

Package ‘polyCub’

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Title Cubature over Polygonal Domains

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Description Numerical integration of continuously differentiable functions $f(x,y)$ over simple closed polygonal domains. The following cubature methods are implemented: product Gauss cubature (Sommariva and Vianello, 2007, <doi:10.1007/s10543-007-0131-2>), the simple two-dimensional midpoint rule (wrapping 'spatstat' functions), adaptive cubature for radially symmetric functions via line integrate() along the polygon boundary (Meyer and Held, 2014, <doi:10.1214/14-AOAS743>, Supplement B), and integration of the bivariate Gaussian density based on polygon triangulation. For simple integration along the axes, the 'cubature' package is more appropriate.

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URL <https://github.com/bastistician/polyCub>

BugReports <https://github.com/bastistician/polyCub/issues>

Depends R (>= 2.15.0), methods

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Author Sebastian Meyer [aut, cre, trl]
(<<https://orcid.org/0000-0002-1791-9449>>),
Leonhard Held [ths],
Michael Hoehle [ths]

Maintainer Sebastian Meyer <seb.meyer@fau.de>

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polyCub-package	<i>Cubature over Polygonal Domains</i>
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Description

The R package **polyCub** implements *cubature* (numerical integration) over *polygonal* domains. It solves the problem of integrating a continuously differentiable function $f(x, y)$ over simple closed polygons.

Details

polyCub provides the following cubature methods, which can either be called explicitly or via the generic `polyCub` function:

polyCub.SV: General-purpose *product Gauss cubature* (Sommariva and Vianello, 2007)

polyCub.midpoint: Simple *two-dimensional midpoint rule* based on `as.im.function` from **spatstat** (Baddeley and Turner, 2005)

polyCub.iso: Adaptive cubature for *radially symmetric functions* via line `integrate()` along the polygon boundary (Meyer and Held, 2014, Supplement B, Section 2.4).

polyCub.exact.Gauss: Accurate (but slow) integration of the *bivariate Gaussian density* based on polygon triangulation (via `tristrip` from **gpclib**) and (numerous) evaluations of cumulative densities (via `pmvnorm` from package **mvtnorm**). Note that there is also a function `circleCub.Gauss` to integrate the *isotropic Gaussian density* over a *circular domain*.

A more detailed description and benchmark experiment of the above cubature methods can be found in the vignette("polyCub") and in Meyer (2010, Section 3.2).

References

Abramowitz, M. and Stegun, I. A. (1972). Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. New York: Dover Publications.

Baddeley, A. and Turner, R. (2005). **spatstat**: an R package for analyzing spatial point patterns. *Journal of Statistical Software*, **12** (6), 1-42.

Meyer, S. (2010). Spatio-Temporal Infectious Disease Epidemiology based on Point Processes. Master's Thesis, LMU Munich. Available as <http://epub.ub.uni-muenchen.de/11703/>.

Meyer, S. and Held, L. (2014). Power-law models for infectious disease spread. *The Annals of Applied Statistics*, **8** (3), 1612-1639.

DOI-Link: <https://doi.org/10.1214/14-AOAS743>, [arXiv:1308.5115](https://arxiv.org/abs/1308.5115)

Sommariva, A. and Vianello, M. (2007). Product Gauss cubature over polygons based on Green's integration formula. *BIT Numerical Mathematics*, **47** (2), 441-453.

DOI-Link: <https://doi.org/10.1007/s10543-007-0131-2>

See Also

vignette("polyCub")

For the special case of a rectangular domain along the axes (e.g., a bounding box), the **cubature** package is more appropriate.

checkintrfr

Check the Integral of $r f_r(r)$

Description

This function is auxiliary to [polyCub.iso](#). The (analytical) integral of $r f_r(r)$ from 0 to R is checked against a numeric approximation using [integrate](#) for various values of the upper bound R . A warning is issued if inconsistencies are found.

Usage

```
checkintrfr(intrfr, f, ..., center, control = list(), rs = numeric(0L),
  tolerance = control$rel.tol)
```

Arguments

intrfr a function(R, \dots), which implements the (analytical) antiderivative of $r f_r(r)$ from 0 to R . The first argument must be vectorized but not necessarily named R . If **intrfr** is missing, it will be approximated numerically via [integrate](#)(function(r, \dots) $r * f(\text{cbind}(x0 + r, y0), \dots)$, 0, R, \dots , control=control), where $c(x0, y0)$ is the center of isotropy. Note that f will *not* be checked for isotropy.

f	a two-dimensional real-valued function. As its first argument it must take a coordinate matrix, i.e., a numeric matrix with two columns, and it must return a numeric vector of length the number of coordinates.
...	further arguments for f or intrfr.
center	numeric vector of length 2, the center of isotropy.
control	list of arguments passed to integrate , the quadrature rule used for the line integral along the polygon boundary.
rs	numeric vector of upper bounds for which to check the validity of intrfr. If it has length 0 (default), no checks are performed.
tolerance	of all.equal.numeric when comparing intrfr results with numerical integration. Defaults to the relative tolerance used for integrate.

Value

The intrfr function. If it was not supplied, its quadrature version using integrate is returned.

circleCub.Gauss	<i>Integration of the Isotropic Gaussian Density over Circular Domains</i>
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Description

This function calculates the integral of the bivariate, isotropic Gaussian density (i.e., $\Sigma = \text{sd}^2 \cdot \text{diag}(2)$) over a circular domain via the cumulative distribution function `pchisq` of the (non-central) Chi-Squared distribution (Abramowitz and Stegun, 1972, Formula 26.3.24).

Usage

```
circleCub.Gauss(center, r, mean, sd)
```

Arguments

center	numeric vector of length 2 (center of the circle).
r	numeric (radius of the circle). Several radii may be supplied.
mean	numeric vector of length 2 (mean of the bivariate Gaussian density).
sd	numeric (common standard deviation of the isotropic Gaussian density in both dimensions).

Value

The integral value (one for each supplied radius).

Note

The non-centrality parameter of the evaluated chi-squared distribution equals the squared distance between the mean and the center. If this becomes too large, the result becomes inaccurate, see [pchisq](#).

References

Abramowitz, M. and Stegun, I. A. (1972). Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. New York: Dover Publications.

Examples

```
circleCub.Gauss(center=c(1,2), r=3, mean=c(4,5), sd=6)

if (requireNamespace("mvtnorm") && gpclibPermit() && requireNamespace("spatstat")) {
  ## compare with cubature over a polygonal approximation of a circle
  disc.poly <- spatstat::disc(radius=3, centre=c(1,2), npoly=32)
  polyCub.exact.Gauss(disc.poly, mean=c(4,5), Sigma=6^2*diag(2))
}
```

coerce-gpc-methods *Conversion between polygonal "owin" and "gpc.poly"*

Description

Package **polyCub** implements converters between the classes "owin" of package **spatstat** and "gpc.poly" of package **rgeos** (originally from **gpclib**). Support for the "gpc.poly" class was dropped from **spatstat** as of version 1.34-0.

Usage

```
owin2gpc(object)

gpc2owin(object, ...)

as.owin.gpc.poly(W, ...)
```

Arguments

object	an object of class "gpc.poly" or "owin", respectively.
...	further arguments passed to <code>owin</code> .
W	an object of class "gpc.poly".

Value

The converted polygon of class "gpc.poly" or "owin", respectively. If neither package **rgeos** nor **gpclib** are available, `owin2gpc` will just return the `pts` slot of the "gpc.poly" (no formal class) with a warning.

Note

The converter `owin2gpc` requires the package **rgeos** (or **gpclib**) for the formal class definition of a "gpc.poly". It will produce vertices ordered according to the **sp** convention, i.e. clockwise for normal boundaries and anticlockwise for holes, where, however, the first vertex is *not* repeated!

Author(s)

Sebastian Meyer

See Also

[xylist](#), and the package **rgeos** for conversions of "gpc.poly" objects from and to **sp**'s "SpatialPolygons" class.

Examples

```
if (gpclibPermit() && require("spatstat")) {
  ## use example polygons from
  example(plotpolyf, ask = FALSE)

  letterR # a simple "xylist"
  letterR.owin <- owin(poly = letterR)
  letterR.gpc_from_owin <- owin2gpc(letterR.owin)
  letterR.xylist_from_gpc <- xylist(letterR.gpc_from_owin)
  stopifnot(all.equal(letterR, lapply(letterR.xylist_from_gpc, "[", 1:2)))
  letterR.owin_from_gpc <- as.owin(letterR.gpc_from_owin)
  stopifnot(all.equal(letterR.owin, letterR.owin_from_gpc))
}
```

 coerce-sp-methods

 Coerce "SpatialPolygons" to "owin"

Description

Package **polyCub** implements coerce-methods (`as(object, Class)`) to convert "SpatialPolygons" (or "Polygons" or "Polygon") to "owin". They are also available as `as.owin.*` functions to support `polyCub.midpoint`. However, these are no registered S3 methods for `as.owin`, since package **spatstat** is optional. Note that the **maptools** package contains an alternative implementation of coercion from "SpatialPolygons" to "owin" (and reverse), and **R** will use the S4 coerce-method that was loaded last, and prefer the `as.owin.SpatialPolygons` S3-method exported from **maptools** if attached.

Usage

```
as.owin.SpatialPolygons(W, ...)
```

```
as.owin.Polygons(W, ...)
```

```
as.owin.Polygon(W, ...)
```

Arguments

W an object of class "SpatialPolygons", "Polygons", or "Polygon".
... further arguments passed to `owin`.

Author(s)

Sebastian Meyer

Examples

```

if (require("spatstat") && require("sp")) {
  diamond <- list(x = c(1,2,1,0), y = c(1,2,3,2)) # anti-clockwise
  diamond.owin <- owin(poly = diamond)
  diamond.sp <- Polygon(lapply(diamond, rev))      # clockwise
  diamond.owin_from_sp <- as(diamond.sp, "owin")
  stopifnot(all.equal(diamond.owin, diamond.owin_from_sp))

  ## similarly works for Polygons and SpatialPolygons
  diamond.Ps <- as(diamond.sp, "Polygons")
  stopifnot(identical(diamond.owin, as.owin(diamond.Ps)))
  diamond.SpPs <- SpatialPolygons(list(diamond.Ps))
  stopifnot(identical(diamond.owin, as.owin(diamond.SpPs)))
}

```

gpplibPermit

gpplib License Acceptance**Description**

Similar to the handling in package **maptools**, these functions explicitly accept the restricted **gpplib** license (commercial use prohibited) and report its acceptance status, respectively. **gpplib** functionality is only required for [polyCub.exact.Gauss](#).

Usage

```

gpplibPermit()

gpplibPermitStatus()

```

plotpolyf

*Plot Polygonal Domain on Image of Bivariate Function***Description**

Produces a combined plot of a polygonal domain and an image of a bivariate function, using either [lattice::levelplot](#) or [image](#).

Usage

```

plotpolyf(polyregion, f, ..., npixel = 100, cuts = 15,
  col = rev(heat.colors(cuts + 1)), lwd = 3, xlim = NULL,
  ylim = NULL, use.lattice = TRUE, print.args = list())

```

Arguments

polyregion	a polygonal domain. The following classes are supported: "owin" from package spatstat , "gpc.poly" from rgeos (or gpclib), as well as "SpatialPolygons", "Polygons", and "Polygon" from package sp . (For these classes, polyCub knows how to get an xylist .)
f	a two-dimensional real-valued function. As its first argument it must take a coordinate matrix, i.e., a numeric matrix with two columns, and it must return a numeric vector of length the number of coordinates.
...	further arguments for f.
npixel	numeric vector of length 1 or 2 setting the number of pixels in each dimension.
cuts	number of cut points in the <i>z</i> dimension. The range of function values will be divided into cuts+1 levels.
col	color vector used for the function levels.
lwd	line width of the polygon edges.
xlim, ylim	numeric vectors of length 2 setting the axis limits. NULL means using the bounding box of polyregion.
use.lattice	logical indicating if lattice graphics (levelplot) should be used.
print.args	a list of arguments passed to print.trellis for plotting the produced " trellis " object (given use.lattice = TRUE). The latter will be returned without explicit printing if print.args is not a list.

Author(s)

Sebastian Meyer

Examples

```
### a polygonal domain (a rounded version of spatstat.data::letterR$bdry)
letterR <- list(
  list(x = c(3.9, 3.8, 3.7, 3.5, 3.4, 3.5, 3.7, 3.8, 3.8, 3.7,
            3.7, 3.5, 3.3, 2, 2, 2.7, 2.7, 2.9, 3, 3.3, 3.9),
       y = c(0.7, 1.1, 1.3, 1.7, 1.8, 1.9, 2.1, 2.3, 2.5, 2.8, 3,
            3.2, 3.3, 3.3, 0.7, 0.7, 1.7, 1.7, 1.5, 0.7, 0.6)),
  list(x = c(2.6, 2.6, 3, 3.1, 3.2, 3.1, 3.1, 3),
       y = c(2.2, 2.7, 2.7, 2.6, 2.5, 2.4, 2.3, 2.2))
)

### f: isotropic exponential decay
fr <- function(r, rate = 1) dexp(r, rate = rate)
fcenter <- c(2,3)
f <- function (s, rate = 1) fr(sqrt(rowSums(t(t(s)-fcenter)^2)), rate = rate)

### plot
plotpolyf(letterR, f, use.lattice = FALSE)
plotpolyf(letterR, f, use.lattice = TRUE)
```

plot_polyregion	<i>Plots a Polygonal Domain (of Various Classes)</i>
-----------------	--

Description

Plots a Polygonal Domain (of Various Classes)

Usage

```
plot_polyregion(polyregion, lwd = 2, add = FALSE)
```

Arguments

polyregion	a polygonal domain. The following classes are supported: "owin" from package spatstat , "gpc.poly" from rgeos (or gpclib), as well as "SpatialPolygons", "Polygons", and "Polygon" from package sp . (For these classes, polyCub knows how to get an xylist .)
lwd	line width of the polygon edges.
add	logical. Add to existing plot?

polyCub	<i>Wrapper Function for the Various Cubature Methods</i>
---------	--

Description

The wrapper function polyCub can be used to call specific cubature methods via its method argument. It calls `polyCub.SV` by default, which implements general-purpose product Gauss cubature.

Usage

```
polyCub(polyregion, f, method = c("SV", "midpoint", "iso",
  "exact.Gauss"), ..., plot = FALSE)
```

Arguments

polyregion	a polygonal domain. The following classes are supported: "owin" from package spatstat , "gpc.poly" from rgeos (or gpclib), as well as "SpatialPolygons", "Polygons", and "Polygon" from package sp . (For these classes, polyCub knows how to get an xylist .)
f	a two-dimensional real-valued function to be integrated over polyregion. As its first argument it must take a coordinate matrix, i.e., a numeric matrix with two columns, and it must return a numeric vector of length the number of coordinates. For the "exact.Gauss" method, f is ignored since it is specific to the bivariate normal density.

method	choose one of the implemented cubature methods (partial argument matching is applied), see <code>help("polyCub-package")</code> for an overview. Defaults to using product Gauss cubature implemented in <code>polyCub.SV</code> .
...	arguments of <code>f</code> or of the specific method.
plot	logical indicating if an illustrative plot of the numerical integration should be produced.

Value

The approximated integral of `f` over `polyregion`.

See Also

Details and examples in the vignette("polyCub") and on the method-specific help pages.

Other polyCub-methods: `polyCub.SV`, `polyCub.exact.Gauss`, `polyCub.iso`, `polyCub.midpoint`

`polyCub.exact.Gauss` *Quasi-Exact Cubature of the Bivariate Normal Density*

Description

The bivariate Gaussian density can be integrated based on a triangulation of the (transformed) polygonal domain, using formulae from the Abramowitz and Stegun (1972) handbook (Section 26.9, Example 9, pp. 956f.). This method is quite cumbersome because the A&S formula is only for triangles where one vertex is the origin (0,0). For each triangle of the `tristrip` we have to check in which of the 6 outer regions of the triangle the origin (0,0) lies and adapt the signs in the formula appropriately: $(AOB + BOC - AOC)$ or $(AOB - AOC - BOC)$ or $(AOB + AOC - BOC)$ or $(AOC + BOC - AOB)$ or However, the most time consuming step is the evaluation of `pmvnorm`.

Usage

```
polyCub.exact.Gauss(polyregion, mean = c(0, 0), Sigma = diag(2),
  plot = FALSE)
```

Arguments

polyregion	a " <code>gpc.poly</code> " polygon or something that can be coerced to this class, e.g., an "owin" polygon (converted via <code>owin2gpc</code> and – given <code>rgeos</code> is available – "SpatialPolygons" also work.
mean, Sigma	mean and covariance matrix of the bivariate normal density to be integrated.
plot	logical indicating if an illustrative plot of the numerical integration should be produced. Note that the <code>polyregion</code> will be transformed (shifted and scaled).

Description

polyCub.iso numerically integrates a radially symmetric function $f(x, y) = f_r(\|(x, y) - \mu\|)$, with μ being the center of isotropy, over a polygonal domain. It internally approximates a line integral along the polygon boundary using `integrate`. The integrand requires the antiderivative of $r f_r(r)$, which should be supplied as argument `intrfr` (f itself is only required if `check.intrfr=TRUE`). The two-dimensional integration problem thereby reduces to an efficient adaptive quadrature in one dimension. If `intrfr` is not available analytically, polyCub.iso can use a numerical approximation (meaning `integrate` within `integrate`), but the general-purpose cubature method `polyCub.SV` might be more efficient in this case. See Meyer and Held (2014, Supplement B, Section 2.4) for mathematical details.

`.polyCub.iso` is a “bare-bone” version of `polyCub.iso`.

Usage

```
polyCub.iso(polyregion, f, intrfr, ..., center, control = list(),
  check.intrfr = FALSE, plot = FALSE)
```

```
.polyCub.iso(polys, intrfr, ..., center, control = list(),
  .witherror = FALSE)
```

Arguments

polyregion	a polygonal domain. The following classes are supported: “ <code>owin</code> ” from package <code>spatstat</code> , “ <code>gpc.poly</code> ” from <code>rgeos</code> (or <code>gpclib</code>), as well as “ <code>SpatialPolygons</code> ”, “ <code>Polygons</code> ”, and “ <code>Polygon</code> ” from package <code>sp</code> . (For these classes, <code>polyCub</code> knows how to get an <code>xylist</code> .)
f	a two-dimensional real-valued function. As its first argument it must take a coordinate matrix, i.e., a numeric matrix with two columns, and it must return a numeric vector of length the number of coordinates.
intrfr	a function(<code>R, ...</code>), which implements the (analytical) antiderivative of $r f_r(r)$ from 0 to <code>R</code> . The first argument must be vectorized but not necessarily named <code>R</code> . If <code>intrfr</code> is missing, it will be approximated numerically via <code>integrate(function(r, ...) r * f(cbind(x0 + r, y0), ...), 0, R, ..., control=control)</code> , where <code>c(x0, y0)</code> is the center of isotropy. Note that <code>f</code> will <i>not</i> be checked for isotropy.
...	further arguments for <code>f</code> or <code>intrfr</code> .
center	numeric vector of length 2, the center of isotropy.
control	list of arguments passed to <code>integrate</code> , the quadrature rule used for the line integral along the polygon boundary.

check.intrfr	logical (or numeric vector) indicating if (for which r's) the supplied intrfr function should be checked against a numeric approximation. This check requires f to be specified. If TRUE, the set of test r's defaults to a <code>seq</code> of length 20 from 1 to the maximum absolute x or y coordinate of any edge of the polyregion.
plot	logical indicating if an image of the function should be plotted together with the polygonal domain, i.e., <code>plotpolyf(polyregion, f, ...)</code> .
polys	something like <code>owin\$bdry</code> , but see <code>xylist</code> .
.witherror	logical indicating if an upper bound for the absolute integration error should be attached as an attribute to the result?

Value

The approximate integral of the isotropic function `f` over `polyregion`.

If the `intrfr` function is provided (which is assumed to be exact), an upper bound for the absolute integration error is appended to the result as attribute `"abs.error"`. It equals the sum of the absolute errors reported by all `integrate` calls (there is one for each edge of `polyregion`).

Author(s)

Sebastian Meyer

The basic mathematical formulation of this efficient integration for radially symmetric functions was ascertained with great support by Emil Hedevang (2013), Dept. of Mathematics, Aarhus University, Denmark.

References

Hedevang, E. (2013). Personal communication at the Summer School on Topics in Space-Time Modeling and Inference (May 2013, Aalborg, Denmark).

Meyer, S. and Held, L. (2014). Power-law models for infectious disease spread. *The Annals of Applied Statistics*, **8** (3), 1612-1639.

DOI-Link: <https://doi.org/10.1214/14-AOAS743>, [arXiv:1308.5115](https://arxiv.org/abs/1308.5115)

See Also

`system.file("include", "polyCubAPI.h", package = "polyCub")` for a full C-implementation of this cubature method (for a *single* polygon). The corresponding C-routine `polyCub_iso` can be used by other R packages, notably **surveillance**, via `'LinkingTo: polyCub'` (in the `'DESCRIPTION'`) and `'#include <polyCubAPI.h>'` (in suitable `'/src'` files). Note that the `intrfr` function must then also be supplied as a C-routine. An example can be found in the package tests.

Other `polyCub`-methods: `polyCub.SV`, `polyCub.exact.Gauss`, `polyCub.midpoint`, `polyCub`

Examples

```
## we use the example polygon and f (exponential decay) from
example(plotpolyf)
```

```
## numerical approximation of 'intrfr' (not recommended)
(intISOnum <- polyCub.iso(letterR, f, center = fcenter))
```

```

## analytical 'intrfr'
## intrfr(R) = int_0^R r*f(r) dr, for f(r) = dexp(r), gives
intrfr <- function (R, rate = 1) pgamma(R, 2, rate) / rate
(intISOana <- polyCub.iso(letterR, f, intrfr = intrfr, center = fcenter,
                        check.intrfr = TRUE))
## f is only used to check 'intrfr' against a numerical approximation

stopifnot(all.equal(intISOana, intISOnum, check.attributes = FALSE))

### polygon area: f(r) = 1, f(x,y) = 1, center does not really matter

## intrfr(R) = int_0^R r*f(r) dr = int_0^R r dr = R^2/2
intrfr.const <- function (R) R^2/2
(area.ISO <- polyCub.iso(letterR, intrfr = intrfr.const, center = c(0,0)))

if (require("spatstat")) { # check against area.owin()
  stopifnot(all.equal(area.owin(owin(poly = letterR)),
                    area.ISO, check.attributes = FALSE))
}

```

polyCub.midpoint

Two-Dimensional Midpoint Rule

Description

The surface is converted to a binary pixel image using the `as.im.function` method from package **spatstat** (Baddeley and Turner, 2005). The integral under the surface is then approximated as the sum over (pixel area * f(pixel midpoint)).

Usage

```
polyCub.midpoint(polyregion, f, ..., eps = NULL, dimyx = NULL,
                 plot = FALSE)
```

Arguments

polyregion	a polygonal integration domain. It can be any object coercible to the spatstat class "owin" via a corresponding <code>as.owin</code> -method. Note that this includes polygons of the classes "gpc.poly" and "SpatialPolygons", because polyCub defines methods <code>as.owin.gpc.poly</code> and <code>as.owin.SpatialPolygons</code> , respectively.
f	a two-dimensional real-valued function. As its first argument it must take a coordinate matrix, i.e., a numeric matrix with two columns, and it must return a numeric vector of length the number of coordinates.
...	further arguments for f.
eps	width and height of the pixels (squares), see <code>as.mask</code> .

dimyx number of subdivisions in each dimension, see [as.mask](#).
 plot logical indicating if an illustrative plot of the numerical integration should be produced.

Value

The approximated value of the integral of f over polyregion.

References

Baddeley, A. and Turner, R. (2005). **spatstat**: an R package for analyzing spatial point patterns. *Journal of Statistical Software*, **12** (6), 1-42.

See Also

Other polyCub-methods: [polyCub.SV](#), [polyCub.exact.Gauss](#), [polyCub.iso](#), [polyCub](#)

Examples

```
## a function to integrate (here: isotropic zero-mean Gaussian density)
f <- function(s, sigma = 5)
  exp(-rowSums(s^2)/2/sigma^2) / (2*pi*sigma^2)

## a simple polygon as integration domain
hexagon <- list(
  list(x = c(7.33, 7.33, 3, -1.33, -1.33, 3),
       y = c(-0.5, 4.5, 7, 4.5, -0.5, -3))
)

if (require("spatstat")) {
  hexagon.owin <- owin(poly = hexagon)

  show_midpoint <- function(eps)
  {
    plotpolyf(hexagon.owin, f, xlim = c(-8,8), ylim = c(-8,8),
              use.lattice = FALSE)
    ## add evaluation points to plot
    with(as.mask(hexagon.owin, eps = eps),
         points(expand.grid(xcol, yrow), col = t(m), pch = 20))
    title(main = paste("2D midpoint rule with eps =", eps))
  }

  ## show nodes (eps = 0.5)
  show_midpoint(0.5)

  ## show pixel image (eps = 0.5)
  polyCub.midpoint(hexagon.owin, f, eps = 0.5, plot = TRUE)

  ## use a decreasing pixel size (increasing number of nodes)
  for (eps in c(5, 3, 1, 0.5, 0.3, 0.1))
    cat(sprintf("eps = %.1f: %.7f\n", eps,
               polyCub.midpoint(hexagon.owin, f, eps = eps)))
}
```

}

polyCub.SV

*Product Gauss Cubature over Polygonal Domains***Description**

Product Gauss cubature over polygons as proposed by Sommariva and Vianello (2007).

Usage

```
polyCub.SV(polyregion, f, ..., nGQ = 20, alpha = NULL,
           rotation = FALSE, engine = "C", plot = FALSE)
```

Arguments

polyregion	a polygonal domain. The following classes are supported: "owin" from package spatstat , "gpc.poly" from rgeos (or gpclib), as well as "SpatialPolygons", "Polygons", and "Polygon" from package sp . (For these classes, polyCub knows how to get an xylist .)
f	a two-dimensional real-valued function to be integrated over polyregion (or NULL to only compute nodes and weights). As its first argument it must take a coordinate matrix, i.e., a numeric matrix with two columns, and it must return a numeric vector of length the number of coordinates.
...	further arguments for f.
nGQ	degree of the one-dimensional Gauss-Legendre quadrature rule (default: 20) as implemented in function gauss.quad of package statmod . Nodes and weights up to nGQ=60 are cached in polyCub , for larger degrees statmod is required.
alpha	base-line of the (rotated) polygon at $x = \alpha$ (see Sommariva and Vianello (2007) for an explication). If NULL (default), the midpoint of the x-range of each polygon is chosen if no rotation is performed, and otherwise the x-coordinate of the rotated point "P" (see rotation). If f has its maximum value at the origin (0,0), e.g., the bivariate Gaussian density with zero mean, $\alpha = 0$ is a reasonable choice.
rotation	logical (default: FALSE) or a list of points "P" and "Q" describing the preferred direction. If TRUE, the polygon is rotated according to the vertices "P" and "Q", which are farthest apart (see Sommariva and Vianello, 2007). For convex polygons, this rotation guarantees that all nodes fall inside the polygon.
engine	character string specifying the implementation to use. Up to polyCub version 0.4-3, the two-dimensional nodes and weights were computed by R functions and these are still available by setting <code>engine = "R"</code> . The new C-implementation is now the default (<code>engine = "C"</code>) and requires approximately 30% less computation time. The special setting <code>engine = "C+reduce"</code> will discard redundant nodes at (0,0) with zero weight resulting from edges on the base-line $x = \alpha$ or orthogonal to it.

This extra cleaning is only worth its cost for computationally intensive functions f over polygons which really have some edges on the baseline or parallel to the x -axis. Note that the old R implementation does not have such unset zero nodes and weights.

`plot` logical indicating if an illustrative plot of the numerical integration should be produced.

Value

The approximated value of the integral of f over `polyregion`.
 In the case $f = \text{NULL}$, only the computed nodes and weights are returned in a list of length the number of polygons of `polyregion`, where each component is a list with nodes (a numeric matrix with two columns), weights (a numeric vector of length `nrow(nodes)`), the rotation angle, and `alpha`.

Author(s)

Sebastian Meyer

These R and C implementations of product Gauss cubature are based on the original MATLAB implementation `polygauss` by Sommariva and Vianello (2007), which is available under the GNU GPL (≥ 2) license from <http://www.math.unipd.it/~alvise/software.html>.

References

Sommariva, A. and Vianello, M. (2007): Product Gauss cubature over polygons based on Green's integration formula. *BIT Numerical Mathematics*, **47** (2), 441-453.
 DOI-Link: <https://doi.org/10.1007/s10543-007-0131-2>

See Also

Other `polyCub`-methods: [polyCub.exact.Gauss](#), [polyCub.iso](#), [polyCub.midpoint](#), [polyCub](#)

Examples

```
## a function to integrate (here: isotropic zero-mean Gaussian density)
f <- function (s, sigma = 5)
  exp(-rowSums(s^2)/2/sigma^2) / (2*pi*sigma^2)

## a simple polygon as integration domain
hexagon <- list(
  list(x = c(7.33, 7.33, 3, -1.33, -1.33, 3),
       y = c(-0.5, 4.5, 7, 4.5, -0.5, -3))
)

## image of the function and integration domain
plotpolyf(hexagon, f, xlim = c(-8,8), ylim = c(-8,8))

## use a degree of nGQ = 3 and show the corresponding nodes
polyCub.SV(hexagon, f, nGQ = 3, plot = TRUE)

## extract nodes and weights
```

```

nw <- polyCub.SV(hexagon, f = NULL, nGQ = 3)[[1]]
nrow(nw$nodes)

## manually apply the cubature rule
sum(nw$weights * f(nw$nodes))

## use an increasing number of nodes
for (nGQ in c(1:5, 10, 20, 60))
  cat(sprintf("nGQ = %2i: %.16f\n", nGQ,
             polyCub.SV(hexagon, f, nGQ = nGQ)))

## polyCub.SV() is the default method used by the polyCub() wrapper
polyCub(hexagon, f, nGQ = 3) # calls polyCub.SV()

### now using a simple *rectangular* integration domain

rectangle <- list(list(x = c(-1, 7, 7, -1), y = c(-3, -3, 7, 7)))

## try rotation (may improve accuracy)
opar <- par(mfrow = c(1,3))
polyCub.SV(rectangle, f, nGQ = 4, rotation = FALSE, plot = TRUE)
  title(main = "without rotation")
polyCub.SV(rectangle, f, nGQ = 4, rotation = TRUE, plot = TRUE)
  title(main = "default rotation")
polyCub.SV(rectangle, f, nGQ = 4,
           rotation = list(P = c(0,0), Q = c(2,-3)), plot = TRUE)
  title(main = "custom rotation")
par(opar)

## comparison with cubature::adaptIntegrate()
if (require("cubature")) {
  fc <- function (s, sigma = 5)
    exp(-sum(s^2)/2/sigma^2) / (2*pi*sigma^2)
  adaptIntegrate(f = fc, lowerLimit = c(-1, -3), upperLimit = c(7, 7))
}

```

Description

Different packages concerned with spatial data use different polygon specifications, which sometimes becomes very confusing (see Details below). To be compatible with the various polygon classes, package **polyCub** uses an S3 class "xylist", which represents polygons by their core feature only, a list of lists of vertex coordinates (see the "Value" section below). The generic function xylist can deal with the following polygon classes:

- "owin" from package **spatstat**
- "gpc.poly" from package **rgeos** (or **gpclib**)

- "Polygons" from package **sp** (as well as "Polygon" and "SpatialPolygons")

The (somehow useless) default xylist-method does not perform any transformation but only ensures that the polygons are not closed (first vertex not repeated).

Usage

```
xylist(object, ...)

## S3 method for class 'owin'
xylist(object, ...)

## S3 method for class 'gpc.poly'
xylist(object, ...)

## S3 method for class 'SpatialPolygons'
xylist(object, reverse = TRUE, ...)

## S3 method for class 'Polygons'
xylist(object, reverse = TRUE, ...)

## S3 method for class 'Polygon'
xylist(object, reverse = TRUE, ...)

## Default S3 method:
xylist(object, ...)
```

Arguments

object	an object of one of the supported spatial classes.
...	(unused) argument of the generic.
reverse	logical (TRUE) indicating if the vertex order of the sp classes should be reversed to get the xylist/owin convention.

Details

Different packages concerned with spatial data use different polygon specifications with respect to:

- do we repeat the first vertex?
- which direction represents holes?

Package overview:

sp: *Repeat* first vertex at the end (closed), anticlockwise = hole, clockwise = normal boundary

spatstat: do *not repeat* first vertex, anticlockwise = normal boundary, clockwise = hole. This convention is also used in xylist.

gpclib: Unfortunately, there seems to be no convention for the specification of polygons of class "gpc.poly".

Value

Applying `xylist` to a polygon object, one gets a simple list, where each component (polygon) is a list of "x" and "y" coordinates. These represent vertex coordinates following **spatstat**'s "owin" convention (anticlockwise order without repeating any vertex). The opposite vertex order can be retained for the **sp**-classes by the non-default use with `reverse=FALSE`.

Author(s)

Sebastian Meyer

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