

# Package ‘MPS’

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**Type** Package

**Title** Estimating Through the Maximum Product Spacing Approach

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**Description** Developed for computing the probability density function, computing the cumulative distribution function, computing the quantile function, random generation, and estimating the parameters of 24 G-family of statistical distributions via the maximum product spacing approach introduced in <<https://www.jstor.org/stable/2345411>>. The set of families contains: beta G distribution, beta exponential G distribution, beta extended G distribution, exponentiated G distribution, exponentiated exponential Poisson G distribution, exponentiated generalized G distribution, exponentiated Kumaraswamy G distribution, gamma type I G distribution, gamma type II G distribution, gamma uniform G distribution, gamma-X generated of log-logistic family of G distribution, gamma-X family of modified beta exponential G distribution, geometric exponential Poisson G distribution, generalized beta G distribution, generalized transmuted G distribution, Kumaraswamy G distribution, log gamma type I G distribution, log gamma type II G distribution, Marshall Olkin G distribution, Marshall Olkin Kumaraswamy G distribution, modified beta G distribution, odd log-logistic G distribution, truncated-exponential skew-symmetric G distribution, and Weibull G distribution.

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MPS-package	<i>Developed for computing pdf, cdf, quantile, random generation, and estimating the parameters of 24 G-family of statistical distributions.</i>
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## Description

Developed for computing the probability density function, computing the cumulative distribution function, computing the quantile function, random generation, and estimating the parameters of 24 G-family of statistical distributions via the maximum product spacing approach introduced in <<https://www.jstor.org/stable/2345411>>. These families are: beta G distribution due to Eugene et al. (2002), beta exponential G distribution due to Alzaatreh et al. (2013), beta extended G distribution due to Alzaatreh et al. (2013), exponentiated G distribution due to Gupta et al. (1998), exponentiated Kumaraswamy G distribution due to Lemonte et al. (2013), exponentiated exponential Poisson G distribution due to Ristic and Nadarajah (2014), exponentiated generalized G distribution due to Cordeiro et al. (2013), gamma type I G distribution due to Zografos and Balakrishnan (2009), gamma type II G distribution due to Ristic and Balakrishnan (2012), gamma uniform G distribution due to Torabi and Montazeri (2012), gamma-X generated of log-logistic family of G distribution due to Alzaatreh et al. (2013), gamma-X family of modified beta exponential G distribution due to Alzaatreh et al. (2013), geometric exponential Poisson G distribution due to Nadarajah et al. (2013), generalized beta G distribution due to Alexander et al. (2012), generalized transmuted G distribution due to Merovci et al. (2017), Kumaraswamy G distribution due to Cordeiro and Castro (2011), log gamma type I G distribution due to Amini et al. (2013), log gamma type II G distribution due to Amini et al. (2013), Marshall-Olkin G distribution due to Marshall and Olkin (1997), Marshall-Olkin Kumaraswamy G distribution due to Roshini and Thobias (2017), modified beta

G distribution due to Nadarajah et al. (2013), odd log-logistic G distribution due to Gauss et al. (2017), truncated-exponential skew-symmetric G distribution due to Nadarajah et al. (2014), and Weibull G distribution due to Alzaatreh et al. (2013).

### Details

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### Author(s)

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 betaexpG

 beta exponential G distribution
 

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### Description

Computes the pdf, cdf, quantile, and random numbers of the beta exponential G distribution. The General form for the probability density function (pdf) of the beta exponential G distribution due to Alzaatreh et al. (2013) is given by

$$f(x, \Theta) = \frac{d g(x - \mu, \theta) \left[ 1 - (1 - G(x - \mu, \theta))^d \right]^{b-1} (1 - G(x - \mu, \theta))^{ad-1}}{B(a, b)},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ ,  $d > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the beta exponential G distribution is given by

$$F(x, \Theta) = 1 - \frac{\int_0^{(1-G(x-\mu,\theta))^d} y^{a-1} (1-y)^{b-1} dy}{B(a, b)}.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, d, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$ ,  $b$ , and  $d$  are the first, second, and the third shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

**Usage**

```

dbetaexpg(mydata, g, param, location = TRUE, log=FALSE)
pbetaexpg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qbetaexpg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rbetaexpg(n, g, param, location = TRUE)
mpsbetaexpg(mydata, g, location = TRUE, method, sig.level)

```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, b, d, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

**References**

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Alzaatreh, A., Lee, C., and Famoye, F. (2013). A new method for generating families of continuous distributions, *Metron*, 71, 63-79.

**Examples**

```
x<-rweibull(100,shape=2,scale=2)+3
dbetaexp(x, "weibull", c(1,1,1,2,2,3))
pbetaexp(x, "weibull", c(1,1,1,2,2,3))
qbetaexp(runif(100), "weibull", c(1,1,1,2,2,3))
rbetaexp(100, "weibull", c(1,1,1,2,2,3))
mpsbetaexp(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

betag

*beta G distribution***Description**

Computes the pdf, cdf, quantile, and random numbers of the beta G distribution. General form for the probability density function (pdf) of beta G distribution due to Eugene et al. (2002) is given by

$$f(x, \Theta) = \frac{g(x - \mu, \theta)(G(x - \mu, \theta))^{a-1}(1 - G(x - \mu, \theta))^{b-1}}{B(a, b)},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the beta G distribution is given by

$$F(x, \Theta) = \frac{\int_0^{G(x-\mu, \theta)} y^{a-1}(1-y)^{b-1} dy}{B(a, b)}.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  and  $b$  are the first and second shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

**Usage**

```

dbetag(mydata, g, param, location = TRUE, log=FALSE)
pbetag(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qbetag(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rbetag(n, g, param, location = TRUE)
mpsbetag(mydata, g, location = TRUE, method, sig.level)

```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, b, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

**References**

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Eugene, N., Lee, C., and Famoye, F. (2002). Beta-normal distribution and its applications, *Communications in Statistics-Theory and Methods*, 31, 497-512.

**Examples**

```
x<-rweibull(100,shape=2,scale=2)+3
dbetag(x, "weibull", c(1,1,2,2,3))
pbetag(x, "weibull", c(1,1,2,2,3))
qbetag(runif(100), "weibull", c(1,1,2,2,3))
rbetag(100, "weibull", c(1,1,2,2,3))
mpsbetag(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

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 expexppg

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*exponentiated exponential Poisson G distribution*


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**Description**

Computes the pdf, cdf, quantile, and random numbers of the exponentiated exponential Poisson G distribution. The general form for the probability density function (pdf) of the the exponentiated exponential Poisson G distribution due to Ristic and Nadarajah (2014) is given by

$$f(x, \Theta) = \frac{a b g(x - \mu, \theta) (G(x - \mu, \theta))^{a-1} e^{-b(G(x-\mu, \theta))^a}}{1 - e^{-b}}$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the exponentiated exponential Poisson G distribution is given by

$$F(x, \Theta) = \frac{1 - e^{-b(G(x-\mu, \theta))^a}}{1 - e^{-b}}$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  and  $b$  are the first and second shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

**Usage**

```
dexpexppg(mydata, g, param, location = TRUE, log=FALSE)
pexpexppg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qexpexppg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rexpexppg(n, g, param, location = TRUE)
mpsexpexppg(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, b, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

**References**

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Ristic, M. M. and Nadarajah, S. (2014). A new lifetime distribution, *Journal of Statistical Computation and Simulation*, 84 (1), 135-150.

**Examples**

```
x<-rweibull(100,shape=2,scale=2)+3
dexpexppg(x, "weibull", c(1,1,2,2,3))
pexpexppg(x, "weibull", c(1,1,2,2,3))
qexpexppg(runif(100), "weibull", c(1,1,2,2,3))
rexpexppg(100, "weibull", c(1,1,2,2,3))
mpsexpexppg(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

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 expg

---

*exponentiated G distribution*


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**Description**

Computes the pdf, cdf, quantile, and random numbers of the exponentiated G distribution. General form for the probability density function (pdf) of the exponentiated G distribution due to Gupta et al. (1998) is given by

$$f(x, \Theta) = a g(x - \mu, \theta) (G(x - \mu, \theta))^{a-1},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$  and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the exponentiated G distribution is given by

$$F(x, \Theta) = (G(x - \mu, \theta))^a.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  is the shape parameter). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

**Usage**

```
dexpg(mydata, g, param, location = TRUE, log=FALSE)
pexpg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qexpg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rexpg(n, g, param, location = TRUE)
mpsexpexp(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

g	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
p	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
n	number of realizations to be generated.
mydata	Vector of observations.
param	parameter vector $\Theta = (a, \theta, \mu)$
location	If FALSE, then the location parameter will be omitted.
log	If TRUE, then log(pdf) is returned.
log.p	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
lower.tail	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
method	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
sig.level	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as sig.level.

**Value**

1. A vector of the same length as mydata, giving the pdf values computed at mydata.
2. A vector of the same length as mydata, giving the cdf values computed at mydata.
3. A vector of the same length as p, giving the quantile values computed at p.
4. A vector of the same length as n, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

- Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.
- Gupta, R. C., Gupta, P. L., and Gupta, R. D. (1998). Modeling failure time data by Lehman alternatives, *Communications in Statistics-Theory and Methods*, 27, 887-904.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dexpg(x, "weibull", c(1,2,2,3))
pexpg(x, "weibull", c(1,2,2,3))
qexpg(runif(100), "weibull", c(1,2,2,3))
rexp(100, "weibull", c(1,2,2,3))
mpsexpg(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

expgg

*exponentiated generalized G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the exponentiated generalized G distribution. The General form for the probability density function (pdf) of the exponentiated generalized G distribution due to Cordeiro et al. (2013) is given by

$$f(x, \Theta) = a b g(x - \mu, \theta) (1 - G(x - \mu, \theta))^{a-1} [1 - (1 - G(x - \mu, \theta))^a]^{b-1},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the exponentiated generalized G distribution is given by

$$F(x, \Theta) = [1 - (1 - G(x - \mu, \theta))^a]^b.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  and  $b$  are the first and second shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dexpgg(mydata, g, param, location = TRUE, log=FALSE)
pexpgg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qexpgg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rexp(100, g, param, location = TRUE)
mpsexpg(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, b, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

- Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.
- Cordeiro, G. M., Ortega, E. M. M., and da Cunha, D. C. C. (2013). The exponentiated generalized class of distributions, *Journal of Data Science*, 11, 1-27.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dexpkg(x, "weibull", c(1,1,2,2,3))
pexpkg(x, "weibull", c(1,1,2,2,3))
qexpkg(runif(100), "weibull", c(1,1,2,2,3))
rexpkg(100, "weibull", c(1,1,2,2,3))
mpsexpkg(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

expkumg

*exponentiated Kumaraswamy G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the exponentiated Kumaraswamy G distribution. The General form for the probability density function (pdf) of exponentiated Kumaraswamy G distribution due to Lemonte et al. (2013) is given by

$$f(x, \Theta) = a b d g(x - \mu, \theta) (G(x - \mu, \theta))^{a-1} [1 - (G(x - \mu, \theta))^a]^{b-1} \left\{ 1 - [1 - (G(x - \mu, \theta))^a]^b \right\}^{d-1},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ ,  $d > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the exponentiated Kumaraswamy G distribution is given by

$$F(x, \Theta) = \left\{ 1 - [1 - (G(x - \mu, \theta))^a]^b \right\}^d.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, d, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$ ,  $b$ , and  $d$  are the first, second, and the third shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dexpkumg(mydata, g, param, location = TRUE, log=FALSE)
pexpkumg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qexpkumg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rexpkumg(n, g, param, location = TRUE)
mpsexpkg(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

g	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
p	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
n	number of realizations to be generated.
mydata	Vector of observations.
param	parameter vector $\Theta = (a, b, d, \theta, \mu)$
location	If FALSE, then the location parameter will be omitted.
log	If TRUE, then log(pdf) is returned.
log.p	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
lower.tail	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
method	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
sig.level	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as sig.level.

**Value**

1. A vector of the same length as mydata, giving the pdf values computed at mydata.
2. A vector of the same length as mydata, giving the cdf values computed at mydata.
3. A vector of the same length as p, giving the quantile values computed at p.
4. A vector of the same length as n, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Lemonte, A. J., Barreto-Souza, W., and Cordeiro, G. M. (2013). The exponentiated Kumaraswamy distribution and its log-transform, *Brazilian Journal of Probability and Statistics*, 27, 31-53.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dexpkumg(x, "weibull", c(1,1,1,2,2,3))
pexpkumg(x, "weibull", c(1,1,1,2,2,3))
qexpkumg(runif(100), "weibull", c(1,1,1,2,2,3))
rexpkmg(100, "weibull", c(1,1,1,2,2,3))
mpsexpkumg(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

gammag

*gamma uniform G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the gamma uniform G distribution. General form for the probability density function (pdf) of the gamma uniform G distribution due to Torabi and Montazeri (2012) is given by

$$f(x, \Theta) = \frac{g(x - \mu, \theta)}{\Gamma(a)(1 - G(x - \mu, \theta))^2} \left( \frac{G(x - \mu, \theta)}{1 - G(x - \mu, \theta)} \right)^{a-1} e^{-\frac{G(x - \mu, \theta)}{1 - G(x - \mu, \theta)}},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$  and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the gamma uniform G distribution is given by

$$F(x, \Theta) = \int_0^{\frac{G(x - \mu, \theta)}{1 - G(x - \mu, \theta)}} \frac{y^{a-1} e^{-y}}{\Gamma(a)} dy.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  is the shape parameter). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dgammag(mydata, g, param, location = TRUE, log=FALSE)
pgammag(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qgammag(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rgammag(n, g, param, location = TRUE)
mpsgammag(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (`log`), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

- Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.
- Torabi, H. and Montazeri, N. H. (2012). The gamma uniform distribution and its applications, *Kybernetika*, 48, 16-30.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dgamma(x, "weibull", c(1,2,2,3))
pgamma(x, "weibull", c(1,2,2,3))
qgamma(runif(100), "weibull", c(1,2,2,3))
rgamma(100, "weibull", c(1,2,2,3))
mpsgamma(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

gammag1

*gamma uniform type I G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the gamma uniform type I G distribution. General form for the probability density function (pdf) of the gamma uniform type I G distribution due to Zografos and Balakrishnan (2009) is given by

$$f(x, \Theta) = \frac{g(x - \mu, \theta)}{\Gamma(a)} [-\log(1 - G(x - \mu, \theta))]^{a-1},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$  and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the gamma uniform type I G distribution is given by

$$F(x, \Theta) = \int_0^{-\log(1-G(x-\mu,\theta))} \frac{y^{a-1} e^{-y}}{\Gamma(a)} dy.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  is the shape parameter). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dgamma(mydata, g, param, location = TRUE, log=FALSE)
pgamma(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qgamma(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rgamma(n, g, param, location = TRUE)
mpsgamma(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

g	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
p	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
n	number of realizations to be generated.
mydata	Vector of observations.
param	parameter vector $\Theta = (a, \theta, \mu)$
location	If FALSE, then the location parameter will be omitted.
log	If TRUE, then log(pdf) is returned.
log.p	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
lower.tail	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
method	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
sig.level	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as sig.level.

**Value**

1. A vector of the same length as mydata, giving the pdf values computed at mydata.
2. A vector of the same length as mydata, giving the cdf values computed at mydata.
3. A vector of the same length as p, giving the quantile values computed at p.
4. A vector of the same length as n, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

- Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.
- Zografos, K. and Balakrishnan, N. (2009). On families of beta- and generalized gamma-generated distributions and associated inference, *Statistical Methodology*, 6, 344-362.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dgamma2(x, "weibull", c(1,2,2,3))
pgamma2(x, "weibull", c(1,2,2,3))
qgamma2(runif(100), "weibull", c(1,2,2,3))
rgamma2(100, "weibull", c(1,2,2,3))
mpsgamma2(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

gammag2

*gamma uniform type II G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the gamma uniform type II G distribution . General form for the probability density function (pdf) of the gamma uniform type II G distribution due to Ristic and Balakrishnan (2012) is given by

$$f(x, \Theta) = \frac{g(x - \mu, \theta)}{\Gamma(a)} [-\log(G(x - \mu, \theta))]^{a-1},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$  and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the gamma uniform type II G distribution is given by

$$F(x, \Theta) = \int_0^{-\log(G(x-\mu, \theta))} \frac{y^{a-1} e^{-y}}{\Gamma(a)} dy.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  is the shape parameter). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dgamma2(mydata, g, param, location = TRUE, log=FALSE)
pgamma2(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qgamma2(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rgamma2(n, g, param, location = TRUE)
mpsgamma2(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (`log`), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

- Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.
- Ristic, M. M. and Balakrishnan, N. (2012). The gamma exponentiated exponential distribution, *Journal of Statistical Computation and Simulation*, 82, 1191-1206.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dgammag2(x, "weibull", c(1,2,2,3))
pgammag2(x, "weibull", c(1,2,2,3))
qgammag2(runif(100), "weibull", c(1,2,2,3))
rgammag2(100, "weibull", c(1,2,2,3))
mpsgammag2(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

gbetag

*generalized beta G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the generalized beta G distribution. General form for the probability density function (pdf) of the generalized beta G distribution due to Alexander et al. (2012) is given by

$$f(x, \Theta) = \frac{d g(x - \mu, \theta) (G(x - \mu, \theta))^{ad-1} [1 - (G(x - \mu, \theta))^d]^{b-1}}{B(a, b)},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ ,  $d > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the generalized beta G distribution is given by

$$F(x, \Theta) = \frac{\int_0^{(G(x-\mu, \theta))^d} y^{a-1} (1-y)^{b-1} dy}{B(a, b)}.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, d, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$ ,  $b$ , and  $d$  are the first, second, and the third shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dgbetag(mydata, g, param, location = TRUE, log=FALSE)
pgbetag(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qgbetag(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rgbetag(n, g, param, location = TRUE)
mpsgbetag(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, b, d, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12*m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Mises statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Alexander, C., Cordeiro, G. M., and Ortega, E. M. M. (2012). Generalized beta-generated distributions, *Computational Statistics and Data Analysis*, 56, 1880-1897.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dgbetag(x, "weibull", c(1,1,1,2,2,3))
pgbetag(x, "weibull", c(1,1,1,2,2,3))
qgbetag(runif(100), "weibull", c(1,1,1,2,2,3))
rgbetag(100, "weibull", c(1,1,1,2,2,3))
mpsgbetag(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

gexppg

*geometric exponential Poisson G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the geometric exponential Poisson G distribution. General form for the probability density function (pdf) of the geometric exponential Poisson G distribution due to Nadarajah et al. (2013) is given by

$$f(x, \Theta) = \frac{a(1-b)g(x-\mu, \theta)(1-e^{-a})e^{-a+aG(x-\mu, \theta)}}{(1-e^{-a}-b+be^{-a+aG(x-\mu, \theta)})^2},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a>0$ ,  $0<b<1$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the geometric exponential Poisson G distribution is given by

$$F(x, \Theta) = \frac{e^{-a+aG(x-\mu, \theta)} - e^{-a}}{1 - e^{-a} - b + be^{-a+aG(x-\mu, \theta)}}.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  and  $b$  are the first and second shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dgexppg(mydata, g, param, location = TRUE, log=FALSE)
pgexppg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qgexppg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rgexppg(n, g, param, location = TRUE)
mpsgexppg(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

g	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
p	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
n	number of realizations to be generated.
mydata	Vector of observations.
param	parameter vector $\Theta = (a, b, \theta, \mu)$
location	If FALSE, then the location parameter will be omitted.
log	If TRUE, then log(pdf) is returned.
log.p	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
lower.tail	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
method	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
sig.level	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as sig.level.

**Value**

1. A vector of the same length as mydata, giving the pdf values computed at mydata.
2. A vector of the same length as mydata, giving the cdf values computed at mydata.
3. A vector of the same length as p, giving the quantile values computed at p.
4. A vector of the same length as n, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

- Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.
- Nadarajah, S., Cancho, V. G., and Ortega, E. M. M. (2013). The geometric exponential Poisson distribution, *Statistical Methods & Applications*, 22, 355-380.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dgexppg(x, "weibull", c(1,0.5,2,2,3))
pgexppg(x, "weibull", c(1,0.5,2,2,3))
qgexppg(runif(100), "weibull", c(1,0.5,2,2,3))
rgexppg(100, "weibull", c(1,0.5,2,2,3))
mpsgexppg(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

gmbetaexpG

*gamma-X family of modified beta exponential G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the gamma-X family of modified beta exponential G distribution. The General form for the probability density function (pdf) of the gamma-X family of the modified beta exponential G distribution due to Alzaatreh et al. (2013) is given by

$$f(x, \Theta) = abg(x - \mu, \theta)(1 - G(x - \mu, \theta))^{-2} e^{-b \frac{G(x - \mu, \theta)}{1 - G(x - \mu, \theta)}} \left[ 1 - e^{-b \frac{G(x - \mu, \theta)}{1 - G(x - \mu, \theta)}} \right]^{a-1},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the gamma-X family of modified beta exponential G distribution is given by

$$F(x, \Theta) = \left( 1 - e^{-b \frac{G(x - \mu, \theta)}{1 - G(x - \mu, \theta)}} \right)^a.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  and  $b$  are the first and second shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dgmbetaexpG(mydata, g, param, location = TRUE, log=FALSE)
pgmbetaexpG(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qgmbetaexpG(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rgmbetaexpG(n, g, param, location = TRUE)
mpsgmbetaexpG(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, b, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Alzaatreh, A., Lee, C., and Famoye, F. (2013). A new method for generating families of continuous distributions, *Metron*, 71, 63-79.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dgbetaexp(x, "weibull", c(1,1,2,2,3))
pgbetaexp(x, "weibull", c(1,1,2,2,3))
qgbetaexp(runif(100), "weibull", c(1,1,2,2,3))
rgbetaexp(100, "weibull", c(1,1,2,2,3))
mpsgbetaexp(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

gtransg

*exponentiated exponential Poisson G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the generalized transmuted G distribution. The general form for the probability density function (pdf) of the generalized transmuted G distribution due to Merovci et al. (2017) is given by

$$f(x, \Theta) = a g(x - \mu, \theta) (G(x - \mu, \theta))^{a-1} [1 + b - 2bG(x - \mu, \theta)] [1 + b(1 - G(x - \mu, \theta))]^{a-1},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b < 1$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the generalized transmuted G distribution is given by

$$F(x, \Theta) = (G(x - \mu, \theta))^a [1 + b(1 - G(x - \mu, \theta))]^a.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  and  $b$  are the first and second shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dgtransg(mydata, g, param, location = TRUE, log=FALSE)
pgtransg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qgtransg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rgtransg(n, g, param, location = TRUE)
mpsgtransg(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

g	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
p	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
n	number of realizations to be generated.
mydata	Vector of observations.
param	parameter vector $\Theta = (a, b, \theta, \mu)$
location	If FALSE, then the location parameter will be omitted.
log	If TRUE, then log(pdf) is returned.
log.p	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
lower.tail	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
method	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
sig.level	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as sig.level.

**Value**

1. A vector of the same length as mydata, giving the pdf values computed at mydata.
2. A vector of the same length as mydata, giving the cdf values computed at mydata.
3. A vector of the same length as p, giving the quantile values computed at p.
4. A vector of the same length as n, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

- Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.
- Merovcia, F., Alizadeh, M., Yousof, H. M., and Hamedani, G. G. (2017). The exponentiated transmuted-G family of distributions: Theory and applications, *Communications in Statistics-Theory and Methods*, 46(21), 10800-10822.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dgtransg(x, "weibull", c(1,0.5,2,2,3))
pgtransg(x, "weibull", c(1,0.5,2,2,3))
qgtransg(runif(100), "weibull", c(1,0.5,2,2,3))
rgtransg(100, "weibull", c(1,0.5,2,2,3))
mpsgtransg(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

gxlogisticg

*gamma-X generated of log-logistic-X family of G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the log-logistic-X family of G distribution. General form for the probability density function (pdf) of gamma-X generated of the log-logistic-X family of G distribution due to Alzaatreh et al. (2013) is given by

$$f(x, \Theta) = \frac{ag(x - \mu, \theta)[- \log(1 - G(x - \mu, \theta))]^{-a-1}}{(1 - G(x, \theta)) \{1 + [- \log(1 - G(x, \theta))]^a\}^2},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$  and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . It should be noted that here we set  $W(G(x, \theta)) = -\log(1 - G(x, \theta))$ . The general form for the cumulative distribution function (cdf) of the gamma-X generated of log-logistic family of G distribution is given by

$$F(x, \Theta) = \frac{1}{1 + [- \log(1 - G(x, \theta))]^{-a}}.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  is the shape parameter). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dgxlogisticg(mydata, g, param, location = TRUE, log=FALSE)
pgxlogisticg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qgxlogisticg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rgxlogisticg(n, g, param, location = TRUE)
mpsgxlogisticg(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

g	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
p	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
n	number of realizations to be generated.
mydata	Vector of observations.
param	parameter vector $\Theta = (a, \theta, \mu)$
location	If FALSE, then the location parameter will be omitted.
log	If TRUE, then log(pdf) is returned.
log.p	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
lower.tail	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
method	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
sig.level	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as sig.level.

**Value**

1. A vector of the same length as mydata, giving the pdf values computed at mydata.
2. A vector of the same length as mydata, giving the cdf values computed at mydata.
3. A vector of the same length as p, giving the quantile values computed at p.
4. A vector of the same length as n, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Alzaatreh, A., Lee, C., and Famoye, F. (2013). A new method for generating families of continuous distributions, *Metron*, 71, 63-79.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dgxlogisticg(x, "weibull", c(1,2,2,3))
pgxlogisticg(x, "weibull", c(1,2,2,3))
qgxlogisticg(runif(100), "weibull", c(1,2,2,3))
rgxlogisticg(100, "weibull", c(1,2,2,3))
mpsgxlogisticg(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

kumg

*Kumaraswamy G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the Kumaraswamy G distribution. General form for the probability density function (pdf) of the Kumaraswamy G distribution due to Cordeiro and Castro (2011) is given by

$$f(x, \Theta) = a b g(x - \mu, \theta) (G(x - \mu, \theta))^{a-1} [1 - (G(x - \mu, \theta))^a]^{b-1},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the Kumaraswamy G distribution is given by

$$F(x, \Theta) = 1 - [1 - (G(x - \mu, \theta))^a]^b.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  and  $b$  are the first and second shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dkumg(mydata, g, param, location = TRUE, log=FALSE)
pkumg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qkumg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rkumg(n, g, param, location = TRUE)
mpskumg(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, b, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

- Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.
- Cordeiro, G. M. and Castro, M. (2011). A new family of generalized distributions, *Journal of Statistical Computation and Simulation*, 81, 883-898.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dkumg(x, "weibull", c(1,1,2,2,3))
pkumg(x, "weibull", c(1,1,2,2,3))
qkumg(runif(100), "weibull", c(1,1,2,2,3))
rkumg(100, "weibull", c(1,1,2,2,3))
mpskumg(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

loggammag1

log gamma G type I distribution

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the log gamma type I G distribution. General form for the probability density function (pdf) of the log gamma type I G distribution due to Amini et al. (2013) is given by

$$f(x, \Theta) = \frac{b^a}{\Gamma(a)} g(x - \mu, \theta) [-\log(1 - G(x - \mu, \theta))]^{a-1} (1 - G(x - \mu, \theta))^{b-1},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the log gamma type I G distribution is given by

$$F(x, \Theta) = \int_0^{-b \log(1 - G(x - \mu, \theta))} \frac{y^{a-1} e^{-y}}{\Gamma(a)} dy.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  and  $b$  are the first and second shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dloggammag1(mydata, g, param, location = TRUE, log=FALSE)
ploggammag1(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qloggammag1(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rloggammag1(n, g, param, location = TRUE)
mpslloggammag1(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

g	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
p	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
n	number of realizations to be generated.
mydata	Vector of observations.
param	parameter vector $\Theta = (a, b, \theta, \mu)$
location	If FALSE, then the location parameter will be omitted.
log	If TRUE, then log(pdf) is returned.
log.p	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
lower.tail	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
method	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
sig.level	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as sig.level.

**Value**

1. A vector of the same length as mydata, giving the pdf values computed at mydata.
2. A vector of the same length as mydata, giving the cdf values computed at mydata.
3. A vector of the same length as p, giving the quantile values computed at p.
4. A vector of the same length as n, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Amini, M., MirMostafaei, S. M. T. K., and Ahmadi, J. (2013). Log-gamma-generated families of distributions, *Statistics*, 48 (4), 913-932.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dloggammag1(x, "weibull", c(1,1,2,2,3))
ploggammag1(x, "weibull", c(1,1,2,2,3))
qloggammag1(runif(100), "weibull", c(1,1,2,2,3))
rloggammag1(100, "weibull", c(1,1,2,2,3))
mpsloggammag1(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

loggammag2

*log gamma G type II distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the log gamma type II G distribution. General form for the probability density function (pdf) of the log gamma type II G distribution due to Amini et al. (2013) is given by

$$f(x, \Theta) = \frac{b^a}{\Gamma(a)} g(x - \mu, \theta) [-\log(G(x - \mu, \theta))]^{a-1} (G(x - \mu, \theta))^{b-1},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ ,  $d > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the log gamma type II G distribution is given by

$$F(x, \Theta) = 1 - \int_0^{-b \log(G(x - \mu, \theta))} \frac{y^{a-1} e^{-y}}{\Gamma(a)} dy.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  and  $b$  are the first and second shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dloggammag2(mydata, g, param, location = TRUE, log=FALSE)
ploggammag2(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qloggammag2(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rloggammag2(n, g, param, location = TRUE)
mpsloggammag2(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

g	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
p	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
n	number of realizations to be generated.
mydata	Vector of observations.
param	parameter vector $\Theta = (a, b, \theta, \mu)$
location	If FALSE, then the location parameter will be omitted.
log	If TRUE, then log(pdf) is returned.
log.p	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
lower.tail	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
method	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
sig.level	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as sig.level.

**Value**

1. A vector of the same length as mydata, giving the pdf values computed at mydata.
2. A vector of the same length as mydata, giving the cdf values computed at mydata.
3. A vector of the same length as p, giving the quantile values computed at p.
4. A vector of the same length as n, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

- Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.
- Amini, M., MirMostafaei, S. M. T. K., and Ahmadi, J. (2013). Log-gamma-generated families of distributions, *Statistics*, 48 (4), 913-932.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dloggammag2(x, "weibull", c(1,1,2,2,3))
ploggammag2(x, "weibull", c(1,1,2,2,3))
qloggammag2(runif(100), "weibull", c(1,1,2,2,3))
rloggammag2(100, "weibull", c(1,1,2,2,3))
mpsloggammag2(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

mbetag

*modified beta G distribution*


---

## Description

Computes the pdf, cdf, quantile, and random numbers of the modified beta G distribution. General form for the probability density function (pdf) of the modified beta G distribution due to Nadarajah et al. (2013) is given by

$$f(x, \Theta) = \frac{d^a g(x - \mu, \theta) (G(x - \mu, \theta))^{a-1} (1 - G(x - \mu, \theta))^{b-1}}{B(a, b) [1 - (1 - d) G(x - \mu, \theta)]^{a+b}},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ ,  $d > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cdf of the modified beta G distribution is given by

$$F(x, \Theta) = \frac{\int_0^{1 - (1-d)G(x-\mu, \theta)} y^{a-1} (1-y)^{b-1} dy}{B(a, b)}.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, d, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$ ,  $b$ , and  $d$  are the first, second, and the third shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dmbetag(mydata, g, param, location = TRUE, log=FALSE)
pmbetag(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qmbetag(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rmbetag(n, g, param, location = TRUE)
mpsmbetag(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, b, d, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

- Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.
- Nadarajah, S., Teimouri, M., and Shih, S. H. (2014). Modified beta distributions, *Sankhya*, 76 (1), 19-48.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dmbetag(x, "weibull", c(1,1,1,2,2,3))
pmbetag(x, "weibull", c(1,1,1,2,2,3))
qmbetag(runif(100), "weibull", c(1,1,1,2,2,3))
rmbetag(100, "weibull", c(1,1,1,2,2,3))
mpsmbetag(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

mog

*Marshall-Olkin G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the Marshall-Olkin G distribution. General form for the probability density function (pdf) of the Marshall-Olkin G distribution due to Marshall and Olkin (1997) is given by

$$f(x, \Theta) = \frac{ag(x - \mu, \theta)}{[1 - (1 - a)(1 - G(x - \mu, \theta))]^2},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$  and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the Marshall-Olkin G distribution is given by

$$F(x, \Theta) = 1 - \frac{a(1 - G(x - \mu, \theta))}{[1 - (1 - a)(1 - G(x - \mu, \theta))]}.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter. Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dmog(mydata, g, param, location = TRUE, log=FALSE)
pmog(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qmog(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rmog(n, g, param, location = TRUE)
mpsmog(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Marshall, A. W. and Olkin, I. (1997). A new method for adding a parameter to a family of distributions with application to the exponential and Weibull families, *Biometrika*, 84, 641-652.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dmog(x, "weibull", c(0.5,2,2,3))
pmog(x, "weibull", c(0.5,2,2,3))
qmog(runif(100), "weibull", c(0.5,2,2,3))
rmog(100, "weibull", c(0.5,2,2,3))
mpsmog(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

mokumg

*Marshall-Olkin Kumaraswamy G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the Marshall-Olkin Kumaraswamy G distribution. General form for the probability density function (pdf) of the Marshall-Olkin Kumaraswamy G distribution due to Roshini and Thobias (2017) is given by

$$f(x, \Theta) = \frac{abd g(x - \mu, \theta) (G(x - \mu, \theta))^{a-1} [1 - (G(x - \mu, \theta))^a]^{b-1}}{[1 - (1 - d) [1 - (G(x - \mu, \theta))^a]^b]^2},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ ,  $d > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the Marshall-Olkin Kumaraswamy G distribution is given by

$$F(x, \Theta) = 1 - \frac{d [1 - (G(x - \mu, \theta))^a]^b}{1 - (1 - d) [1 - (G(x - \mu, \theta))^a]^b}.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, d, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$ ,  $b$ , and  $d$  are the first, second, and the third shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

## Usage

```
dmokumg(mydata, g, param, location = TRUE, log=FALSE)
pmokumg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qmokumg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rmokumg(n, g, param, location = TRUE)
mpsmokumg(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, b, d, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (`log`), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

## References

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Roshini, G. and Thobias, S. (2017). Marshall-Olkin Kumaraswamy Distribution, *International Mathematical Forum*, 12 (2), 47-69.

## Examples

```
x<-rweibull(100,shape=2,scale=2)+3
dmokumg(x, "weibull", c(1,1,1,2,2,3))
pmokumg(x, "weibull", c(1,1,1,2,2,3))
qmokumg(runif(100), "weibull", c(1,1,1,2,2,3))
rmokumg(100, "weibull", c(1,1,1,2,2,3))
mpsmokumg(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

ologlogg

*odd log-logistic G distribution*

---

## Description

Computes the pdf, cdf, quantile, and random numbers of the odd log-logistic G distribution. General form for the probability density function (pdf) of the odd log-logistic G distribution due to Gauss et al. (2017) is given by

$$f(x, \Theta) = \frac{a b d g(x - \mu, \theta) (G(x - \mu, \theta))^{a d - 1} [\bar{G}(x - \mu, \theta)]^{d - 1}}{\left[ (G(x - \mu, \theta))^d - (\bar{G}(x - \mu, \theta))^d \right]^{a + 1}} \left\{ 1 - \left[ \frac{(G(x - \mu, \theta))^d}{(G(x - \mu, \theta))^d - (\bar{G}(x - \mu, \theta))^d} \right]^a \right\}^{b - 1},$$

with  $\bar{G}(x - \mu, \theta) = 1 - G(x - \mu, \theta)$  where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ ,  $d > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the odd log-logistic G distribution is given by

$$F(x, \Theta) = 1 - \left\{ 1 - \left[ \frac{(G(x - \mu, \theta))^d}{(G(x - \mu, \theta))^d - (\bar{G}(x - \mu, \theta))^d} \right]^a \right\}^b.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, d, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$ ,  $b$ , and  $d$  are the first, second, and the third shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

**Usage**

```
dologlogg(mydata, g, param, location = TRUE, log=FALSE)
pologlogg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qologlogg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rologlogg(n, g, param, location = TRUE)
mpsologlogg(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, b, d, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

**References**

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Gauss, M. C., Alizadeh, M., Ozel, G., Hosseini, B. Ortega, E. M. M., and Altunc, E. (2017). The generalized odd log-logistic family of distributions: properties, regression models and applications, *Journal of Statistical Computation and Simulation*, 87(5), 908-932.

**Examples**

```
x<-rweibull(100,shape=2,scale=2)+3
dologlogg(x, "weibull", c(1,1,1,2,2,3))
pologlogg(x, "weibull", c(1,1,1,2,2,3))
qologlogg(runif(100), "weibull", c(1,1,1,2,2,3))
rologlogg(100, "weibull", c(1,1,1,2,2,3))
mpsologlogg(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

---

 texpsg

---

*truncated-exponential skew-symmetric G distribution*


---

**Description**

Computes the pdf, cdf, quantile, and random numbers of the truncated-exponential skew-symmetric G distribution. General form for the probability density function (pdf) of the truncated-exponential skew-symmetric G distribution due to Nadarajah et al. (2014) is given by

$$f(x, \Theta) = \frac{a}{1 - e^{-a}} g(x - \mu, \theta) e^{-aG(x - \mu, \theta)},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$  and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the truncated-exponential skew-symmetric G distribution is given by

$$F(x, \Theta) = \frac{1 - e^{-aG(x - \mu, \theta)}}{1 - e^{-a}}.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter. Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

**Usage**

```

dtexpsg(mydata, g, param, location = TRUE, log=FALSE)
ptexpsg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qtexpsg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rtexpsg(n, g, param, location = TRUE)
mpstexpsg(mydata, g, location = TRUE, method, sig.level)

```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

**References**

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Nadarajah, S., Nassiri, V., and Mohammadpour, A. (2014). Truncated-exponential skew-symmetric distributions, *Statistics*, 48 (4), 872-895.

**Examples**

```
x<-rweibull(100,shape=2,scale=2)+3
dtexpsg(x, "weibull", c(1,2,2,3))
ptexpsg(x, "weibull", c(1,2,2,3))
qtexpsg(runif(100), "weibull", c(1,2,2,3))
rtexpsg(100, "weibull", c(1,2,2,3))
mpstexpsg(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

weibullextg

*T-X{log-logistic} G distribution***Description**

Computes the pdf, cdf, quantile, and random numbers of the Weibull extended or T-X{log-logistic} G distribution. General form for the probability density function (pdf) of the Weibull extended G distribution due to Alzaatreh et al. (2013) is given by

$$f(x, \Theta) = \frac{a g(x - \mu, \theta)}{b(1 - G(x - \mu, \theta))^2} \left( \frac{G(x - \mu, \theta)}{1 - G(x - \mu, \theta)} \right)^{\frac{1}{b}-1} \exp \left\{ -a \left( \frac{G(x - \mu, \theta)}{1 - G(x - \mu, \theta)} \right)^{\frac{1}{b}} \right\},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the Weibull extended G distribution is given by

$$F(x, \Theta) = 1 - \exp \left\{ -a \left( \frac{G(x - \mu, \theta)}{1 - G(x - \mu, \theta)} \right)^{\frac{1}{b}} \right\}.$$

Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  and  $b$  are the first and second shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

**Usage**

```
dweibullextg(mydata, g, param, location = TRUE, log=FALSE)
pweibullextg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qweibullextg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rweibullextg(n, g, param, location = TRUE)
mpsweibullextg(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, b, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

Mahdi Teimouri

**References**

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Alzaatreh, A., Lee, C., and Famoye, F. (2013). A new method for generating families of continuous distributions, *Metron*, 71, 63-79.

**Examples**

```
x<-rweibull(100, shape=2, scale=2)+3
dweibullextg(x, "weibull", c(1,1,2,2,3))
pweibullextg(x, "weibull", c(1,1,2,2,3))
qweibullextg(runif(100), "weibull", c(1,1,2,2,3))
rweibullextg(100, "weibull", c(1,1,2,2,3))
mpsweibullextg(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```

weibullg

*Weibull G distribution***Description**

Computes the pdf, cdf, quantile, and random numbers of the Weibull G distribution. General form for the probability density function (pdf) of the Weibull G distribution due to Alzaatreh et al. (2013) is given by

$$f(x, \Theta) = \frac{a}{b^a} \frac{g(x - \mu, \theta)}{1 - G(x - \mu, \theta)} [-\log(1 - G(x - \mu, \theta))]^{a-1} e^{-\left(\frac{-\log(1 - G(x - \mu, \theta))}{b}\right)^a},$$

where  $\theta$  is the baseline family parameter vector. Also,  $a > 0$ ,  $b > 0$ , and  $\mu$  are the extra parameters induced to the baseline cumulative distribution function (cdf)  $G$  whose pdf is  $g$ . The general form for the cumulative distribution function (cdf) of the Weibull G distribution is given by

$$F(x, \Theta) = 1 - e^{-\left(\frac{-\log(1 - G(x - \mu, \theta))}{b}\right)^a}.$$

The `weibullg` is the special case (Weibull-X) of the Alzaatreh et al. (2013) families of distributions. Here, the baseline  $G$  refers to the cdf of famous families such as: Birnbaum-Saunders, Burr type XII, Exponential, Chen, Chisquare, F, Frechet, Gamma, Gompertz, Linear failure rate (lfr), Log-normal, Log-logistic, Lomax, Rayleigh, and Weibull. The parameter vector is  $\Theta = (a, b, \theta, \mu)$  where  $\theta$  is the baseline  $G$  family's parameter space. If  $\theta$  consists of the shape and scale parameters, the last component of  $\theta$  is the scale parameter (here,  $a$  and  $b$  are the first and second shape parameters). Always, the location parameter  $\mu$  is placed in the last component of  $\Theta$ .

**Usage**

```
dweibullg(mydata, g, param, location = TRUE, log=FALSE)
pweibullg(mydata, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
qweibullg(p, g, param, location = TRUE, log.p = FALSE, lower.tail = TRUE)
rweibullg(n, g, param, location = TRUE)
mpsweibullg(mydata, g, location = TRUE, method, sig.level)
```

**Arguments**

<code>g</code>	The name of family's pdf including: "birnbaum-saunders", "burrxii", "chisq", "chen", "exp", "f", "frechet", "gamma", "gompertz", "lfr", "log-normal", "log-logistic", "lomax", "rayleigh", and "weibull".
<code>p</code>	a vector of value(s) between 0 and 1 at which the quantile needs to be computed.
<code>n</code>	number of realizations to be generated.
<code>mydata</code>	Vector of observations.
<code>param</code>	parameter vector $\Theta = (a, b, \theta, \mu)$
<code>location</code>	If FALSE, then the location parameter will be omitted.
<code>log</code>	If TRUE, then log(pdf) is returned.
<code>log.p</code>	If TRUE, then log(cdf) is returned and quantile is computed for $\exp(-p)$ .
<code>lower.tail</code>	If FALSE, then 1-cdf is returned and quantile is computed for 1-p.
<code>method</code>	The used method for maximizing the sum of log-spacing function. It will be "BFGS", "CG", "L-BFGS-B", "Nelder-Mead", or "SANN".
<code>sig.level</code>	Significance level for the Chi-square goodness-of-fit test.

**Details**

It can be shown that the Moran's statistic follows a normal distribution. Also, a chi-square approximation exists for small samples whose mean and variance approximately are  $m(\log(m)+0.57722)-0.5-1/(12m)$  and  $m(\pi^2/6-1)-0.5-1/(6m)$ , respectively, with  $m=n+1$ , see Cheng and Stephens (1989). So, a hypothesis testing can be constructed based on a sample of  $n$  independent realizations at the given significance level, indicated in above as `sig.level`.

**Value**

1. A vector of the same length as `mydata`, giving the pdf values computed at `mydata`.
2. A vector of the same length as `mydata`, giving the cdf values computed at `mydata`.
3. A vector of the same length as `p`, giving the quantile values computed at `p`.
4. A vector of the same length as `n`, giving the random numbers realizations.
5. A sequence of goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer-von Misses statistic (CM), Anderson Darling statistic (AD), log-likelihood statistic (log), and Moran's statistic (M). The Kolmogorov-Smirnov (KS) test statistic and corresponding p-value. The Chi-square test statistic, critical upper tail Chi-square distribution, related p-value, and the convergence status.

**Author(s)**

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**References**

Cheng, R. C. H. and Stephens, M. A. (1989). A goodness-of-fit test using Moran's statistic with estimated parameters, *Biometrika*, 76 (2), 385-392.

Alzaatreh, A., Lee, C., and Famoye, F. (2013). A new method for generating families of continuous distributions, *Metron*, 71, 63-79.

**Examples**

```
x<-rweibull(100,shape=2,scale=2)+3
dweibullg(x, "weibull", c(1,1,2,2,3))
pweibullg(x, "weibull", c(1,1,2,2,3))
qweibullg(runif(100), "weibull", c(1,1,2,2,3))
rweibullg(100, "weibull", c(1,1,2,2,3))
mpsweibullg(x, "weibull", TRUE, "Nelder-Mead", 0.05)
```



- pexp (exp), 10
- pexp (exp), 12
- pexpkumg (expkumg), 14
- pgammag (gammag), 16
- pgammag1 (gammag1), 18
- pgammag2 (gammag2), 20
- pgbetag (gbetag), 22
- pgexppg (gexppg), 24
- pgmbetaexp (gmbetaexp), 26
- pgtransg (gtransg), 28
- pgxlogisticg (gxlogisticg), 30
- pkumg (kumg), 32
- ploggammag1 (loggammag1), 34
- ploggammag2 (loggammag2), 36
- pmbetag (mbetag), 38
- pmog (mog), 40
- pmokumg (mokumg), 42
- pologlogg (ologlogg), 44
- ptexpsg (texpsg), 46
- pweibullextg (weibullextg), 48
- pweibullg (weibullg), 50
  
- qbetaexp (betaexp), 4
- qbetag (betag), 6
- qexpexppg (expexppg), 8
- qexp (exp), 10
- qexp (exp), 12
- qexpkumg (expkumg), 14
- qgammag (gammag), 16
- qgammag1 (gammag1), 18
- qgammag2 (gammag2), 20
- qgbetag (gbetag), 22
- qgexppg (gexppg), 24
- qgmbetaexp (gmbetaexp), 26
- qgtransg (gtransg), 28
- qgxlogisticg (gxlogisticg), 30
- qkumg (kumg), 32
- qloggammag1 (loggammag1), 34
- qloggammag2 (loggammag2), 36
- qmbetag (mbetag), 38
- qmog (mog), 40
- qmokumg (mokumg), 42
- qologlogg (ologlogg), 44
- qtexpsg (texpsg), 46
- qweibullextg (weibullextg), 48
- qweibullg (weibullg), 50
  
- rbetaexp (betaexp), 4
- rbetag (betag), 6
  
- rexpexppg (expexppg), 8
- rexp (exp), 10
- rexp (exp), 12
- rexp (exp), 14
- rgammag (gammag), 16
- rgammag1 (gammag1), 18
- rgammag2 (gammag2), 20
- rgbetag (gbetag), 22
- rgexppg (gexppg), 24
- rgmbetaexp (gmbetaexp), 26
- rgtransg (gtransg), 28
- rgxlogisticg (gxlogisticg), 30
- rkumg (kumg), 32
- rloggammag1 (loggammag1), 34
- rloggammag2 (loggammag2), 36
- rmbetag (mbetag), 38
- rmog (mog), 40
- rmokumg (mokumg), 42
- rologlogg (ologlogg), 44
- rtexpsg (texpsg), 46
- rweibullextg (weibullextg), 48
- rweibullg (weibullg), 50
  
- texpsg, 46
  
- weibullextg, 48
- weibullg, 50