Package ‘CholWishart’

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Type Package

Title Cholesky Decomposition of the Wishart Distribution

Version 0.9.2

Description Sampling from the Cholesky factorization of a Wishart random variable, sampling from the inverse Wishart distribution, sampling from the Cholesky factorization of an inverse Wishart random variable, computing densities for the Wishart and inverse Wishart distributions, and computing the multivariate gamma and digamma functions.

License GPL (>= 3)

Encoding UTF-8

LazyData true

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URL https://github.com/gzt/CholWishart

BugReports https://github.com/gzt/CholWishart/issues

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VignetteBuilder knitr

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R topics documented:

`CholWishart` *Cholesky Factor of a Wishart or Inverse Wishart*

Description

A package for fast computation of various functions related to the Wishart distribution, such as sampling from the Cholesky factor of the Wishart, sampling from the inverse Wishart, sampling from the Cholesky factor of the inverse Wishart, computing densities for the Wishart and inverse Wishart, and computing the multivariate gamma and digamma functions. Many of these functions are written in C to maximize speed.

`dWishart` *Density for Random Wishart Distributed Matrices*

Description

Compute the density of an observation of a random Wishart distributed matrix (`dWishart`) or an observation from the inverse Wishart distribution (`dInvWishart`).

Usage

```
dWishart(x, df, Sigma, log = TRUE)

dInvWishart(x, df, Sigma, log = TRUE)
```

Arguments

`x`	positive definite $p \times p$ observations for density estimation - either one matrix or a 3-D array.
`df`	numeric parameter, "degrees of freedom".
`Sigma`	positive definite $p \times p$ "scale" matrix, the matrix parameter of the distribution.
`log`	logical, whether to return value on the log scale.

Details

Note there are different ways of parameterizing the Inverse Wishart distribution, so check which one you need. Here, If $X \sim IW_p(\Sigma, \nu)$ then $X^{-1} \sim W_p(\Sigma^{-1}, \nu)$. Dawid (1981) has a different definition: if $X \sim W_p(\Sigma^{-1}, \nu)$ and $\nu > p - 1$, then $X^{-1} = Y \sim IW(\Sigma, \delta)$, where $\delta = \nu - p + 1$.

Value

Density or log of density

Functions

- `dInvWishart`: density for the inverse Wishart distribution.

References

Dawid, A. (1981). Some Matrix-Variate Distribution Theory: Notational Considerations and a Bayesian Application. *Biometrika*, 68(1), 265-274. doi: 10.2307/2335827

Gupta, A. K. and D. K. Nagar (1999). *Matrix variate distributions*. Chapman and Hall.

Mardia, K. V., J. T. Kent, and J. M. Bibby (1979) *Multivariate Analysis*, London: Academic Press.

Examples

```
set.seed(20180222)
A <- rWishart(1,10,diag(4))[,,1]
A
dWishart(x = A, df = 10,Sigma = diag(4), log=TRUE)
dInvWishart(x = solve(A), df = 10,Sigma = diag(4), log=TRUE)
```

`lmvgamma` *Multivariate Gamma Function*

Description

A special mathematical function related to the gamma function, generalized for multivariate gammas. `lmvgamma` if the log of the multivariate gamma, `mvgamma`.

The multivariate gamma function for a dimension p is defined as:

$$\Gamma_p(a) = \pi^{p(p-1)/4} \prod j = 1^p \Gamma[a + (1-j)/2]$$

For $p = 1$, this is the same as the usual gamma function.

Usage

```
lmvgamma(x, p)

mvgamma(x, p)
```

Arguments

`x`	non-negative numeric vector, matrix, or array
`p`	positive integer, dimension of a square matrix

Value

For `lmvgamma` log of multivariate gamma of dimension `p` for each entry of `x`. For non-log variant, use `mvgamma`.

Functions

- `mvgamma`: Multivariate gamma function.

References

A. K. Gupta and D. K. Nagar 1999. *Matrix variate distributions*. Chapman and Hall.

Multivariate gamma function. In *Wikipedia, The Free Encyclopedia*,from https://en.wikipedia.org/w/index.php?title=Multivariate_gamma_function&oldid=808084916

See Also

`gamma` and `lgamma`

Examples

```
lgamma(1:12)
lmvgamma(1:12,1)
mvgamma(1:12,1)
gamma(1:12)
```

`mvdigamma` *Multivariate Digamma Function*

Description

A special mathematical function related to the gamma function, generalized for multivariate distributions. The multivariate digamma function is the derivative of the log of the multivariate gamma function; for $p = 1$ it is the same as the univariate digamma function.

$$\psi_p(a) = \sum_{i=1}^{p} \psi(a + (1 - i)/2)$$

where ψ is the univariate digamma function (the derivative of the log-gamma function).

Usage

```
mvdigamma(x, p)
```

Arguments

`x`	non-negative numeric vector, matrix, or array
`p`	positive integer, dimension of a square matrix

Value

vector of values of multivariate digamma function.

References

A. K. Gupta and D. K. Nagar 1999. *Matrix variate distributions*. Chapman and Hall.

Multivariate gamma function. In *Wikipedia, The Free Encyclopedia*,from https://en.wikipedia.org/w/index.php?title=Multivariate_gamma_function&oldid=808084916

See Also

gamma, lgamma, digamma, and mvgamma

Examples

```
digamma(1:10)
mvdigamma(1:10,1)
```

`rCholWishart` *Cholesky Factor of Random Wishart Distributed Matrices*

Description

Generate n random matrices, distributed according to the Cholesky factorization of a Wishart distribution with parameters `Sigma` and `df`, $W_p(Sigma, df)$ (known as the Bartlett decomposition in the context of Wishart random matrices).

Usage

```
rCholWishart(n, df, Sigma)
```

Arguments

`n`	integer sample size.
`df`	numeric parameter, "degrees of freedom".
`Sigma`	positive definite $p \times p$ "scale" matrix, the matrix parameter of the distribution.

Value

a numeric array, say `R`, of dimension $p \times p \times n$, where each `R[,,i]` is a Cholesky decomposition of a sample from the Wishart distribution $W_p(Sigma, df)$. Based on a modification of the existing code for the `rWishart` function.

References

Anderson, T. W. (2003). *An Introduction to Multivariate Statistical Analysis* (3rd ed.). Hoboken, N. J.: Wiley Interscience.

Mardia, K. V., J. T. Kent, and J. M. Bibby (1979) *Multivariate Analysis*, London: Academic Press.

A. K. Gupta and D. K. Nagar 1999. *Matrix variate distributions*. Chapman and Hall.

See Also

rWishart, rInvCholWishart

Examples

```
# How it is parameterized:
set.seed(20180211)
A <- rCholWishart(1,10,3*diag(5))[,,1]
A
set.seed(20180211)
B <- rInvCholWishart(1,10,1/3*diag(5))[,,1]
B
crossprod(A) %*% crossprod(B)

set.seed(20180211)
C <- chol(stats::rWishart(1,10,3*diag(5))[,,1])
C
```

---

rInvCholWishart *Cholesky Factor of Random Inverse Wishart Distributed Matrices*

Description

Generate n random matrices, distributed according to the Cholesky factor of an inverse Wishart distribution with parameters Sigma and df, $W_p(Sigma, df)$.

Note there are different ways of parameterizing the Inverse Wishart distribution, so check which one you need. Here, If $X \sim IW_p(\Sigma, \nu)$ then $X^{-1} \sim W_p(\Sigma^{-1}, \nu)$. Dawid (1981) has a different definition: if $X \sim W_p(\Sigma^{-1}, \nu)$ and $\nu > p - 1$, then $X^{-1} = Y \sim IW(\Sigma, \delta)$, where $\delta = \nu - p + 1$.

Usage

```
rInvCholWishart(n, df, Sigma)
```

Arguments

n	integer sample size.
df	numeric parameter, "degrees of freedom".
Sigma	positive definite $p \times p$ "scale" matrix, the matrix parameter of the distribution.

Value

a numeric array, say R, of dimension $p \times p \times n$, where each `R[,,i]` is a Cholesky decomposition of a realization of the Wishart distribution $W_p(Sigma, df)$. Based on a modification of the existing code for the `rWishart` function

References

Anderson, T. W. (2003). *An Introduction to Multivariate Statistical Analysis* (3rd ed.). Hoboken, N. J.: Wiley Interscience.

Dawid, A. (1981). Some Matrix-Variate Distribution Theory: Notational Considerations and a Bayesian Application. *Biometrika*, 68(1), 265-274. doi: 10.2307/2335827

Gupta, A. K. and D. K. Nagar (1999). *Matrix variate distributions*. Chapman and Hall.

Mardia, K. V., J. T. Kent, and J. M. Bibby (1979) *Multivariate Analysis*, London: Academic Press.

See Also

`rWishart` and `rCholWishart`

Examples

```
# How it is parameterized:
set.seed(20180211)
A <- rCholWishart(1,10,3*diag(5))[,,1]
A
set.seed(20180211)
B <- rInvCholWishart(1,10,1/3*diag(5))[,,1]
B
crossprod(A) %*% crossprod(B)

set.seed(20180211)
C <- chol(stats::rWishart(1,10,3*diag(5))[,,1])
C
```

`rInvWishart` *Random Inverse Wishart Distributed Matrices*

Description

Generate n random matrices, distributed according to the inverse Wishart distribution with parameters `Sigma` and `df`, $W_p(Sigma, df)$.

Note there are different ways of parameterizing the Inverse Wishart distribution, so check which one you need. Here, If $X \sim IW_p(\Sigma, \nu)$ then $X^{-1} \sim W_p(\Sigma^{-1}, \nu)$. Dawid (1981) has a different definition: if $X \sim W_p(\Sigma^{-1}, \nu)$ and $\nu > p - 1$, then $X^{-1} = Y \sim IW(\Sigma, \delta)$, where $\delta = \nu - p + 1$.

Usage

```
rInvWishart(n, df, Sigma)
```

Arguments

n	integer sample size.
df	numeric parameter, "degrees of freedom".
Sigma	positive definite $p \times p$ "scale" matrix, the matrix parameter of the distribution.

Value

a numeric array, say R, of dimension $p \times p \times n$, where each R[,,i] is a realization of the inverse Wishart distribution $IW_p(Sigma, df)$. Based on a modification of the existing code for the rWishart function.

References

Dawid, A. (1981). Some Matrix-Variate Distribution Theory: Notational Considerations and a Bayesian Application. *Biometrika*, 68(1), 265-274. doi: 10.2307/2335827

Gupta, A. K. and D. K. Nagar (1999). *Matrix variate distributions*. Chapman and Hall.

Mardia, K. V., J. T. Kent, and J. M. Bibby (1979) *Multivariate Analysis*, London: Academic Press.

See Also

rWishart, rCholWishart, and rInvCholWishart

Examples

```
set.seed(20180221)
A<-rInvWishart(1,10,5*diag(5))[,,1]
set.seed(20180221)
B<-rWishart(1,10,.2*diag(5))[,,1]

A %*% B
```

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