

Package ‘mlergm’

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Title Multilevel Exponential-Family Random Graph Models

Version 0.3

Description Estimates exponential-family random graph models for multilevel network data, assuming the multilevel structure is observed. The scope, at present, covers multilevel models where the set of nodes is nested within known blocks. The estimation method uses Monte-Carlo maximum likelihood estimation (MCMLE) methods to estimate a variety of canonical or curved exponential family models for binary random graphs. MCMLE methods for curved exponential-family random graph models can be found in Hunter and Handcock (2006) <DOI: 10.1198/106186006X133069>. The package supports parallel computing, and provides methods for assessing goodness-of-fit of models and visualization of networks.

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classes	<i>Polish school classes data set.</i>
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Description

The Polish school classes data set `classes` is a subset of a larger data set which was generated as part of a Polish study on adolescent youth. The network data, obtained via a nomination processes, results in a binary, directed random graph where a directed edge from i to j indicates that student i nominated student j as a playmate. A further description of the data as well as a demonstration of an analysis with curved ERGMs can be found in Stewart, Schweinberger, Bojanowski, and Morris (2018).

Usage

```
data(classes)
```

Format

An `mlnet` object.

Details

A dataset containing network data for 9 school classes as part of a Polish educational study. The nodes of the network are students with nodal covariate `sex` and known class membership of the students.

References

Dolata, R. (ed). (2014). *Czy szkoła ma znaczenie? Zróżnicowanie wyników nauczania po pierwszym etapie edukacyjnym oraz jego pozaszkolne i szkolne uwarunkowania*. Vol. 1. Warsaw: Instytut Badań Edukacyjnych.

Dolata, R. and Rycielski, P. (2014). Wprowadzenie: założenia i cele badania szkolnych uwarunkowań efektywności kształcenia SUEK.

Stewart, J., Schweinberger, M., Bojanowski, M., and M. Morris (2019). Multilevel network data facilitate statistical inference for curved ERGMs with geometrically weighted terms. *Social Networks*, to appear.

`gof.mlergm`*Evaluate the goodness-of-fit of an estimated model.*

Description

Performs a Goodness-of-Fit procedure along the lines of Hunter, Goodreau, and Handcock (2008). Statistics are simulated from an fitted `mlergm` object, which can then be plotted and visualized. An example is given in the documentation of `mlergm`.

Usage

```
## S3 method for class 'mlergm'  
gof(object, ..., options = set_options(), seed = NULL,  
     gof_form = NULL)
```

Arguments

<code>object</code>	An object of class <code>mlergm</code> , likely produced by function <code>mlergm</code> .
<code>...</code>	Additional arguments to be passed if necessary.
<code>options</code>	See <code>set_options</code> for details.
<code>seed</code>	A seed to be provided to ensure reproducibility of results.
<code>gof_form</code>	A formula object of the form <code>~ term1 + term2 + ...</code> for statistics to compute for the GOF procedure.

Value

`gof.mlergm` returns an object of class `gof_mlergm` which is a list containing:

<code>obs_stats</code>	The GOF statistic values of the observed network.
<code>gof_stats</code>	The GOF statistic values simulated from the the estimated <code>mlergm</code> object provided.

References

Hunter, D. R., Goodreau, S. M., and Handcock, M. S. (2008). Goodness of fit of social network models. *Journal of the American Statistical Association*, 103(481), 248-258.

See Also

`plot.gof_mlergm`

is.gof_mlerngm	<i>Check if object is of class gof_mlerngm</i>
----------------	------------------------------------------------

Description

Function checks if a provided object is of class gof_mlerngm (see [gof.mlerngm](#) for details).

Usage

```
is.gof_mlerngm(x)
```

Arguments

x An object to be checked.

Value

TRUE if the provided object x is of class gof_mlerngm, FALSE otherwise.

See Also

[mlerngm](#), [gof.mlerngm](#)

is.mlerngm	<i>Check if the object is of class mlerngm</i>
------------	------------------------------------------------

Description

Function checks if a provided object is of class mlerngm (see [mlerngm](#) for details).

Usage

```
is.mlerngm(x)
```

Arguments

x An object to be checked.

Value

TRUE if the provided object x is of class mlerngm, FALSE otherwise.

See Also

[mlerngm](#)

is.mlnet	<i>Check if object is of class mlnet</i>
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Description

Function checks if a provided object is of class `mlnet` (see [mlnet](#) for details).

Usage

```
is.mlnet(x)
```

Arguments

`x` An object to be checked.

Value

TRUE if the provided object `x` is of class `mlnet`, FALSE otherwise.

See Also

[mlnet](#)

mloggm	<i>Multilevel Exponential-Family Random Graph Models</i>
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Description

This function estimates an exponential-family random graph model for multilevel network data. At present, `mloggm` covers network data where the set of nodes is nested within known blocks (see, e.g., Schweinberger and Handcock, 2015). An example is groups of students nested within classrooms, which is covered in the [classes](#) data set. It is assumed that the node membership, that to which block each node is associated, is known (or has been previously estimated).

Usage

```
mloggm(form, node_memb, parameterization = "standard",
        options = set_options(), theta_init = NULL, verbose = 0,
        eval_loglik = TRUE, seed = NULL)
```

```
## S3 method for class 'mloggm'
print(x, ...)
```

```
## S3 method for class 'mloggm'
summary(object, ...)
```

Arguments

form	Formula of the form: $\text{network} \sim \text{term1} + \text{term2} + \dots$; allowable model terms are a subset of those in R package <code>ergm</code> , see ergm.terms .
node_memb	Vector (length equal to the number of nodes in the network) indicating to which block or group the nodes belong. If the network provided in <code>form</code> is an object of class <code>m1net</code> , then <code>node_memb</code> can be extracted directly from the network and need not be provided.
parameterization	Parameterization options include 'standard', 'offset', or 'size'. <ul style="list-style-type: none"> • 'standard' : Does not adjust the individual block parameters for size. • 'offset' : The offset parameterization uses edge and mutual offsets along the lines of Krivitsky, Handcock, and Morris (2011) and Krivitsky and Kocaczyk (2015). The edge parameter is offset by $-\log n(k)$ and the mutual parameter is offset by $+\log n(k)$, where $n(k)$ is the size of the kth block. • 'size' : Multiplies the block parameters by $\log n(k)$, where $n(k)$ is the size of the kth block.
options	See set_options for details.
theta_init	Parameter vector of initial estimates for theta to be used.
verbose	Controls the level of output. A value of 0 corresponds to no output, except for warnings; a value of 1 corresponds to minimal output, and a value of 2 corresponds to full output.
eval_loglik	(Logical TRUE or FALSE) If set to TRUE, the bridge estimation procedure of Hunter and Handcock (2006) is used to estimate the loglikelihood for BIC calculations, otherwise the loglikelihood and therefore the BIC is not estimated.
seed	For reproducibility, an integer-valued seed may be specified.
x	An object of class <code>mlergm</code> , probably produced by mlergm .
...	Additional arguments to be passed if necessary.
object	An object of class <code>mlergm</code> , probably produced by mlergm .

Details

The estimation procedure performs Monte-Carlo maximum likelihood for the specified ERGM using a version of the Fisher scoring method detailed by Hunter and Handcock (2006). Settings governing the MCMC procedure (such as `burnin`, `interval`, and `sample_size`) as well as more general settings for the estimation procedure can be adjusted through [set_options](#). The estimation procedure uses the stepping algorithm of Hummel, et al., (2012) for added stability.

Value

`mlergm` returns an object of class `mlergm` which is a list containing:

theta	Estimated parameter vector of the exponential-family random graph model.
between_theta	Estimated parameter vector of the between group model.
se	Standard error vector for theta.

between_se	Standard error vector for between_theta.
pvalue	A vector of p-values for the estimated parameter vector.
between_pvalue	A vector of p-values for the estimated parameter vector.
logLikval	The loglikelihood for at the estimated MLE.
bic	The BIC for the estimated model.
mcmc_chain	The MCMC sample used in the final estimation step, which can be used to diagnose non-convergence.
estimation_status	Indicator of whether the estimation procedure had success or failed.
parameterization	The model parameterization (either standard or offset).
formula	The model formula.
network	The network for which the model is estimated.
node_memb	Vector indicating to which group or block the nodes belong.
size_quantiles	The quantiles of the block sizes.

Methods (by generic)

- `print`: Print method for objects of class `mlergm`. Indicates whether the model was successfully estimated, as well as the model formula provided.
- `summary`: Prints a summary of the estimated `mlergm` model.

References

- Schweinberger, M. and Stewart, J. (2019) Concentration and consistency results for canonical and curved exponential-family random graphs. *The Annals of Statistics*, to appear.
- Schweinberger, M. and Handcock, M. S. (2015). Local dependence in random graph models: characterization, properties and statistical inference. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 77(3), 647-676.
- Hunter, D. R., and Handcock, M. S. (2006). Inference in curved exponential family models for networks. *Journal of Computational and Graphical Statistics*, 15(3), 565-583.
- Hummel, R. M., Hunter, D. R., and Handcock, M. S. (2012). Improving simulation-based algorithms for fitting ERGMs. *Journal of Computational and Graphical Statistics*, 21(4), 920-939.
- Krivitsky, P. N., Handcock, M. S., & Morris, M. (2011). Adjusting for network size and composition effects in exponential-family random graph models. *Statistical methodology*, 8(4), 319-339.
- Krivitsky, P.N, and Kolaczyk, E. D. (2015). On the question of effective sample size in network modeling: An asymptotic inquiry. *Statistical science: a review journal of the Institute of Mathematical Statistics*, 30(2), 184.
- Hunter D., Handcock M., Butts C., Goodreau S., and Morris M. (2008). `ergm`: A Package to Fit, Simulate and Diagnose Exponential-Family Models for Networks. *Journal of Statistical Software*, 24(3), 1-29.
- Butts, C. (2016). `sna`: Tools for Social Network Analysis. R package version 2.4. <https://CRAN.R-project.org/package=sna>.

Butts, C. (2008). network: a Package for Managing Relational Data in R. Journal of Statistical Software, 24(2). <http://www.jstatsoft.org/v24/i02/paper>.

Stewart, J., Schweinberger, M., Bojanowski, M., and M. Morris (2019). Multilevel network data facilitate statistical inference for curved ERGMs with geometrically weighted terms. Social Networks, to appear.

Schweinberger, M., Krivitsky, P. N., Butts, C.T. and J. Stewart (2018). Exponential-family models of random graphs: Inference in finite-, super-, and infinite-population scenarios. <https://arxiv.org/abs/1707.04800>

See Also

[gof.mlerngm](#), [mlnet](#)

Examples

```
### Load the school classes data-set
data(classes)

# Estimate a curved multilevel ergm model with offset parameter
# Approximate run time (2 cores): 1.2m, Run time (3 cores): 55s
model_est <- mlerngm(classes ~ edges + mutual + nodematch("sex") + gwesp(fixed = FALSE),
                    seed = 123,
                    options = set_options(number_cores = 2))

# To access a summary of the fitted model, call the 'summary' function
summary(model_est)

# Goodness-of-fit can be run by calling the 'gof.mlerngm' method
# Approximate run time (2 cores): 48s, Run time (3 cores): 34s
gof_res <- gof(model_est, options = set_options(number_cores = 2))
plot(gof_res, cutoff = 15)
```

mlnet

Multilevel Network

Description

Function creates a multilevel network object of class `mlnet`. The object inherits the `network` class, with additional information concerning the multilevel structure.

Usage

```
mlnet(network, node_memb, directed = FALSE)

## S3 method for class 'mlnet'
plot(x, node_size = 2.5, palette = NULL,
     memb_colors = NULL, arrow.gap = 0.015, arrow.size = 4,
     color_legend_title = "", legend = TRUE, legend.position = "right",
     layout_type = "kamadakawai", ...)
```


Arguments

network	Either a network object, an adjacency matrix, or an edge list.
node_memb	Vector (length equal to the number of nodes in the network) indicating to which block or group the nodes belong.
directed	(TRUE or FALSE) Indicates whether the supplied network is directed or undirected. Default is FALSE.
x	An object of class <code>mlnet</code> , possibly produced by mlnet or simulate_mlnet .
node_size	Controls the size of nodes.
palette	If package <code>RColorBrewer</code> is installed, then the name of an R color brewer palette can be specified and used for the block colors. See <code>brewer.pal</code> for details on <code>RColorBrewer</code> palletes.
memb_colors	Specifies the named colors to be used for the membership colors.
arrow.gap	(Directed graphs only) Controls the amount of space between arrowheads and the nodes.
arrow.size	(Directed graphs only) Controls the size of the arrowhead.
color_legend_title	Name for the node color legend title.
legend	(TRUE or FALSE) Controls whether the block membership legend is printed.
legend.position	The position of the legend in the plot. Defaults to the "right" position.
layout_type	Viable layout options. See <code>gplot.layout</code> for options.
...	Additional arguments to be passed to ggnet2 .

Details

The `mlnet` function creates an object of class `mlnet` which is used to access methods designed specifically for multilevel networks, including visualization methods as well as direct interface with some of the main functions, such as [mlergm](#). Presently, the `mlnet` function and object class cover multilevel structure where the set of nodes is nested within known block structure.

Value

`mlnet` returns an object of class `mlnet` which inherits the [network](#) class, with the additional vector attribute `node_memb`, which encodes the block membership of the multilevel network.

Methods (by generic)

- `plot`: Plots network objects of type `mlnet`.

Examples

```
# Show how the sampson dataset can be turned into an mlnet object
data(sampson)
net <- mlnet(samplike, get.vertex.attribute(samplike, "group"))
```

plot.gof_mlergm	<i>Plot goodness-of-fit results</i>
-----------------	-------------------------------------

Description

Produces goodness-of-fit plots for a `gof_mlergm` object in order to visualize and assess the fit of an estimated model produced by `mlergm`.

Usage

```
## S3 method for class 'gof_mlergm'
plot(x, ..., individual_plots = FALSE,
     save_plots = FALSE, show_plots = TRUE, width = 8, height = 4.5,
     cutoff = NULL, x_labels = NULL, x_angle = 0, x_axis_label = NULL,
     y_axis_label = "Count", plot_title = "", title_size = 18,
     axis_label_size = 14, axis_size = 10, line_size = 1,
     x_axis_label_size = NULL, y_axis_label_size = NULL,
     x_axis_size = NULL, y_axis_size = NULL, pretty_x = TRUE)
```

Arguments

<code>x</code>	An object of class <code>gof_mlergm</code> , produced by <code>gof.mlergm</code> .
<code>...</code>	Additional argument to be passed if necessary.
<code>individual_plots</code>	(Logical TRUE or FALSE) If TRUE, individual gof plots are produced. Defaults to FALSE.
<code>save_plots</code>	(Logical TRUE or FALSE) If TRUE, the individual GOF plots are saved.
<code>show_plots</code>	(Logical TRUE or FALSE) If TRUE, the plots are printed to the screen, and if FALSE no plots are displayed. This may be helpful when the only desire is to save the individual GOF plots.
<code>width</code>	If <code>save_plots == TRUE</code> , controls the plot width dimension saved.
<code>height</code>	If <code>save_plots == TRUE</code> , controls the plot height dimension saved.
<code>cutoff</code>	For statistics that are distributions (e.g., degree distributions), specifies a cutoff point. Dimensions past the cutoff are ignored and not plotted.
<code>x_labels</code>	Character vector specifying the statistic names or labels.
<code>x_angle</code>	Adjusts the angle of the x axis tick labels (typically the statistic names).
<code>x_axis_label</code>	Label for the x axis.
<code>y_axis_label</code>	Label for the y axis.
<code>plot_title</code>	Title for the plot.
<code>title_size</code>	Font size for the plot title.
<code>axis_label_size</code>	Font size for the axis labels. Individual axes label sizes can be changed using <code>x_axis_label_size</code> and <code>y_axis_label_size</code> which are detailed below.

axis_size	Font size for the axis tick labels. Individual axes tick label sizes can be changed using x_axis_size and y_axis_size which are detailed below.
line_size	(Numeric, non-negative) If line_size is positive, then a red line will be plotted to indicate the observed network value of the statistic. If line_size is equal to zero, then the observed data line will not be plotted.
x_axis_label_size	The font size of the x axis label. When NULL, axis_label_size is used. Defaults to NULL.
y_axis_label_size	The font size of the y axis label. When NULL, axis_label_size is used. Defaults to NULL.
x_axis_size	The font size of the x axis tick labels. When NULL, axis_size is used. Defaults to NULL.
y_axis_size	The font size of the y axis tick labels. When NULL, axis_size is used. Defaults to NULL.
pretty_x	(Logical TRUE or FALSE) If set to TRUE, the link{pretty} function will be called to format the x-axis breaks. This can be useful for when the x-axis range is large.

print.gof_mlergm *Print summary of a gof_mlergm object.*

Description

Prints a formatted summary output for gof_mlergm object which was produced by [gof.mlergm](#).

Usage

```
## S3 method for class 'gof_mlergm'  
print(x, ...)
```

Arguments

x An object of class gof_mlergm, probably produced by [gof.mlergm](#).
... Additional arguments to be passed if necessary.

See Also

[gof.mlergm](#)

set_options *Set and adjust options and settings.*

Description

Function allows for specification of options and settings for simulation and estimation procedures.

Usage

```
set_options(burnin = 10000, interval = 1000, sample_size = 1000,
  NR_tol = 1e-04, NR_max_iter = 50, MCMLE_max_iter = 10,
  do_parallel = TRUE, number_cores = detectCores(all.tests = FALSE,
  logical = TRUE) - 1, adaptive_step_len = TRUE,
  step_len_multiplier = 0.5, step_len = 1, bridge_num = 10,
  bridge_burnin = 10000, bridge_interval = 500,
  bridge_sample_size = 5000)
```

Arguments

burnin	The burnin length for MCMC chains.
interval	The sampling interval for MCMC chains.
sample_size	The number of points to sample from MCMC chains for the MCMLE procedure.
NR_tol	The convergence tolerance for the Newton-Raphson optimization (implemented as Fisher scoring).
NR_max_iter	The maximum number of Newton-Raphson updates to perform.
MCMLE_max_iter	The maximum number of MCMLE steps to perform.
do_parallel	(logical) Whether or not to use parallel processing (defaults to TRUE).
number_cores	The number of parallel cores to use for parallel computations.
adaptive_step_len	(logical) If TRUE, an adaptive steplength procedure is used for the Newton-Raphson procedure. Arguments NR_step_len and NR_step_len_multiplier are ignored when adaptive_step_len is TRUE.
step_len_multiplier	The step_len adjustment multiplier when convergence fails.
step_len	The step length adjustment default to be used for the Newton-Raphson updates.
bridge_num	The number of bridges to use for likelihood computations.
bridge_burnin	The burnin length for the bridge MCMC chain for approximate likelihood computation.
bridge_interval	The sampling interval for the bridge MCMC chain for approximate likelihood computation.
bridge_sample_size	The number of points to sample from the bridge MCMC chain for approximate likelihood computation.

Details

The main simulation settings are `burnin`, `interval`, and `sample_size`. For estimation of the log-likelihood value, options include `bridge_num` which controls the number of bridges to be used for approximating the loglikelihood (see, e.g., Hunter and Handcock (2006) for a discussion). The main estimation settings and options include `NR_tol`, `NR_max_iter`, `MCMLE_max_iter`, `adaptive_step_len`, and `step_len`. Parameters `NR_tol` and `NR_max_iter` control the convergence tolerance and maximum number of iterations for the Newton-Raphson, or Fisher scoring, optimization. When the L2 norm of the increment in the Newton-Raphson procedure is under the specified tolerance `NR_tol` convergence is reached; and, no more than `NR_max_iter` iterations are performed. The MCMLE procedure uses the stepping algorithm of Hummel, et al., (2012) to give stability to the estimation procedure. Each MCMLE iteration draws samples from an MCMC chain, and `MCMLE_max_iter` controls how many iterations are performed before termination. Most functions support parallel computing for efficiency; by default `do_parallel` is TRUE. The number of computing cores can be adjusted by `number_cores`, and the default is one less than the number of cores available.

References

- Hunter, D. R., and Handcock, M. S. (2006). Inference in curved exponential family models for networks. *Journal of Computational and Graphical Statistics*, 15(3), 565-583.
- Hummel, R. M., Hunter, D. R., and Handcock, M. S. (2012). Improving simulation-based algorithms for fitting ERGMs. *Journal of Computational and Graphical Statistics*, 21(4), 920-939.

simulate_mlnet	<i>Simulate a multilevel network</i>
----------------	--------------------------------------

Description

Function simulates a multilevel network by specifying a network size, node block memberships, and within-block and between-block models. The function currently only supports block-models where between-block edges are dyad-independent.

Usage

```
simulate_mlnet(form, node_memb, theta, parameterization = "standard",
  seed = NULL, between_form = NULL, between_theta = NULL,
  between_prob = NULL, options = set_options())
```

Arguments

- | | |
|------------------|---------------------------------------------------------------------------------------------------------------------------------------------|
| form | A formula object of the form <code>network ~ model terms</code> which specifies how the within-block subgraphs are modeled. |
| node_memb | Vector of node block memberships. |
| theta | A vector of model parameters (coefficients) for the ERGM governing the within-subgraph edges. |
| parameterization | Parameterization options include 'standard', 'offset', or 'size'. |

- 'standard' : Does not adjust the individual block parameters for size.
- 'offset' : The offset parameterization uses edge and mutual offsets along the lines of Krivitsky, Handcock, and Morris (2011) and Krivitsky and Kocaczyk (2015). The edge parameter is offset by $-\log n(k)$ and the mutual parameter is offset by $+\log n(k)$, where $n(k)$ is the size of the k th block.
- 'size' : Multiplies the block parameters by $\log n(k)$, where $n(k)$ is the size of the k th block.

seed	Seed to be provided for reproducibility.
between_form	A formula object of the form \sim model terms which specifies how the within-block subgraphs are modeled.
between_theta	A vector of model parameters (coefficients) for the ERGM governing the between-subgraph edges.
between_prob	A probability which specifies how edges between blocks are governed. An ERGM (between_form and between_theta) cannot be specified together with between_prob.
options	Use set_options to change the simulation options. Note that some options are only valid for estimation using mlergm .

Details

Simulation of multilevel block networks is done with a Monte-Carlo Markov chain (MCMC) and can be done in parallel where [set_options](#) can be used to adjust the simulation settings (such as burnin, interval, and sample_size). Each within-block subgraph is given its own Markov chain, and so these settings are the settings to be used for each within-block chain.

Value

simulate_mlnet returns an objects of class [mlnet](#).

Examples

```
# Create a K = 2 block network with edge + gwesp term
net <- simulate_mlnet(form = network.initialize(30, directed = FALSE) ~ edges + gwesp,
  node_memb = c(rep(1, 15), rep(2, 15)),
  theta = c(-3, 0.5, 1.0),
  between_prob = 0.01,
  options = set_options(number_cores = 2, burnin = 2000))

# Simulate a K = 2 block directed network, specifying a formula for between edges
net <- simulate_mlnet(form = network.initialize(30, directed = TRUE) ~ edges + gwesp,
  node_memb = c(rep(1, 15), rep(2, 15)),
  theta = c(-3, 0.5, 1.0),
  between_form = ~ edges + mutual,
  between_theta = c(-4, 2),
  options = set_options(number_cores = 2, burnin = 2000))
```

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