

Package ‘JADE’

March 2, 2018

Type Package

Title Blind Source Separation Methods Based on Joint Diagonalization
and Some BSS Performance Criteria

Version 2.0-1

Date 2018-03-02

Author

Klaus Nordhausen, Jean-Francois Cardoso, Jari Miettinen, Hannu Oja, Esa Ollila, Sara Taskinen

Maintainer Klaus Nordhausen <klaus.nordhausen@tuwien.ac.at>

Imports clue, graphics

Suggests ICS, ICSNP

Description Cardoso's JADE algorithm as well as his functions for joint diagonalization are ported to R. Also several other blind source separation (BSS) methods, like AMUSE and SOBI, and some criteria for performance evaluation of BSS algorithms, are given. The package is described in Miettinen, Nordhausen and Taskinen (2017) <doi:10.18637/jss.v076.i02>.

License GPL (>= 2)

LazyData true

NeedsCompilation yes

Repository CRAN

Date/Publication 2018-03-02 15:49:31 UTC

R topics documented:

JADE-package	2
amari.error	3
AMUSE	4
bss.components	6
cjd	7
coef.bss	8
ComonGAP	9
CPPdata	10

djd	11
FG	13
FOBI	14
JADE	15
k_JADE	17
MD	19
multscatter	20
NSS.JD	21
NSS.SD	23
NSS.TD.JD	25
plot.bss	27
print.bss	28
rjd	28
SIR	30
SOBI	31

Index **34**

JADE-package	<i>Blind Source Separation Methods Based on Joint Diagonalization and Some BSS Performance Criteria</i>
--------------	---

Description

Cardoso's JADE algorithm as well as his functions for joint diagonalization are ported to R. Also several other blind source separation (BSS) methods, like AMUSE and SOBI, and some criteria for performance evaluation of BSS algorithms, are given. The package is described in Miettinen, Nordhausen and Taskinen (2017) doi: [10.18637/jss.v076.i02](https://doi.org/10.18637/jss.v076.i02).

Details

Package: JADE
 Type: Package
 Version: 2.0-1
 Date: 2018-03-02
 License: GPL (>= 2)

Author(s)

Klaus Nordhausen, Jean-Francois Cardoso, Jari Miettinen, Hannu Oja, Esa Ollila, Sara Taskinen

Maintainer: Klaus Nordhausen <klaus.nordhausen@tuwien.ac.at>

References

Miettinen, J., Nordhausen, K. and Taskinen, S. (2017), *Blind Source Separation Based on Joint Diagonalization in R: The Packages JADE and BSSasymp*, Journal of Statistical Software, **76**, 1–31, <doi:10.18637/jss.v076.i02>.

amari.error

Amari Error

Description

Computes the Amari Error to evaluate the performance of an ICA algorithm.

Usage

```
amari.error(W.hat, A, standardize = F)
```

Arguments

W.hat	The estimated square unmixing matrix W.
A	The true square mixing matrix A.
standardize	Logical value if A and W.hat need to be standardized. Default is FALSE.

Details

The Amari Error can be used in simulation studies to evaluate the performance of an ICA algorithm. The Amari error is permutation invariant but not scale invariant. Therefore if different algorithms should be compared the matrices should be scaled in the same way. If `standardize` is TRUE, this will be done by the function by standardizing 'W.hat' and the inverse of 'A' in such a way, that every row has length 1, the largest absolute value of the row has a positive sign and the rows are ordered decreasingly according to their largest values.

Note that this function assumes the ICA model is $X = SA'$, as is assumed by [JADE](#) and [ics](#). However [fastICA](#) and [PearsonICA](#) assume $X = SA$. Therefore matrices from those functions have to be transposed first.

The Amari Error is scaled in such a way, that it takes a value between 0 and 1. And 0 corresponds to an optimal separation.

Value

The value of the Amari Error.

Author(s)

Klaus Nordhausen

References

Amari, S., Cichocki, A. and Yang, H.H. (1996), A new learning algorithm for blind signal separation, *Advances in Neural Information Processing Systems*, **8**, 757–763.

Nordhausen, K., Ollila, E. and Oja, H. (2011), On the Performance Indices of ICA and Blind Source Separation. In *the Proceedings of 2011 IEEE 12th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC 2011)*, 486–490.

See Also

[ComonGAP](#), [SIR](#)

Examples

```
S <- cbind(rt(1000, 4), rnorm(1000), runif(1000))
A <- matrix(rnorm(9), ncol = 3)
X <- S %*% t(A)

W.hat <- JADE(X, 3)$W
amari.error(W.hat, A)
amari.error(W.hat, A, TRUE)
```

AMUSE

AMUSE Method for Blind Source Separation

Description

AMUSE method for the second order blind source separation problem. The function estimates the unmixing matrix in a second order stationary source separation model by jointly diagonalizing the covariance matrix and an autocovariance matrix at lag k .

Usage

```
AMUSE(x, ...)

## Default S3 method:
AMUSE(x, k = 1, ...)
## S3 method for class 'ts'
AMUSE(x, ...)
```

Arguments

<code>x</code>	a numeric matrix or a multivariate time series object of class <code>ts</code> . Missing values are not allowed.
<code>k</code>	integer lag for the autocovariance matrix, must be larger than 0. Default is 1.
<code>...</code>	further arguments to be passed to or from methods.

Details

The lag k has a huge effect on the performance and it should be chosen so that the eigenvalues of autocovariance matrix are distinct. The function assumes always as many sources as there are time series.

Value

A list with class 'bss' containing the following components:

W	estimated unmixing matrix.
EV	eigenvectors of autocovariance matrix.
k	lag of the autocovariance matrix used.
S	estimated sources as time series objected standardized to have mean 0 and unit variances.

Author(s)

Klaus Nordhausen

References

Tong, L., Soon, V.C., Huang, Y.F. and Liu, R. (1990), *AMUSE: a new blind identification algorithm*, in *Proceedings of IEEE International Symposium on Circuits and Systems 1990*, 1784–1787.

Miettinen, J., Nordhausen, K., Oja, H. and Taskinen, S. (2012), *Statistical properties of a blind source separation estimator for stationary time series*, *Statistics & Probability Letters*, 82, 1865–1873.

Miettinen, J., Nordhausen, K. and Taskinen, S. (2017), *Blind Source Separation Based on Joint Diagonalization in R: The Packages JADE and BSSasymp*, *Journal of Statistical Software*, 76, 1–31, <doi:10.18637/jss.v076.i02>.

See Also

[ts](#)

Examples

```
# creating some toy data
A<- matrix(rnorm(9),3,3)
s1 <- arima.sim(list(ar=c(0.3,0.6)),1000)
s2 <- arima.sim(list(ma=c(-0.3,0.3)),1000)
s3 <- arima.sim(list(ar=c(-0.8,0.1)),1000)

S <- cbind(s1,s2,s3)
X <- S %**% t(A)

res1<-AMUSE(X)
res1
coef(res1)
plot(res1) # compare to plot.ts(S)
```

```
MD(coef(res1),A)

# input of a time series
X2<- ts(X, start=c(1961, 1), frequency=12)
plot(X2)
res2<-AMUSE(X2, k=2)
plot(res2)
```

bss.components

Function to Extract Estimated Sources from an Object of Class bss

Description

Extracts the sources estimated by an bss method.

Usage

```
bss.components(object)
```

Arguments

object object of class bss

Author(s)

Klaus Nordhausen

Examples

```
A<- matrix(rnorm(9),3,3)
s1 <- arima.sim(list(ar=c(0.3,0.6)),1000)
s2 <- arima.sim(list(ma=c(-0.3,0.3)),1000)
s3 <- arima.sim(list(ar=c(-0.8,0.1)),1000)

S <- cbind(s1,s2,s3)
X <- S %*% t(A)

res1<-AMUSE(X)
head(bss.components(res1))
colMeans(bss.components(res1))
cov(bss.components(res1))
```

Description

This is an **R** version of Cardoso's `joint_diag` matlab function for joint diagonalization of k complex-valued square matrices.

Usage

```
cjd(X, eps = 1e-06, maxiter = 100)
```

Arguments

<code>X</code>	A matrix of k stacked $p \times p$ complex matrices with dimension $c(kp, p)$ or an array with dimension $c(p, p, k)$.
<code>eps</code>	Convergence tolerance.
<code>maxiter</code>	Maximum number of iterations.

Value

<code>V</code>	An orthogonal matrix.
<code>D</code>	A stacked matrix with the diagonal matrices or an array with the diagonal matrices. The form of the output depends on the form of the input.

Author(s)

Jean-Francois Cardoso. Ported to **R** by Klaus Nordhausen.

References

Cardoso, J.-F. and Souloumiac, A., (1996), *Jacobi angles for simultaneous diagonalization*, SIAM J. Mat. Anal. Appl., **17**, 161–164.

See Also

[rjd](#), [rjd.fortran](#)

Examples

```
D1 <- diag(complex(real=runif(3,0,2), imaginary=runif(3)))
D2 <- diag(complex(real=runif(3,0,2), imaginary=runif(3)))
D3 <- diag(complex(real=runif(3,0,2), imaginary=runif(3)))
D4 <- diag(complex(real=runif(3,0,2), imaginary=runif(3)))

Z <- matrix(runif(9), ncol = 3)
V <- eigen(Z %*% t(Z))$vectors
```

```

M1 <- t(V)%*%D1%*%V
M2 <- t(V)%*%D2%*%V
M3 <- t(V)%*%D3%*%V
M4 <- t(V)%*%D4%*%V
MS <- rbind(M1,M2,M3,M4)
Ms <- array(0,dim=c(3,3,4))
Ms[, ,1]<-M1
Ms[, ,3]<-M3
Ms[, ,2]<-M2
Ms[, ,4]<-M4
res.array <- cjd(Ms)
res.mat <- cjd(MS)
Re(res.array$V)
V
round(V%*%Re(res.array$V),2)
round(V%*%Re(res.mat$V),2)

```

coef.bss

Coefficients of a bss Object

Description

Extracts the estimated unmixing matrix from an object of class bss.

Usage

```

## S3 method for class 'bss'
coef(object, ...)

```

Arguments

object	object of class bss.
...	further arguments to be passed to or from methods.

Author(s)

Klaus Nordhausen

Examples

```

A<- matrix(rnorm(9),3,3)
s1 <- arima.sim(list(ar=c(0.3,0.6)),1000)
s2 <- arima.sim(list(ma=c(-0.3,0.3)),1000)
s3 <- arima.sim(list(ar=c(-0.8,0.1)),1000)

S <- cbind(s1,s2,s3)
X <- S %*% t(A)

res1<-AMUSE(X)
coef(res1)
coef(res1) %*% A # should be a matrix with one dominant element in each row and column

```

`ComonGAP`*Comon's Gap*

Description

Comon's GAP criterion to evaluate the performance of an ICA algorithm.

Usage

```
ComonGAP(A, A.hat)
```

Arguments

A	The true square mixing matrix.
A.hat	The estimated square mixing matrix.

Details

Comon's GAP criterion is permutation and scale invariant. It can take every positive value and 0 corresponds to an optimal separation. If A is however nearly singular the values of the criterion can be huge.

Note that this function assumes the ICA model is $X = SA'$, as is assumed by [JADE](#) and [ics](#). However [fastICA](#) and [PearsonICA](#) assume $X = SA$. Therefore matrices from those functions have to be transposed first.

Value

The value of the Comon's GAP.

Author(s)

Klaus Nordhausen

References

Comon, P., (1994), *Independent Component Analysis, A new concept?*, *Signal Processing*, **36**, 287–314.

See Also

[amari.error](#), [SIR](#)

Examples

```
S <- cbind(rt(1000, 4), rnorm(1000), runif(1000))
A <- matrix(rnorm(9), ncol = 3)
X <- S %*% t(A)

A.hat <- JADE(X, 3)$A
ComonGAP(A, A.hat)
```

CPPdata

Cocktail Party Problem Data

Description

This data set is a toy example for the so called cocktail party problem. In this case three sounds are mixed together with one noise source using four microphones.

Usage

```
data("CPPdata")
```

Format

A data frame with 50000 observations on the following 4 variables.

Mic1 the mixture recorded by the first microphone.

Mic2 the mixture recorded by the second microphone.

Mic3 the mixture recorded by the third microphone.

Mic4 the mixture recorded by the fourth microphone.

Details

The three original source files were kindly provided by Ella Bingham and are also available online at the following locations: research.ics.aalto.fi/ica/cocktail/source5.wav, research.ics.aalto.fi/ica/cocktail/source7.wav and research.ics.aalto.fi/ica/cocktail/source9.wav.

Note that the original sound files are included in the package's subfolder datafiles. In the example section we illustrate how the CPPdata was created. An example analysis of the data is given in Miettinen et al. (2017).

Source

http://research.ics.aalto.fi/ica/cocktail/cocktail_en.cgi

References

Miettinen, J., Nordhausen, K. and Taskinen, S. (2017), *Blind Source Separation Based on Joint Diagonalization in R: The Packages JADE and BSSasyp*, Journal of Statistical Software, **76**, 1–31, <doi:10.18637/jss.v076.i02>.

Examples

```
## Not run:
# the data was created as follows:
library("tuneR")
S1 <- readWave(system.file("datafiles/source5.wav", package = "JADE"))
S2 <- readWave(system.file("datafiles/source7.wav", package = "JADE"))
S3 <- readWave(system.file("datafiles/source9.wav", package = "JADE"))

set.seed(321)
NOISE <- noise("white", duration = 50000)
S <- cbind(S1@left, S2@left, S3@left, NOISE@left)
S <- scale(S, center = FALSE, scale = apply(S, 2, sd))
St <- ts(S, start = 0, frequency = 8000)
p <- 4
A <- matrix(runif(p^2, 0, 1), p, p)
A

X <- tcrossprod(St, A)
Xt <- as.ts(X)

colnames(X) <- c("Mic1", "Mic2", "Mic3", "Mic4")
CPPdata <- as.data.frame(X)

## End(Not run)
```

djd

Function for Joint Diagonalization of k Square Matrices in a Deflation Based Manner

Description

This function jointly diagonalizes k real-valued square matrices by searching an orthogonal matrix in a deflation based manner.

Usage

```
djd(X, G = "max", r = 2, eps = 1e-06, maxiter = 500)
```

Arguments

X	an array containing the k p times p real valued matrices of dimension $c(p, p, k)$.
G	criterion function used for the the algorithm. Options are max, pow and log. See details.
r	power value used if $G="pow"$ or $G="max"$. 0 is not meaningful for this value. See details.
eps	convergence tolerance.
maxiter	maximum number of iterations.

Details

Denote the square matrices as A_i , $i = 1, \dots, k$. This algorithm searches then an orthogonal matrix W so that $D_i = W'A_iW$ is diagonal for all i . If the A_i commute then there is an exact solution. If not, the function will perform an approximate joint diagonalization by maximizing $\sum G(w_j'A_iw_j)$ where w_j are the orthogonal vectors in W .

The function G can be chosen to be of the form $G(x) = |x|^r$ or $G(x) = \log(x)$. If $G="max"$ is chosen, the function G is of the form $G(x) = |x|^r$, and the diagonalization criterion will be maximized globally at each stage by choosing an appropriate initial value from a set of random vectors. If $G="pow"$ or $G="log"$ are chosen, the initial values are the eigenvectors of A_1 which plays hence a special role.

Value

The matrix W

Author(s)

Klaus Nordhausen, Jari Miettinen

References

Nordhausen, K., Gutch, H. W., Oja, H. and Theis, F.J. (2012): *Joint Diagonalization of Several Scatter Matrices for ICA*, in *LVA/ICA 2012, LNCS 7191*, pp. 172–179.

Miettinen, J., Nordhausen, K., Oja, H. and Taskinen, S. (2014), *Deflation-based Separation of Uncorrelated Stationary Time Series*, *Journal of Multivariate Analysis*, 123, 214–227.

Miettinen, J., Nordhausen, K. and Taskinen, S. (2017), *Blind Source Separation Based on Joint Diagonalization in R: The Packages JADE and BSSasymp*, *Journal of Statistical Software*, 76, 1–31, <doi:10.18637/jss.v076.i02>.

Examples

```
Z <- matrix(runif(9), ncol = 3)
U <- eigen(Z %*% t(Z))$vectors
D1 <- diag(runif(3))
D2 <- diag(runif(3))
D3 <- diag(runif(3))
D4 <- diag(runif(3))

X.matrix <- array(0, dim=c(3, 3, 4))
X.matrix[, ,1] <- t(U) %*% D1 %*% U
X.matrix[, ,2] <- t(U) %*% D2 %*% U
X.matrix[, ,3] <- t(U) %*% D3 %*% U
X.matrix[, ,4] <- t(U) %*% D4 %*% U

W1 <- djd(X.matrix)
round(U %*% W1, 4) # should be a signed permutation
# matrix if W1 is correct.

W2 <- djd(X.matrix, r=1)
```

```

round(U %%% W2, 4) # should be a signed permutation
                  # matrix if W2 is correct.

W3 <- djd(X.matrix, G="1")
round(U %%% W3, 4) # should be a signed permutation
                  # matrix if W3 is correct.

```

FG

Joint Diagonalization of Real Positive-definite Matrices

Description

This is a slightly modified version of Flury's FG algorithm for the joint diagonalization of k positive-definite matrices. The underlying function is written in C.

Usage

```
FG(X, weight = NULL, init = NULL, maxiter = 100, eps = 1e-06, na.action = na.fail)
```

Arguments

<code>X</code>	A matrix of k stacked $p \times p$ matrices with dimension $c(kp,p)$ or an array with dimension $c(p,p,k)$.
<code>weight</code>	A vector of length k to give weight to the different matrices, if NULL, all matrices have equal weight.
<code>init</code>	Initial value for the orthogonal matrix to be estimated, if NULL, the identity matrix is used.
<code>maxiter</code>	Maximum number of iterations.
<code>eps</code>	Convergence tolerance.
<code>na.action</code>	A function which indicates what should happen when the data contain 'NA's. Default is to fail.

Value

A list with the components

<code>V</code>	An orthogonal matrix.
<code>D</code>	A stacked matrix with the diagonal matrices or an array with the diagonal matrices. The form of the output depends on the form of the input.
<code>iter</code>	The Fortran function returns also the number of iterations.

Author(s)

Jari Miettinen

References

Flury, B. D. (1998), *Common principal components and related models*, Wiley, New York.

See Also

[rjd](#), [rjd.fortran](#)

FOBI

Function to perform FOBI for ICA

Description

The FOBI method for independent component analysis (ICA). We assume that all components have different kurtosis values.

Usage

```
FOBI(X, na.action = na.fail)
```

Arguments

X	a numeric matrix.
na.action	A function which indicates what should happen when the data contain 'NA's. Default is to fail.

Value

A list with class 'bss' containing the following components:

W	estimated unmixing matrix.
EV	eigenvectors of autocovariance matrix.
Xmu	the original mean of the data.
S	estimated sources as time series objected standardized to have mean 0 and unit variances.

Note

More general is the function [ics](#) in the **ICS** package.

Author(s)

Klaus Nordhausen

References

Cardoso, J.-F. (1989), *Source separation using higher order moments*, in *Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing*, 2109–2112.

Miettinen, J., Taskinen S., Nordhausen, K. and Oja, H. (2015), *Fourth Moments and Independent Component Analysis*, *Statistical Science*, 30, 372–390.

Miettinen, J., Nordhausen, K. and Taskinen, S. (2017), *Blind Source Separation Based on Joint Diagonalization in R: The Packages JADE and BSSasymp*, *Journal of Statistical Software*, 76, 1–31, <doi:10.18637/jss.v076.i02>.

See Also

[ics](#)

Examples

```
# 3 source and 3 signals

S <- cbind(rt(1000, 4), rnorm(1000), runif(1000))
A <- matrix(rnorm(9), ncol = 3)
X <- S %*% t(A)
res<-FOBI(X)
MD(coef(res),A)
```

JADE

JADE Algorithm for ICA

Description

This is an **R** version of Cardoso's JADE ICA algorithm (for real data) ported from matlab. The ported version is 1.5. Some minor changes compared to the matlab function are explained in the details section. The matlab code can be found for example on the ICA central homepage.

The function uses [frjd](#) for the joint diagonalization.

Usage

```
JADE(X, n.comp = NULL, eps = 1e-06, maxiter = 100, na.action = na.fail)
```

Arguments

X	Numeric data matrix or dataframe.
n.comp	Number of components to extract.
eps	Convergence tolerance.
maxiter	Maximum number of iterations.
na.action	A function which indicates what should happen when the data contain 'NA's. Default is to fail.

Details

Some minor modifications were done when porting the function to **R**, and they are:

- 1 the model assumed here is $X = SA' + \mu$. Therefore S and X have one row per observation. Note that this still differs from the model definition in **R** of FastICA and PearsonICA but agrees with `ics`.
- 2 The whitening covariance matrix is divided by $n-1$ and not n (n = number of observations).
- 3 The initial value for the joint diagonalisation is always I .
- 4 The original `eps` would be $\frac{1}{100\sqrt{n}}$.

It is also worth mentioning that the estimated independent components S are scaled to unit variance and are ordered in such a way, that their fourth moments are in the decreasing order. The signs of the unmixing matrix W are fixed so that the sum of the elements on each row is positive.

For further details see also the documentation of the original matlab code ("MatlabjadeR.m") on the ICA central homepage (<http://www.tsi.enst.fr/icacentral/>).

Value

A list with class 'bss' containing the following components:

A	The estimated mixing matrix.
W	The estimated unmixing matrix.
S	Dataframe with the estimated independent components.
Xmu	The location of the original data.

Author(s)

Jean-Francois Cardoso. Ported to **R** by Klaus Nordhausen

References

Cardoso, J.-F. and Souloumiac, A., (1993), *Blind beamforming for non Gaussian signals*, IEE Proceedings-F, **140**, 362–370.

Miettinen, J., Taskinen S., Nordhausen, K. and Oja, H. (2015), *Fourth Moments and Independent Component Analysis*, Statistical Science, **30**, 372–390.

Miettinen, J., Nordhausen, K. and Taskinen, S. (2017), *Blind Source Separation Based on Joint Diagonalization in R: The Packages JADE and BSSasymp*, Journal of Statistical Software, **76**, 1–31, <doi:10.18637/jss.v076.i02>.

Examples

```
# 3 source and 3 signals

S <- cbind(rt(1000, 4), rnorm(1000), runif(1000))
A <- matrix(rnorm(9), ncol = 3)
X <- S %*% t(A)
res<-JADE(X,3)
```



```

res$A
res$W
res$S[1:10,]
(sweep(X,2,res$Xmu) %*% t(res$W))[1:10,]
round(res$W %*% A,4)

# 2 sources and 3 signals

S2 <- cbind(rt(1000, 4), rnorm(1000))
A2 <- matrix(rnorm(6), ncol = 2)
X2 <- S2 %*% t(A2)
res2 <- JADE(X2,2)
res2$A
res2$W
res2$S[1:10,]
(sweep(X2,2,res2$Xmu) %*% t(res2$W))[1:10,]
SIR(S2,res2$S)

```

k_JADE

*Fast Equivariant k-JADE Algorithm for ICA***Description**

This algorithm generalizes the [JADE](#) algorithm, as it provides [JADE](#) when k is set to the number of dimensions. Otherwise k can be considered as a way to reduce the number of cumulant matrices to be jointly diagonalized. Hence small values of k speed up the method considerably in high-dimensional cases. In general, k can be considered as maximum number of underlying identical sources.

The function uses [FOBI](#) as an initial estimate and [frjd](#) for the joint diagonalization.

Usage

```
k_JADE(X, k = 1, eps = 1e-06, maxiter = 100, na.action = na.fail)
```

Arguments

X	Numeric data matrix or dataframe.
k	integer value between 1 and the number of columns of X. Default is 1.
eps	Convergence tolerance.
maxiter	Maximum number of iterations.
na.action	A function which indicates what should happen when the data contain 'NA's. Default is to fail.

Details

The order of the estimated components is fixed so that their fourth moments are in the decreasing order.

Value

A list with class 'bss' containing the following components:

A	The estimated mixing matrix.
W	The estimated unmixing matrix.
S	Matrix with the estimated independent components.
Xmu	The location of the original data.

Note

The function uses [FOBI](#) as initial estimate and [frjd](#) for the joint diagonalization.

Author(s)

Jari Miettinen

References

Miettinen, J., Nordhausen, K., Oja, H. and Taskinen, S. (2013), *Fast Equivariant JADE*, In the *Proceedings of 38th IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP 2013)*, 6153–6157.

Miettinen, J., Nordhausen, K. and Taskinen, S. (2017), *Blind Source Separation Based on Joint Diagonalization in R: The Packages JADE and BSSasymp*, *Journal of Statistical Software*, **76**, 1–31, <doi:10.18637/jss.v076.i02>.

See Also

[JADE](#), [FOBI](#), [frjd](#)

Examples

```
# 3 source and 3 signals

S <- cbind(rt(1000, 4), rnorm(1000), runif(1000))
A <- matrix(rnorm(9), ncol = 3)
X <- S %*% t(A)
res_k1 <- k_JADE(X, 1)
res_k1$A
res_k1$W
res_k1$S[1:10,]

MD(coef(res_k1), A)
```

Description

Computes the Minimum Distance index MD to evaluate the performance of an ICA algorithm.

Usage

MD(W.hat, A)

Arguments

W.hat The estimated square unmixing matrix W.
A The true square mixing matrix A.

Details

$$MD(\hat{W}, A) = \frac{1}{\sqrt{p-1}} \inf_{PD} \|PD\hat{W}A - I\|,$$

where P is a permutation matrix and D a diagonal matrix with nonzero diagonal entries.

The step that minimizes the index of the set over all permutation matrix can be expressed as a linear sum assignment problem (LSAP) for which we use as solver the Hungarian method implemented as `solve_LASP` in the **clue** package.

Note that this function assumes the ICA model is $X = SA'$, as is assumed by [JADE](#) and [ics](#). However [fastICA](#) and [PearsonICA](#) assume $X = SA$. Therefore matrices from those functions have to be transposed first.

The MD index is scaled in such a way, that it takes a value between 0 and 1. And 0 corresponds to an optimal separation.

Value

The value of the MD index.

Author(s)

Klaus Nordhausen

References

Ilmonen, P., Nordhausen, K., Oja, H. and Ollila, E. (2010), A New Performance Index for ICA: Properties, Computation and Asymptotic Analysis. In Vigneron, V., Zarzoso, V., Moreau, E., Grillonval, R. and Vincent, E. (editors) Latent Variable Analysis and Signal Separation, 229–236, Springer.

Nordhausen, K., Ollila, E. and Oja, H. (2011), *On the Performance Indices of ICA and Blind Source Separation*. In the Proceedings of 2011 IEEE 12th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC 2011), 486–490.

Miettinen, J., Nordhausen, K. and Taskinen, S. (2017), *Blind Source Separation Based on Joint Diagonalization in R: The Packages JADE and BSSasymp*, Journal of Statistical Software, **76**, 1–31, <doi:10.18637/jss.v076.i02>.

See Also

[ComonGAP](#), [SIR](#), [amari.error](#), [solve_LSAP](#)

Examples

```
S <- cbind(rt(1000, 4), rnorm(1000), runif(1000))
A <- matrix(rnorm(9), ncol = 3)
X <- S %*% t(A)
```

```
W.hat <- JADE(X, 3)$W
MD(W.hat, A)
```

multscatter

Function to Compute Several Scatter Matrices for the Same Data

Description

The function can be used to compute several scatter matrices for the same data.

Usage

```
multscatter(scatterlist, X, toshape = TRUE)
```

Arguments

<code>scatterlist</code>	a vector with the names of the scatter matrices to be computed. Note that each of these functions should only return a matrix of size p times p .
<code>X</code>	the n times p data matrix for which the scatter should be computed.
<code>toshape</code>	logical, whether scatter matrices should be converted to shape matrices. If TRUE, all matrices will have determinant 1.

Details

It is important that the functions do not need any additional input and that they return only the p times p scatter matrix. Hence it might be sometimes necessary to write wrappers for some of the functions. See examples.

Value

An array of dimension $c(p,p,k)$ where k is the number of scatter matrices.

Author(s)

Klaus Nordhausen

Examples

```
# example requires the packages ICS and ICSNP
library(ICSNP)
X <- cbind(rexp(1000), rt(1000,6), runif(1000))

my.tM1 <- function(X,df=1) tM(X,)$V
my.tM2 <- function(X,df=2) tM(X,)$V

multscatter(c("cov", "cov4", "HP1.shape", "my.tM1", "my.tM2"), X)
multscatter(c("cov", "cov4", "HP1.shape", "my.tM1", "my.tM2"), X, toshape=FALSE)
```

NSS.JD

*NSS.JD Method for Nonstationary Blind Source Separation***Description**

The NSS.JD method for nonstationary blind source separation. The method first whitens the complete data and then divides it into K time intervals. Then `frjd` is used to jointly diagonalize the covariance matrices computed for the individual time intervals to find the sources.

Usage

```
NSS.JD(X, ...)
```

Default S3 method:
 NSS.JD(X, K=12, Tau=0, n.cuts=NULL, eps = 1e-06, maxiter = 100, ...)
 ## S3 method for class 'ts'
 NSS.JD(X, ...)

Arguments

X	a numeric matrix or a multivariate time series object of class <code>ts</code> . Missing values are not allowed.
K	number of intervals to be used.
Tau	By default 0 which means covariance are computed of each time interval, if Tau is an integer > 0 then rather autocovariance matrices at lag Tau are used for the joint diagonaliation.
n.cuts	if NULL, then the time series is divided into K equally long intervals. To specify intervals n.cuts should be given in the form <code>c(1,n.cut.1,...,n.cut.k, nrow(X))</code> to specify where to split the time series.
eps	maximum number of iterations for <code>frjd</code> .
maxiter	convergence tolerance for <code>frjd</code> .
...	further arguments to be passed to or from methods.

Details

The model assumes that the mean of the p-variate time series is constant but the variances change over time.

Value

A list with class 'bss' containing the following components:

W	estimated unmixing matrix.
k	the lag used for the autocovariance matrix.
n.cut	specifying the intervals where data is split
K	the number of intervals used
S	estimated sources as time series objected standardized to have mean 0 and that the variance of the sources are 1.

Author(s)

Klaus Nordhausen

References

Choi S. and Cichocki A. (2000), *Blind separation of nonstationary sources in noisy mixtures*, Electronics Letters, 36, 848–849.

Choi S. and Cichocki A. (2000), *Blind separation of nonstationary and temporally correlated sources from noisy mixtures*, Proceedings of the 2000 IEEE Signal Processing Society Workshop Neural Networks for Signal Processing X, 1, 405–414.

Nordhausen K. (2014), *On robustifying some second order blind source separation methods for nonstationary time series*, Statistical Papers, 55, 141–156.

Miettinen, J., Nordhausen, K. and Taskinen, S. (2017), *Blind Source Separation Based on Joint Diagonalization in R: The Packages JADE and BSSasymp*, Journal of Statistical Software, 76, 1–31, <doi:10.18637/jss.v076.i02>.

See Also

[ts](#), [NSS.SD](#), [NSS.TD.JD](#)

Examples

```
n <- 1000
s1 <- rnorm(n)
s2 <- 2*sin(pi/200*1:n)* rnorm(n)
s3 <- c(rnorm(n/2), rnorm(100,0,2), rnorm(n/2-100,0,1.5))
S <- cbind(s1,s2,s3)
plot.ts(S)
A<-matrix(rnorm(9),3,3)
X<- S%*%t(A)

NSS2 <- NSS.JD(X)
```

```

NSS2
MD(coef(NSS2),A)
plot(NSS2)
cor(NSS2$S,S)

NSS2b <- NSS.JD(X, Tau=1)
MD(coef(NSS2b),A)

NSS2c <- NSS.JD(X, n.cuts=c(1,300,500,600,1000))
MD(coef(NSS2c),A)

```

NSS.SD

NSS.SD Method for Nonstationary Blind Source Separation

Description

The NSS.SD method for nonstationary blind source separation. The function estimates the unmixing matrix in a nonstationary source separation model by simultaneously diagonalizing two covariance matrices computed for different time intervals.

Usage

```

NSS.SD(X, ...)

## Default S3 method:
NSS.SD(X, n.cut=NULL, ...)
## S3 method for class 'ts'
NSS.SD(X, ...)

```

Arguments

X	a numeric matrix or a multivariate time series object of class <code>ts</code> . Missing values are not allowed.
n.cut	either an integer between 1 and <code>nrow(X)</code> or a vector of length 3 of the form <code>c(1,n.cut,nrow(X))</code> to specify where to split the time series. If <code>NULL</code> , then <code>c(1,floor(nrow(X)/2),nrow(X))</code> is used.
...	further arguments to be passed to or from methods.

Details

The model assumes that the mean of the p-variate time series is constant but the variances change over time.

Value

A list with class 'bss' containing the following components:

W	estimated unmixing matrix.
EV	eigenvalues from the eigenvalue-eigenvector decomposition.
n.cut	specifying the intervals where data is split
S	estimated sources as time series objected standardized to have mean 0 and that the sources in the first interval are 1.

Author(s)

Klaus Nordhausen

References

- Choi S. and Cichocki A. (2000), *Blind separation of nonstationary sources in noisy mixtures*, *Electronics Letters*, 36, 848–849.
- Choi S. and Cichocki A. (2000), *Blind separation of nonstationary and temporally correlated sources from noisy mixtures*, *Proceedings of the 2000 IEEE Signal Processing Society Workshop Neural Networks for Signal Processing X*, 1, 405–414.
- Nordhausen K. (2014), *On robustifying some second order blind source separation methods for nonstationary time series*, *Statistical Papers*, 55, 141–156.
- Miettinen, J., Nordhausen, K. and Taskinen, S. (2017), *Blind Source Separation Based on Joint Diagonalization in R: The Packages JADE and BSSasymp*, *Journal of Statistical Software*, 76, 1–31, <doi:10.18637/jss.v076.i02>.

See Also

[ts](#), [NSS.JD](#), [NSS.TD.JD](#)

Examples

```
n <- 1000
s1 <- rnorm(n)
s2 <- 2*sin(pi/200*1:n)* rnorm(n)
s3 <- c(rnorm(n/2), rnorm(100,0,2), rnorm(n/2-100,0,1.5))
S <- cbind(s1,s2,s3)
plot.ts(S)
A<-matrix(rnorm(9),3,3)
X<- S%*%t(A)

NSS1 <- NSS.SD(X)
NSS1
MD(coef(NSS1),A)
plot(NSS1)
cor(NSS1$S,S)

NSS1b <- NSS.SD(X, n.cut=400)
MD(coef(NSS1b),A)
```



```
NSS1c <- NSS.SD(X, n.cut=c(1,600,1000))
MD(coef(NSS1c),A)
```

NSS.TD.JD

NSS.TD.JD Method for Nonstationary Blind Source Separation

Description

The NSS.TD.JD method for nonstationary blind source separation. The method first whitens the complete data and then divides it into K time intervals. It is then assumed that within each interval the time series is approximately second order stationary and within each interval L autocovariance are computed. The underlying sources are then found by jointly diagonalizing the $K*L$ autocovariance matrices using [frjd](#).

Usage

```
NSS.TD.JD(X, ...)
```

Default S3 method:
 NSS.TD.JD(X, K=12, Tau=0:11, n.cuts=NULL, eps = 1e-06, maxiter = 100, ...)
 ## S3 method for class 'ts'
 NSS.TD.JD(X, ...)

Arguments

X	a numeric matrix or a multivariate time series object of class ts . Missing values are not allowed.
K	number of intervals to be used.
Tau	Lags for the autocovariance matrices to be computed within each interval.
n.cuts	if NULL, then the time series is divided into K equally long intervals. To specify intervals n.cuts should be given in the form <code>c(1,n.cut.1,...,n.cut.k, nrow(X))</code> to specify where to split the time series.
eps	maximum number of iterations for frjd .
maxiter	convergence tolerance for frjd .
...	further arguments to be passed to or from methods.

Details

The model assumes that the mean of the p -variate time series is constant but the variances change over time.

Value

A list with class 'bss' containing the following components:

W	estimated unmixing matrix.
k	the lags used for the autocovariance matrix used in each interval.
n.cut	specifying the intervals where data is split
K	the number of intervals used
S	estimated sources as time series objected standardized to have mean 0 and that the sources 1.

Author(s)

Klaus Nordhausen

References

Choi S. and Cichocki A. (2000), *Blind separation of nonstationary sources in noisy mixtures*, Electronics Letters, 36, 848–849.

Choi S. and Cichocki A. (2000), *Blind separation of nonstationary and temporally correlated sources from noisy mixtures*, Proceedings of the 2000 IEEE Signal Processing Society Workshop Neural Networks for Signal Processing X, 1, 405–414.

Nordhausen K. (2014), *On robustifying some second order blind source separation methods for nonstationary time series*, Statistical Papers, 55, 141–156.

Miettinen, J., Nordhausen, K. and Taskinen, S. (2017), *Blind Source Separation Based on Joint Diagonalization in R: The Packages JADE and BSSasyp*, Journal of Statistical Software, 76, 1–31, <doi:10.18637/jss.v076.i02>.

See Also

[ts](#), [NSS.JD](#), [NSS.JD](#), [SOBI](#)

Examples

```
n <- 1000
s1 <- rnorm(n)
s2 <- 2*sin(pi/200*1:n)* rnorm(n)
s3 <- c(rnorm(n/2), rnorm(100,0,2), rnorm(n/2-100,0,1.5))
S <- cbind(s1,s2,s3)
plot.ts(S)
A<-matrix(rnorm(9),3,3)
X<- S%*%t(A)

NSS3 <- NSS.TD.JD(X)
NSS3
MD(coef(NSS3),A)
plot(NSS3)
cor(NSS3$S,S)
```

```
NSS3b <- NSS.TD.JD(X, Tau=c(0,3,7,12), K=6)
MD(coef(NSS3b),A)

NSS3c <- NSS.TD.JD(X, n.cuts=c(1,300,500,600,1000))
MD(coef(NSS3c),A)
```

plot.bss

*Plotting an Object of Class bss***Description**

Plots the estimated sources resulting from an bss method. If the bss method is based on second order assumptions and returned the sources as a time series object it will plot the sources using `plot.ts`, otherwise it will plot a scatter plot matrix using `pairs` or `plot` if there are only two sources.

Usage

```
## S3 method for class 'bss'
plot(x, ...)
```

Arguments

```
x          object of class bss.
...        further arguments to be passed to or from methods.
```

Author(s)

Klaus Nordhausen

See Also

[plot.ts](#), [pairs](#), [plot](#)

Examples

```
A<- matrix(rnorm(9),3,3)
s1 <- arima.sim(list(ar=c(0.3,0.6)),1000)
s2 <- arima.sim(list(ma=c(-0.3,0.3)),1000)
s3 <- arima.sim(list(ar=c(-0.8,0.1)),1000)

S <- cbind(s1,s2,s3)
X <- S %*% t(A)

res1 <- AMUSE(X)
plot(res1)
# not so useful:
plot(res1, plot.type = "single", col=1:3)

# not meaningful for this data
```

```
res2 <- JADE(X)
plot(res2)
```

```
print.bss
```

Printing an Object of Class bss

Description

Prints an object of class `bss`. It prints all elements of the list of class `bss` except the component `S` which is the source matrix.

Usage

```
## S3 method for class 'bss'
print(x, ...)
```

Arguments

<code>x</code>	object of class <code>bss</code> .
<code>...</code>	further arguments to be passed to or from methods.

Author(s)

Klaus Nordhausen

```
rjd
```

Joint Diagonalization of Real Matrices

Description

This is an **R** version of Cardoso's `rjd` matlab function for joint diagonalization of k real-valued square matrices. A version written in **C** is also available and preferable.

Usage

```
rjd(X, eps = 1e-06, maxiter = 100, na.action = na.fail)
frjd(X, weight = NULL, maxiter = 100, eps = 1e-06, na.action = na.fail)
rjd.fortran(X, weight = NULL, maxiter = 100, eps = 1e-06, na.action = na.fail)
```

Arguments

X	A matrix of k stacked p x p matrices with dimension c(kp,p) or an array with dimension c(p,p,k).
weight	A vector of length k to give weight to the different matrices, if NULL, all matrices have equal weight
eps	Convergence tolerance.
maxiter	Maximum number of iterations.
na.action	A function which indicates what should happen when the data contain 'NA's. Default is to fail.

Details

Denote the square matrices as $A_i, i = 1, \dots, k$. The algorithm searches an orthogonal matrix V so that $D_i = V' A_i V$ is diagonal for all i . If the A_i commute then there is an exact solution. Otherwise, the function will perform an approximate joint diagonalization by trying to make the D_i as diagonal as possible.

Cardoso points out that notion of approximate joint diagonalization is ad hoc and very small values of eps make in that case not much sense since the diagonality criterion is ad hoc itself.

Value

	A list with the components
V	An orthogonal matrix.
D	A stacked matrix with the diagonal matrices or an array with the diagonal matrices. The form of the output depends on the form of the input.
iter	The frjd function returns also the number of iterations.

Author(s)

Jean-Francois Cardoso. Ported to **R** by Klaus Nordhausen. C code by Jari Miettinen

References

- Cardoso, J.-F. and Souloumiac, A., (1996), *Jacobi angles for simultaneous diagonalization*, SIAM J. Mat. Anal. Appl., **17**, 161–164.
- Miettinen, J., Nordhausen, K. and Taskinen, S. (2017), *Blind Source Separation Based on Joint Diagonalization in R: The Packages JADE and BSSasymp*, Journal of Statistical Software, **76**, 1–31, <doi:10.18637/jss.v076.i02>.

Examples

```
Z <- matrix(runif(9), ncol = 3)
U <- eigen(Z %*% t(Z))$vectors
D1 <- diag(runif(3))
D2 <- diag(runif(3))
D3 <- diag(runif(3))
```

```

D4 <- diag(runif(3))

X.matrix <- rbind(t(U) %*% D1 %*% U, t(U) %*% D2 %*% U,
                 t(U) %*% D3 %*% U, t(U) %*% D4 %*% U)
res.matrix <- rjd(X.matrix)
res.matrix$V
round(U %*% res.matrix$V, 4) # should be a signed permutation
                           # matrix if V is correct.

round(res.matrix$D, 4)

# compare to C version

#res.matrix.C <- frjd(X.matrix)
#res.matrix.C$V
#round(U %*% res.matrix.C$V, 4)
#round(res.matrix.C$D, 4)

X.array <- aperm(array(t(X.matrix), dim = c(3,3,4)), c(2,1,3))

res.array <- rjd(X.array)
round(res.array$D, 4)

res.array.C <- frjd(X.array)
round(res.array.C$D, 4)

```

SIR

Signal to Interference Ratio

Description

Computes the signal to interference ratio between true and estimated signals

Usage

```
SIR(S, S.hat)
```

Arguments

S	Matrix or dataframe with the true numeric signals.
S.hat	Matrix or dataframe with the estimated numeric signals.

Details

The signal to interference ratio is measured in dB and values over 20 are thought to be good. It is scale and permutation invariant and can be seen as measuring the correlation between the matched true and estimated signals.

Value

The value of the signal to interference ratio.

Author(s)

Klaus Nordhausen

References

Eriksson, J., Karvanen, J. and Koivunen, V. (2000), Source distribution adaptive maximum likelihood estimation in ICA model, Proceedings of the second international workshop on independent component analysis and blind source separation (ICA 2000), 227–232.

See Also

[amari.error](#), [ComonGAP](#)

Examples

```
S <- cbind(rt(1000, 4), rnorm(1000), runif(1000))
A <- matrix(rnorm(9), ncol = 3)
X <- S %*% t(A)

S.hat <- JADE(X, 3)$S
SIR(S, S.hat)
```

SOBI

SOBI Method for Blind Source Separation

Description

The SOBI method for the second order blind source separation problem. The function estimates the unmixing matrix in a second order stationary source separation model by jointly diagonalizing the covariance matrix and several autocovariance matrices at different lags.

Usage

```
SOBI(X, ...)
```

Default S3 method:

```
SOBI(X, k=12, method="frjd", eps = 1e-06, maxiter = 100, ...)
```

S3 method for class 'ts'

```
SOBI(X, ...)
```

Arguments

X	a numeric matrix or a multivariate time series object of class <code>ts</code> . Missing values are not allowed.
k	if a single integer, then the lags 1:k are used, if an integer vector, then these are used as the lags.
method	method to use for the joint diagonalization, options are <code>djd</code> , <code>rjd</code> and <code>frjd</code> .
eps	maximum number of iterations.
maxiter	convergence tolerance.
...	further arguments to be passed to or from methods.

Details

The order of the estimated components is fixed so that the sums of squared autocovariances are in the decreasing order.

Value

A list with class 'bss' containing the following components:

W	estimated unmixing matrix.
k	lags used.
method	method used for the joint diagonalization.
S	estimated sources as time series objected standardized to have mean 0 and unit variances.

Author(s)

Klaus Nordhausen

References

Belouchrani, A., Abed-Meriam, K., Cardoso, J.F. and Moulines, R. (1997), A blind source separation technique using second-order statistics, IEEE Transactions on Signal Processing, 434–444.

Miettinen, J., Nordhausen, K., Oja, H. and Taskinen, S. (2014), Deflation-based Separation of Uncorrelated Stationary Time Series, Journal of Multivariate Analysis, 123, 214–227.

Miettinen, J., Illner, K., Nordhausen, K., Oja, H., Taskinen, S. and Theis, F.J. (2016), Separation of Uncorrelated Stationary Time Series Using Autocovariance Matrices, Journal of Time Series Analysis, 37, 337–354.

Miettinen, J., Nordhausen, K. and Taskinen, S. (2017), Blind Source Separation Based on Joint Diagonalization in R: The Packages JADE and BSSasyp, Journal of Statistical Software, 76, 1–31, <doi:10.18637/jss.v076.i02>.

See Also

[ts](#)

Examples

```
# creating some toy data
A<- matrix(rnorm(9),3,3)
s1 <- arima.sim(list(ar=c(0.3,0.6)),1000)
s2 <- arima.sim(list(ma=c(-0.3,0.3)),1000)
s3 <- arima.sim(list(ar=c(-0.8,0.1)),1000)

S <- cbind(s1,s2,s3)
X <- S %*% t(A)

res1<-SOBI(X)
res1
coef(res1)
plot(res1) # compare to plot.ts(S)
MD(coef(res1),A)

# input of a time series
X2<- ts(X, start=c(1961, 1), frequency=12)
plot(X2)
res2<-SOBI(X2, k=c(5,10,1,4,2,9,10))
plot(res2)
```

Index

- *Topic **array**
 - [cjd](#), [7](#)
 - [djd](#), [11](#)
 - [FG](#), [13](#)
 - [rjd](#), [28](#)
- *Topic **datasets**
 - [CPPdata](#), [10](#)
- *Topic **methods**
 - [coef.bss](#), [8](#)
 - [plot.bss](#), [27](#)
 - [print.bss](#), [28](#)
- *Topic **multivariate**
 - [amari.error](#), [3](#)
 - [AMUSE](#), [4](#)
 - [bss.components](#), [6](#)
 - [ComonGAP](#), [9](#)
 - [FOBI](#), [14](#)
 - [JADE](#), [15](#)
 - [k_JADE](#), [17](#)
 - [MD](#), [19](#)
 - [multscatter](#), [20](#)
 - [NSS.JD](#), [21](#)
 - [NSS.SD](#), [23](#)
 - [NSS.TD.JD](#), [25](#)
 - [SIR](#), [30](#)
 - [SOBI](#), [31](#)
- *Topic **package**
 - [JADE-package](#), [2](#)
- *Topic **ts**
 - [AMUSE](#), [4](#)
 - [NSS.JD](#), [21](#)
 - [NSS.SD](#), [23](#)
 - [NSS.TD.JD](#), [25](#)
 - [SOBI](#), [31](#)
- [amari.error](#), [3](#), [9](#), [20](#), [31](#)
- [AMUSE](#), [4](#)
- [bss.components](#), [6](#)
- [cjd](#), [7](#)
- [coef.bss](#), [8](#)
- [ComonGAP](#), [4](#), [9](#), [20](#), [31](#)
- [CPPdata](#), [10](#)
- [djd](#), [11](#), [32](#)
- [FG](#), [13](#)
- [FOBI](#), [14](#), [17](#), [18](#)
- [frjd](#), [15](#), [17](#), [18](#), [21](#), [25](#), [32](#)
- [frjd\(rjd\)](#), [28](#)
- [ics](#), [14](#), [15](#)
- [JADE](#), [3](#), [9](#), [15](#), [17–19](#)
- [JADE-package](#), [2](#)
- [k_JADE](#), [17](#)
- [MD](#), [19](#)
- [multscatter](#), [20](#)
- [NSS.JD](#), [21](#), [24](#), [26](#)
- [NSS.SD](#), [22](#), [23](#)
- [NSS.TD.JD](#), [22](#), [24](#), [25](#)
- [pairs](#), [27](#)
- [plot](#), [27](#)
- [plot.bss](#), [27](#)
- [plot.ts](#), [27](#)
- [print.bss](#), [28](#)
- [rjd](#), [7](#), [14](#), [28](#), [32](#)
- [rjd.fortran](#), [7](#), [14](#)
- [SIR](#), [4](#), [9](#), [20](#), [30](#)
- [SOBI](#), [26](#), [31](#)
- [solve_LSAP](#), [20](#)
- [ts](#), [4](#), [5](#), [21–26](#), [32](#)