

# Package ‘ProbYX’

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**Type** Package

**Title** Inference for the Stress-Strength Model  $R = P(Y < X)$

**Version** 1.1-0

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**Depends** R (>= 3.0-0), rootSolve

**Description** Confidence intervals and point estimation for R under various parametric model assumptions; likelihood inference based on classical first-order approximations and higher-order asymptotic procedures.

**License** GPL-2

**LazyLoad** yes

**Imports** stats, graphics

**NeedsCompilation** no

**Repository** CRAN

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ProbYX-package

*Inference on the stress-strength model  $R = P(Y < X)$*

---

## Description

Compute confidence intervals and point estimates for  $R$ , under parametric model assumptions for  $Y$  and  $X$ .  $Y$  and  $X$  are two independent continuous random variables from two different populations.

## Details

Package: ProbYX  
Type: Package  
Version: 1.1  
Date: 2012-03-20  
License: GPL-2  
LazyLoad: yes

The package can be used for computing accurate confidence intervals and point estimates for the stress-strength (reliability) model  $R = P(Y < X)$ ; maximum likelihood estimates, Wald statistic, signed log-likelihood ratio statistic and its modified version can be computed.

The main function is `Prob`, which evaluates confidence intervals and point estimates under different approaches and parametric assumptions.

## Author(s)

Giuliana Cortese

Maintainer: Giuliana Cortese <<gcortese@stat.unipd.it>>

## References

Cortese G., Ventura L. (2013). Accurate higher-order likelihood inference on  $P(Y < X)$ . *Computational Statistics*, 28:1035-1059.

Kotz S, Lumelskii Y, Pensky M. (2003). *The Stress-Strength Model and its Generalizations. Theory and Applications*. World Scientific, Singapore.

## Examples

```
# data from the first population
Y <- rnorm(15, mean=5, sd=1)
# data from the second population
X <- rnorm(10, mean=7, sd=1.5)
level <- 0.01          # \eqn{\alpha} level
# estimate and confidence interval under the assumption of two
# normal variables with different variances.
```

```

Prob(Y, X, "norm_DV", "RPstar", level)
# method has to be set equal to "RPstar".

```

---

loglik *Log-likelihood of the bivariate distribution of (Y,X)*

---

### Description

Computation of the log-likelihood function of the bivariate distribution (Y,X). The log-likelihood is reparametrized with the parameter of interest  $\psi$ , corresponding to the quantity R, and the nuisance parameter  $\lambda$ .

### Usage

```
loglik(ydat, xdat, lambda, psi, distr = "exp")
```

### Arguments

ydat	data vector of the sample measurements from Y.
xdat	data vector of the sample measurements from X.
lambda	nuisance parameter vector, $\lambda$ . Values can be determined from the reparameterisation of the original parameters of the bivariate distribution chosen in <code>distr</code> .
psi	scalar parameter of interest, $\psi$ , for the probability R. Value can be determined from the reparameterisation of the original parameters of the bivariate distribution chosen in <code>distr</code> .
distr	character string specifying the type of distribution assumed for $X_1$ and $X_2$ . Possible choices for <code>distr</code> are "exp" (default) for the one-parameter exponential, "norm_EV" and "norm_DV" for the Gaussian distribution with, respectively, equal or unequal variances assumed for the two random variables.

### Details

For further information on the random variables Y and X, see help on [Prob](#). Reparameterisation in order to determine  $\psi$  and  $\lambda$  depends on the assumed distribution. Here the following relationships have been used:

**Exponential models:**  $\psi = \frac{\alpha}{(\alpha+\beta)}$  and  $\lambda = \alpha + \beta$ , with  $Y \sim e^\alpha$  and  $X \sim e^\beta$ ;

**Gaussian models with equal variances:**  $\psi = \Phi\left(\frac{\mu_2 - \mu_1}{\sqrt{2\sigma^2}}\right)$  and  $\lambda = (\lambda_1, \lambda_2) = \left(\frac{\mu_1}{\sqrt{2\sigma^2}}, \sqrt{2\sigma^2}\right)$ , with  $Y \sim N(\mu_1, \sigma^2)$  and  $X \sim N(\mu_2, \sigma^2)$ ;

**Gaussian models with unequal variances:**  $\psi = \Phi\left(\frac{\mu_2 - \mu_1}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)$  and  $\lambda = (\lambda_1, \lambda_2, \lambda_3) = (\mu_1, \sigma_1^2, \sigma_2^2)$ , with  $Y \sim N(\mu_1, \sigma_1^2)$  and  $X \sim N(\mu_2, \sigma_2^2)$ .

The Standard Normal cumulative distribution function is indicated with  $\Phi$ .

**Value**

Value of the log-likelihood function computed in  $\psi = \text{psi}$  and  $\lambda = \text{lambda}$ .

**Author(s)**

Giuliana Cortese

**References**

Cortese G., Ventura L. (2013). Accurate higher-order likelihood inference on  $P(Y < X)$ . Computational Statistics, 28:1035-1059.

**See Also**

[MLEs](#)

**Examples**

```
# data from the first population
Y <- rnorm(15, mean=5, sd=1)
# data from the second population
X <- rnorm(10, mean=7, sd=1)
mu1 <- 5
mu2 <- 7
sigma <- 1
# parameter of interest, the R probability
interest <- pnorm((mu2-mu1)/(sigma*sqrt(2)))
# nuisance parameters
nuisance <- c(mu1/(sigma*sqrt(2)), sigma*sqrt(2))
# log-likelihood value
loglik(Y, X, nuisance, interest, "norm_EV")
```

---

MLEs

*Maximum likelihood estimates of the stress-strength model  $R = P(Y < X)$ .*

---

**Description**

Compute maximum likelihood estimates of  $R$ , considered as the parameter of interest. Maximum likelihood estimates of the nuisance parameter are also supplied.

**Usage**

MLEs(ydat, xdat, distr)

**Arguments**

ydat	data vector of the sample measurements from Y.
xdat	data vector of the sample measurements from X.
distr	character string specifying the type of distribution assumed for Y and X. Possible choices for <code>distr</code> are "exp" (default) for the one-parameter exponential, "norm_EV" and "norm_DV" for the Gaussian distribution with, respectively, equal or unequal variances assumed for the two random variables.

**Details**

The two independent random variables Y and X with given distribution `distr` are measurements of a certain characteristics on two different populations. For the relationship of the parameter of interest (R) and nuisance parameters with the original parameters of `distr`, look at the details in [loglik](#).

**Value**

Vector of estimates of the nuisance parameters and the R quantity (parameter of interest), respectively.

**Author(s)**

Giuliana Cortese

**References**

Kotz S, Lumelskii Y, Pensky M. (2003). The Stress-Strength Model and its Generalizations. Theory and Applications. World Scientific, Singapore.

**See Also**

[loglik](#), [Prob](#)

**Examples**

```
# data from the first population
Y <- rnorm(15, mean=5, sd=1)
# data from the second population
X <- rnorm(10, mean=7, sd=1.5)
# vector of MLEs for the nuisance parameters and the quantity R
MLEs(Y, X, "norm_DV")
```

---

 Prob

*Estimation of the stress-strength model  $R = P(Y < X)$* 


---

**Description**

Compute confidence intervals and point estimates for the probability  $R$ , under parametric model assumptions for  $Y$  and  $X$ .  $Y$  and  $X$  are two independent continuous random variable from two different populations.

**Usage**

```
Prob(ydat, xdat, distr = "exp", method = "RPstar", level = 0.05)
```

**Arguments**

ydat	data vector of the sample measurements from $Y$ .
xdat	data vector of the sample measurements from $X$ .
distr	character string specifying the type of distribution assumed for $Y$ and $X$ . Possible choices for <code>distr</code> are "exp" (default) for the one-parameter exponential, "norm_EV" and "norm_DV" for the Gaussian distribution with, respectively, equal or unequal variances assumed for the two random variables.
method	character string specifying the methodological approach used for inference (confidence intervals and point estimates) on the AUC. The argument <code>method</code> can be set equal to "Wald", "RP" or "RPstar" (default), according as inference is based on the Wald statistic, the signed log-likelihood ratio statistic (directed likelihood, $r_p$ ) or the modified signed log-likelihood ratio statistic (modified directed likelihood, $r_p^*$ ), respectively.
level	it is the $\alpha$ that supplies the nominal level $(1 - \alpha)$ chosen for the confidence interval.

**Value**

PROB	Point estimate of $R = P(Y < X)$ . This value corresponds to the maximum likelihood estimate if method "Wald" or "RP" is chosen; otherwise, when method "RPstar" is selected, estimate is obtained from the estimating equation $r_p^* = 0$ .
C.Interval	Confidence interval of $R$ at confidence level $(1 - \alpha)$ .

**Author(s)**

Giuliana Cortese

**References**

Cortese G., Ventura L. (2013). Accurate higher-order likelihood inference on  $R = P(Y < X)$ . *Computational Statistics*, 28:1035-1059.

**See Also**

[wald](#), [rp](#), [rpstar](#)

**Examples**

```
# data from the first population
Y <- rnorm(15, mean=5, sd=1)
# data from the second population
X <- rnorm(10, mean=7, sd=1.5)
  level <- 0.01                ## \eqn{\alpha} level
  # estimate and confidence interval under the assumption of two
  # normal variables with different variances.
Prob(Y, X, "norm_DV", "RPstar", level)
# method has to be set equal to "RPstar".
```

---

 ROC.plot

*Estimated ROC curves*


---

**Description**

Plot of ROC curves estimated under parametric model assumptions on the continuous diagnostic marker.

**Usage**

```
ROC.plot(ydat, xdat, distr = "exp", method = "RPstar", mc = 1)
```

**Arguments**

ydat	data vector of the diagnostic marker measurements on the sample of non-diseased individuals (from Y).
xdat	data vector of the diagnostic marker measurements on the sample of diseased individuals (from X).
distr	character string specifying the type of distribution assumed for Y and X. Possible choices for distr are "exp" (default) for the one-parameter exponential, "norm_EV" and "norm_DV" for the Gaussian distribution with, respectively, equal or unequal variances assumed for the two random variables.
method	character string specifying the methodological approach used for estimating the probability R, which is here interpreted as the area under the ROC curve (AUC). The argument method can be set equal to "Wald", "RP" or "RPstar" (default), according as inference is based on the Wald statistic, the signed log-likelihood ratio statistic (directed likelihood, $r_p$ ) or the modified signed log-likelihood ratio statistic (modified directed likelihood, $r_p^*$ ), respectively. For estimating the ROC curve parametrically, methods "Wald" and "RP" are equivalent and supply maximum likelihood estimation (MLE), whereas, by using method "RPstar", estimate of the ROC curve is based on the modified signed log-likelihood ratio statistic ( $r_p^*$ ). See <a href="#">rpstar</a> for details on this statistic.

**mc** a numeric value indicating single or multiple plots in the same figure. In case mc is equal to 1 (default), only the method specified in method is applied and the corresponding estimated ROC curve is plotted. If mc is different from 1, both MLE and  $r_p^*$ -based methods are applied, and two differently estimated ROC curves are plotted.

### Details

If mc is different from 1, method does not need to be specified.

### Value

Plot of ROC curves

### Note

The two independent random variables Y and X with given distribution `distr` are measurements of the diagnostic marker on the diseased and non-diseased subjects, respectively.

In "Wald" method, or equivalently "RP" method, MLEs for parameters of the Y and X distributions are computed and then used to estimate specificity and sensitivity. These measures are evaluated as  $P(Y < t)$  and  $P(X > t)$ , respectively.

In "RPstar" method, parameters of the Y and X distributions are estimated from the  $r_p^*$ -based estimate of the AUC.

### Author(s)

Giuliana Cortese

### References

Cortese G., Ventura L. (2013). Accurate higher-order likelihood inference on  $P(Y < X)$ . Computational Statistics, 28:1035-1059.

### See Also

[Prob](#)

### Examples

```
# data from the non-diseased population
Y <- rnorm(15, mean=5, sd=1)
# data from the diseased population
X <- rnorm(10, mean=7, sd=1.5)
ROC.plot(Y, X, "norm_DV", method = "RP", mc = 2)
```



## References

Cortese G., Ventura L. (2013). Accurate higher-order likelihood inference on  $P(Y < X)$ . *Computational Statistics*, 28:1035-1059.

Severini TA. (2000). *Likelihood Methods in Statistics*. Oxford University Press, New York.

Brazzale AR., Davison AC., Reid N. (2007). *Applied Asymptotics. Case-Studies in Small Sample Statistics*. Cambridge University Press, Cambridge.

## See Also

[wald](#), [rpstar](#), [MLEs](#), [Prob](#)

## Examples

```
# data from the first population
Y <- rnorm(15, mean=5, sd=1)
# data from the second population
X <- rnorm(10, mean=7, sd=1.5)
# value of  $r_p$  for  $\psi=0.9$ 
rp(Y, X, 0.9, "norm_DV")
```

---

rpstar

*Modified signed log-likelihood ratio statistic*

---

## Description

Compute the modified signed log-likelihood ratio statistic ( $r_p^*$ ) for a given value of the stress strength  $R = P(Y < X)$ , that is the parameter of interest, under given parametric model assumptions.

## Usage

```
rpstar(ydat, xdat, psi, distr = "exp")
```

## Arguments

ydat	data vector of the sample measurements from Y.
xdat	data vector of the sample measurements from X.
psi	scalar for the parameter of interest. It is the value of R, treated as a parameter under the parametric model construction.
distr	character string specifying the type of distribution assumed for Y and X. Possible choices for distr are "exp" (default) for the one-parameter exponential, "norm_EV" and "norm_DV" for the Gaussian distribution with, respectively, equal or unequal variances assumed for the two random variables.

**Details**

The two independent random variables Y and X with given distribution `distr` are measurements from two different populations. For the relationship of the parameter of interest (R) and nuisance parameters with the original parameters of `distr`, look at the details in [loglik](#).

**Value**

<code>rp</code>	Value of the signed log-likelihood ratio statistic $r_p$ .
<code>rp_star</code>	Value of the modified signed log-likelihood ratio statistic $r_p^*$ .

**Note**

The statistic  $r_p^*$  is a modified version of  $r_p$  which provides more statistically accurate estimates. The  $r_p^*$  values can be also used for testing statistical hypotheses on the probability R.

**Author(s)**

Giuliana Cortese

**References**

Cortese G., Ventura L. (2013). Accurate higher-order likelihood inference on  $P(Y < X)$ . Computational Statistics, 28:1035-1059.

Severini TA. (2000). Likelihood Methods in Statistics. Oxford University Press, New York.

Brazzale AR., Davison AC., Reid N. (2007). Applied Asymptotics. Case-Studies in Small Sample Statistics. Cambridge University Press, Cambridge.

**See Also**

[wald](#), [rp](#), [MLEs](#), [Prob](#)

**Examples**

```
# data from the first population
Y <- rnorm(15, mean=5, sd=1)
# data from the second population
X <- rnorm(10, mean=7, sd=1.5)
# value of  $r_p^*$  for {psi=0.9}
rpstar(Y, X, 0.9, "norm_DV")
# method has be set equal to "RPstar".
```

---

wald	<i>Wald statistic</i>
------	-----------------------

---

**Description**

Compute the Wald statistic for a given value of the stress-strength  $R = P(Y < X)$ , that is the parameter of interest, under given parametric model assumptions.

**Usage**

```
wald(ydat, xdat, psi, distr = "exp")
```

**Arguments**

ydat	data vector of the sample measurements from Y.
xdat	data vector of the sample measurements from X.
psi	scalar for the parameter of interest. It is the value of the quantity R, treated as a parameter under the parametric model construction.
distr	character string specifying the type of distribution assumed for Y and X. Possible choices for <code>distr</code> are "exp" (default) for the one-parameter exponential, "norm_EV" and "norm_DV" for the Gaussian distribution with, respectively, equal or unequal variances assumed for the two random variables.

**Details**

The two independent random variables Y and X with given distribution `distr` are measurements from two different populations. For the relationship of the parameter of interest (R) and nuisance parameters with the original parameters of `distr`, look at the details in [loglik](#).

**Value**

Wald	Value of the Wald statistic for a given <code>psi</code>
Jphat	Observed profile Fisher information

**Note**

Values of the Wald statistic can be also used for testing statistical hypotheses on the probability R.

**Author(s)**

Giuliana Cortese

**References**

Cortese G., Ventura L. (2013). Accurate higher-order likelihood inference on  $P(Y < X)$ . *Computational Statistics*, 28:1035-1059.

Brazzale AR., Davison AC., Reid N. (2007). *Applied Asymptotics. Case-Studies in Small Sample Statistics*. Cambridge University Press, Cambridge.

**See Also**

[rp](#), [rpstar](#), [MLEs](#), [Prob](#)

**Examples**

```
# data from the first population
Y <- rnorm(15, mean=5, sd=1)
# data from the second population
X <- rnorm(10, mean=7, sd=1.5)
# value of Wald for \code{psi=0.9}
wald(Y, X, 0.9, "norm_DV")
```

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