

# Package ‘Copula.Markov’

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**Type** Package

**Title** Copula-Based Estimation and Statistical Process Control for  
Serially Correlated Time Series

**Version** 2.7

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**Description** Estimation and statistical process control are performed under  
copula-based time-series models.  
Available are statistical methods in Long and Emura (2014 JCSA),  
Emura et al. (2017 Commun Stat-Simul) <DOI:10.1080/03610918.2015.1073303>,  
Huang and Emura (2019 Commun Stat-  
Simul) <DOI:10.1080/03610918.2019.1602647> and Huang, Chen and Emura (2019-  
, in revision).

**License** GPL-2

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Copula.Markov-package *Copula-Based Estimation and Statistical Process Control for Serially Correlated Time Series*

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### Description

Copulas are applied to model a Markov dependence for serially correlated time series. The Clayton and Joe copulas are available to specify the dependence structure. The normal and binomial distributions are available for the marginal model. Maximum likelihood estimation is implemented for estimating parameters, and a Shewhart control chart is drawn for performing statistical process control.

### Details

Package:	Copula.Markov
Type:	Package
Version:	2.7
Date:	2020-1-17
License:	GPL-2

### Author(s)

Emura T, Chen WR, Long TH, Huang X. Maintainer: Takeshi Emura <takeshiemura@gmail.com>

### References

- Chen W (2018) Copula-based Markov chain model with binomial data, NCU Library
- Huang XW, Chen W, Emura T (2019-), Likelihood-based inference for a copula-based Markov chain model with binomial time series, in review
- Emura T, Long TH, Sun LH (2017), R routines for performing estimation and statistical process control under copula-based time series models, *Communications in Statistics - Simulation and Computation*, 46 (4): 3067-87
- Long TH and Emura T (2014), A control chart using copula-based Markov chain models, *Journal of the Chinese Statistical Association* 52 (No.4): 466-96
- Huang XW, Emura T (2019), Model diagnostic procedures for copula-based Markov chain models for statistical process control, *Communications in Statistics - Simulation and Computation*, doi:10.1080/03610918.2019.1602647

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Clayton.Markov.DATA     *Generating Time Series Data Under a Copula-Based Markov Chain Model with the Clayton Copula*

---

### Description

Time-series datasets are generated under a copula-based Markov chain model with the Clayton copula. See Long et al. (2014) and Emura et al. (2017) for the details.

### Usage

```
Clayton.Markov.DATA(n, mu, sigma, alpha)
```

### Arguments

n	sample size
mu	mean
sigma	standard deviation
alpha	association parameter

### Details

-1<alpha<0 for negative association; alpha>0 for positive association

### Value

Time series data of size n.

### Author(s)

Takeshi Emura

### References

Emura T, Long TH, Sun LH (2017), R routines for performing estimation and statistical process control under copula-based time series models, *Communications in Statistics - Simulation and Computation*, 46 (4): 3067-87

Long TH and Emura T (2014), A control chart using copula-based Markov chain models, *Journal of the Chinese Statistical Association* 52 (No.4): 466-96

### Examples

```
set.seed(1)
Y=Clayton.Markov.DATA(n=1000,mu=0,sigma=1,alpha=8)
Clayton.Markov.MLE(Y,plot=TRUE)
```

---

Clayton.Markov.DATA.binom

*Generating Time Series Data Under a Copula-Based Markov Chain Model with the Clayton Copula and Binomial Margin.*

---

## Description

Time-series datasets are generated under a copula-based Markov chain model with the Clayton copula and binomial margin.

## Usage

```
Clayton.Markov.DATA.binom(n, size, prob, alpha)
```

## Arguments

n	number of observations
size	number of binomial trials
prob	binomial probability; $0 < p < 1$
alpha	association parameter

## Details

$-1 < \alpha < 0$  for negative association;  $\alpha > 0$  for positive association

## Value

Time series data of size n (this is different from the number of binomial trials = "size").

## Author(s)

Huang XW, Chen W, Emura T

## References

Chen W (2018) Copula-based Markov chain model with binomial data, NCU Library

Huang XW, Chen W, Emura T (2019-), Likelihood-based inference for a copula-based Markov chain model with binomial time series, in review

## Examples

```
size=50
prob=0.5
alpha=2
set.seed(1)
Y=Clayton.Markov.DATA.binom(n=500,size,prob,alpha)
### sample mean and SD ###
```

```

mean(Y)
sd(Y)
### true mean and SD ###
size*prob
sqrt(size*prob*(1-prob))

```

---

Clayton.Markov.GOF      *A goodness-of-fit test for the marginal normal distribution.*

---

### Description

Perform a parametric bootstrap test based on the Cramer-von Mises and Kolmogorov-Smirnov statistics as proposed by Huang and Emura (2019).

### Usage

```
Clayton.Markov.GOF(Y, k = 3, D = 1, B = 200, GOF.plot=FALSE, method = "Newton")
```

### Arguments

Y	vector of datasets
k	constant determining the length between LCL and UCL (k=3 corresponds to 3-sigma limit)
D	diameter for U(-D, D) used in randomized Newton-Raphson
B	the number of Bootstrap replications
GOF.plot	if TRUE, show the model diagnostic plots for B bootstrap replications
method	Newton-Raphson method or nlm can be chosen

### Value

CM	The Cramer-von Mises statistic and its P-value
KS	The Kolmogorov-Smirnov statistic and its P-value
CM.boot	Bootstrap values of the Cramer-von Mises statistics
KS.boot	Bootstrap values of the Kolmogorov-Smirnov statistics

### Author(s)

Takeshi Emura

## References

- Emura T, Long TH, Sun LH (2017), R routines for performing estimation and statistical process control under copula-based time series models, *Communications in Statistics - Simulation and Computation*, 46 (4): 3067-87
- Long TH and Emura T (2014), A control chart using copula-based Markov chain models, *Journal of the Chinese Statistical Association* 52 (No.4): 466-96
- Huang XW, Emura T (2019), Model diagnostic procedures for copula-based Markov chain models for statistical process control, *Communications in Statistics - Simulation and Computation*, doi:10.1080/03610918.2019.1602647

## Examples

```
set.seed(1)
Y=Clayton.Markov.DATA(n=1000,mu=0,sigma=1,alpha=2)
Clayton.Markov.GOF(Y,B=5,GOF.plot=TRUE)
```

---

Clayton.Markov.GOF.binom

*A goodness-of-fit test for the marginal binomial distribution.*

---

## Description

Perform a parametric bootstrap test based on the Cramer-von Mises and Kolmogorov-Smirnov statistics as proposed by Huang and Emura (2019) and Huang et al. (2019-).

## Usage

```
Clayton.Markov.GOF.binom(Y, k = 3, size, B = 200, GOF.plot=FALSE, method = "Newton")
```

## Arguments

Y	vector of datasets
k	constant determining the length between LCL and UCL (k=3 corresponds to 3-sigma limit)
size	number of binomial trials
B	the number of Bootstrap replications
GOF.plot	if TRUE, show the model diagnostic plots for B bootstrap replications
method	Newton-Raphson method or nlm can be chosen

## Value

CM	The Cramer-von Mises statistic and its P-value
KS	The Kolmogorov-Smirnov statistic and its P-value
CM.boot	Bootstrap values of the Cramer-von Mises statistics
KS.boot	Bootstrap values of the Kolmogorov-Smirnov statistics

**Author(s)**

Huang XW, Emura T

**References**

Huang XW, Emura T (2019), Model diagnostic procedures for copula-based Markov chain models for statistical process control, *Communications in Statistics - Simulation and Computation*, doi:10.1080/03610918.2019.1602647

Huang XW, Chen W, Emura T (2019-), Likelihood-based inference for a copula-based Markov chain model with binomial time series, in review

**Examples**

```
size=50
prob=0.5
alpha=2
set.seed(1)
Y=Clayton.Markov.DATA.binom(n=500,size,prob,alpha)
Clayton.Markov.GOF.binom(Y,size=size,B=5,k=3,GOF.plot=TRUE) ## B=5 to save time
```

---

Clayton.Markov.MLE      *Maximum Likelihood Estimation and Statistical Process Control Under the Clayton Copula*

---

**Description**

The maximum likelihood estimates are produced and the Shewhart control chart is drawn with k-sigma control limits (e.g., 3-sigma). The dependence model follows the Clayton copula and the marginal (stationary) distribution follows the normal distribution.

**Usage**

```
Clayton.Markov.MLE(Y, k = 3, D = 1, plot = TRUE, GOF=FALSE, method = "nlm")
```

**Arguments**

Y	vector of datasets
k	constant determining the length between LCL and UCL (k=3 corresponds to 3-sigma limit)
D	diameter for U(-D, D) used in randomized Newton-Raphson
plot	show the control chart if TRUE
GOF	show the model diagnostic plot if TRUE
method	apply "nlm" or "Newton" method

**Value**

mu	estimate, SE, and 95 percent CI
sigma	estimate, SE, and 95 percent CI
alpha	estimate, SE, and 95 percent CI
Control_Limit	Center = mu, LCL = mu - k*sigma, UCL = mu + k*sigma
out_of_control	IDs for out-of-control points
Gradient	gradients (must be zero)
Hessian	Hessian matrix
Eigenvalue_Hessian	Eigenvalues for the Hessian matrix
KS.test	KS statistics
CM.test	CM statistics
log.likelihood	Log-likelihood value for the estimation

**Author(s)**

Long TH, Huang XW and Emura T

**References**

Emura T, Long TH, Sun LH (2017), R routines for performing estimation and statistical process control under copula-based time series models, *Communications in Statistics - Simulation and Computation*, 46 (4): 3067-87

Long TH and Emura T (2014), A control chart using copula-based Markov chain models, *Journal of the Chinese Statistical Association* 52 (No.4): 466-96

**Examples**

```
set.seed(1)
Y=Clayton.Markov.DATA(n=1000,mu=0,sigma=1,alpha=2)
Clayton.Markov.MLE(Y,plot=TRUE)
```

---

Clayton.Markov.MLE.binom

*Maximum Likelihood Estimation and Statistical Process Control Under the Clayton Copula*

---

**Description**

The maximum likelihood estimates are produced and the Shewhart control chart is drawn with k-sigma control limits (e.g., 3-sigma). The dependence model follows the Clayton copula and the marginal (stationary) distribution follows the normal distribution.



**Usage**

```
Clayton.Markov.MLE.binom(Y, size, k = 3, method="nlm", plot = TRUE, GOF=FALSE)
```

**Arguments**

Y	vector of observations
size	numbe of binomial trials
method	nlm or Newton
k	constant determining the length between LCL and UCL (k=3 corresponds to 3-sigma limit)
plot	show the control chart if TRUE
GOF	show the model diagnostic plot if TRUE

**Value**

p	estimate, SE, and 95 percent CI
alpha	estimate, SE, and 95 percent CI
Control_Limit	Center = $n \cdot p$ , LCL = $\mu - k \cdot \sigma$ , UCL = $\mu + k \cdot \sigma$
out_of_control	IDs for out-of-control points
Gradient	gradients (must be zero)
Hessian	Hessian matrix
Eigenvalue_Hessian	Eigenvalues for the Hessian matrix
KS.test	KS statistics
CM.test	CM statistics
log_likelihood	Log-likelihood value for the estimation

**Author(s)**

Huang XW, Emura T

**References**

Chen W (2018) Copula-based Markov chain model with binomial data, NCU Library  
 Huang XW, Chen W, Emura T (2019-), Likelihood-based inference for a copula-based Markov chain model with binomial time series, in review

**Examples**

```
size=50
prob=0.5
alpha=2
set.seed(1)
Y=Clayton.Markov.DATA.binom(n=500,size,prob,alpha)
Clayton.Markov.MLE.binom(Y,size=size,k=3,plot=TRUE)
```

---

Clayton.Markov2.DATA *Generating Time Series Data Under a Copula-Based 2nd-order Markov Chain Model with the Clayton Copula*

---

### Description

Time-series datasets are generated under a copula-based 2nd order Markov chain model with the Clayton copula.

### Usage

```
Clayton.Markov2.DATA(n, mu, sigma, alpha)
```

### Arguments

n	sample size
mu	mean
sigma	standard deviation
alpha	association parameter

### Details

-1 < alpha < 0 for negative association; alpha > 0 for positive association

### Value

Time series data of size n

### Author(s)

Xinwei Huang and Takeshi Emura

### References

Huang XW, Emura T (2019), Model diagnostic procedures for copula-based Markov chain models for statistical process control, *Communications in Statistics - Simulation and Computation*, doi:10.1080/03610918.2019.1602647

### Examples

```
Clayton.Markov2.DATA(n = 100, mu = 0, sigma = 1, alpha = 2)
```

---

Clayton.Markov2.MLE *Maximum Likelihood Estimation and Statistical Process Control Under the Clayton Copula with a 2nd order Markov chain.*

---

### Description

The maximum likelihood estimates are produced and the Shewhart control chart is drawn with k-sigma control limits (e.g., 3-sigma). The dependence model follows the Clayton copula and the marginal (stationary) distribution follows the normal distribution. The model diagnostic plot is also given (by the option "GOF=TRUE"). See Huang and Emura (2019) for the methodological details.

### Usage

```
Clayton.Markov2.MLE(Y, k = 3, D = 1, plot = TRUE, GOF=FALSE)
```

### Arguments

Y	vector of datasets
k	constant determining the length between LCL and UCL (k=3 corresponds to 3-sigma limit)
D	diameter for U(-D, D) used in randomized Newton-Raphson
plot	show the control chart if TRUE
GOF	show the model diagnostic plot if TRUE

### Value

mu	estimate, SE, and 95 percent CI
sigma	estimate, SE, and 95 percent CI
alpha	estimate, SE, and 95 percent CI
Control_Limit	Center = mu, LCL = mu - k*sigma, UCL = mu + k*sigma
out_of_control	IDs for out-of-control points
Gradient	gradients (must be zero)
Hessian	Hessian matrix
Eigenvalue_Hessian	Eigenvalues for the Hessian matrix
KS.test	KS statistics
CM.test	CM statistics
log.likelihood	Log-likelihood value for the estimation

### Author(s)

Xinwei Huang and Takeshi Emura

## References

Huang XW, Emura T (2019), Model diagnostic procedures for copula-based Markov chain models for statistical process control, Communications in Statistics - Simulation and Computation, doi:10.1080/03610918.2019.1602647

## Examples

```
Y = c(0.265, 0.256, 0.261, 0.261, 0.260, 0.257, 0.258, 0.263, 0.254, 0.254,
      0.258, 0.256, 0.256, 0.265, 0.270, 0.267, 0.270, 0.267, 0.266, 0.271,
      0.270, 0.264, 0.261, 0.264, 0.266, 0.264, 0.269, 0.268, 0.264, 0.262,
      0.257, 0.255, 0.255, 0.253, 0.251, 0.254, 0.255)
```

```
Clayton.Markov2.MLE(Y, k = 1, D = 1, plot = TRUE)
```

```
Y=Clayton.Markov2.DATA(n=1000,mu=0,sigma=1,alpha=8)
```

```
Clayton.Markov2.MLE(Y, plot=TRUE)
```

---

Joe.Markov.DATA

*Generating Time Series Data Under a Copula-Based Markov Chain Model with the Joe Copula*

---

## Description

Time-series datasets are generated under a copula-based Markov chain model with the Joe copula.

## Usage

```
Joe.Markov.DATA(n, mu, sigma, alpha)
```

## Arguments

n	sample size
mu	mean
sigma	standard deviation
alpha	association parameter

## Details

alpha $\geq$ 1 for positive association

## Value

Time series data of size n

## Author(s)

Takeshi Emura

**References**

Emura T, Long TH, Sun LH (2017), R routines for performing estimation and statistical process control under copula-based time series models, *Communications in Statistics - Simulation and Computation*, 46 (4): 3067-87

Long TS and Emura T (2014), A control chart using copula-based Markov chain models, *Journal of the Chinese Statistical Association* 52 (No.4): 466-96

**Examples**

```
n=1000
alpha=2.856 ### Kendall's tau =0.5 ###
mu=2
sigma=1
Y=Joe.Markov.DATA(n, mu, sigma, alpha)
mean(Y)
sd(Y)
cor(Y[-1], Y[-n], method="kendall")

Joe.Markov.MLE(Y, k=2)
```

---

Joe.Markov.DATA.binom *Generating Time Series Data Under a Copula-Based Markov Chain Model with the Joe Copula and Binomial Margin.*

---

**Description**

Time-series datasets are generated under a copula-based Markov chain model with the Joe copula and binomial margin.

**Usage**

```
Joe.Markov.DATA.binom(n, size, prob, alpha)
```

**Arguments**

n	number of observations
size	number of binomial trials
prob	binomial probability; $0 < p < 1$
alpha	association parameter

**Details**

alpha  $\geq 1$  for positive association

**Value**

time series data

**Author(s)**

Huang X, Emura T

**References**

Chen W (2018) Copula-based Markov chain model with binomial data, NCU Library

Huang X, Chen W, Emura T (2019-) A control chart using a copula-based Markov chain for attribute data, revision

**Examples**

```
size=50
prob=0.5
alpha=2
set.seed(1)
Y=Joe.Markov.DATA.binom(n=500,size,prob,alpha)
### sample mean and SD ###
mean(Y)
sd(Y)
### true mean and SD ###
size*prob
sqrt(size*prob*(1-prob))
```

---

Joe.Markov.MLE

*Maximum Likelihood Estimation and Statistical Process Control Under the Joe Copula*

---

**Description**

The maximum likelihood estimates are produced and the Shewhart control chart is drawn with k-sigma control limits (e.g., 3-sigma). The dependence model follows the Joe copula and the marginal (stationary) distribution follows the normal distribution.

**Usage**

```
Joe.Markov.MLE(Y, k = 3, D = 1, plot = TRUE, GOF=FALSE, method = "nlm")
```

**Arguments**

Y	vector of datasets
k	constant determining the length between LCL and UCL (k=3 corresponds to 3-sigma limit)
D	diameter for U(-D, D) used in randomized Newton-Raphson
plot	show the control chart if TRUE
GOF	show the model diagnostic plot if TRUE
method	apply "nlm" or "Newton" method

**Value**

mu	estimate, SE, and 95 percent CI
sigma	estimate, SE, and 95 percent CI
alpha	estimate, SE, and 95 percent CI
Control_Limit	Center = mu, LCL = mu - k*sigma, UCL = mu + k*sigma
out_of_control	IDs for out-of-control points
Gradient	gradients (must be zero)
Hessian	Hessian matrix
Eigenvalue_Hessian	Eigenvalues for the Hessian matrix
KS.test	KS statistics
CM.test	CM statistics
log.likelihood	Log-likelihood value for the estimation

**Author(s)**

Long TH, Huang XW and Takeshi Emura

**References**

Emura T, Long TH, Sun LH (2017), R routines for performing estimation and statistical process control under copula-based time series models, *Communications in Statistics - Simulation and Computation*, 46 (4): 3067-87

Long TH and Emura T (2014), A control chart using copula-based Markov chain models, *Journal of the Chinese Statistical Association* 52 (No.4): 466-96

**Examples**

```
n=1000
alpha=2.856 ### Kendall's tau =0.5 ###
mu=2
sigma=1
Y=Joe.Markov.DATA(n,mu,sigma,alpha)
mean(Y)
sd(Y)
cor(Y[-1],Y[-n],method="kendall")

Joe.Markov.MLE(Y,k=2)
```

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