
- names, irregFunData-method (irregFunData-class), 27
- names, multiFunData-method (multiFunData-class), 31
- names<-, funData-method (funData-class), 15
- names<-, irregFunData-method (irregFunData-class), 27
- names<-, multiFunData-method (multiFunData-class), 31
- nObs, 17, 28, 34, 35
- nObsPoints, 35
- norm, 13, 17, 28, 36, 43
  
- par, 38, 40, 41
- plot, 38, 40, 41
- plot, funData, missing-method (plot.funData), 37
- plot, irregFunData, missing-method (plot.irregFunData), 39
- plot, multiFunData, missing-method (plot.multiFunData), 41
- plot.default, 40, 41
- plot.funData, 21, 37, 40, 42
- plot.irregFunData, 23, 39
- plot.multiFunData, 25, 41
- plotting, 17, 28
  
- rainbow, 38, 40
  
- scalarProduct, 42
- setArgs (getArgs), 18
- show, funData-method (funData-class), 15
- show, irregFunData-method (irregFunData-class), 27
- simFunData, 3, 10, 44, 48, 49
- simMultiFunData, 3, 10, 46, 49
- sparsify, 45, 48, 49
- stat\_identity, 23
- str, funData-method (funData-class), 15
- str, irregFunData-method (irregFunData-class), 27
- str, multiFunData-method (multiFunData-class), 31
- summary, funData-method (funData-class), 15
- summary, irregFunData-method (irregFunData-class), 27
- summary, multiFunData-method (multiFunData-class), 31
  
- tensorProduct, 50
- tim.colors, 38
  
- zoo, 21, 38