

# Package ‘set6’

February 17, 2021

**Title** R6 Mathematical Sets Interface

**Version** 0.2.1

**Description** An object-oriented package for mathematical sets, upgrading the current gold-standard {sets}. Many forms of mathematical sets are implemented, including (countably finite) sets, tuples, intervals (countably infinite or uncountable), and fuzzy variants. Wrappers extend functionality by allowing symbolic representations of complex operations on sets, including unions, (cartesian) products, exponentiation, and differences (asymmetric and symmetric).

**LinkingTo** Rcpp

**Imports** checkmate, Rcpp, R6, utils

**Suggests** knitr, testthat, devtools, rmarkdown

**License** MIT + file LICENSE

**LazyData** true

**URL** <https://xoops.github.io/set6/>, <https://github.com/xoopsR/set6>

**BugReports** <https://github.com/xoopsR/set6/issues>

**VignetteBuilder** knitr

**Encoding** UTF-8

**RoxygenNote** 7.1.1

**SystemRequirements** C++11

**Collate** 'Properties.R' 'RcppExports.R' 'Set.R' 'SetWrapper.R'  
'SetWrapper\_ComplementSet.R' 'SetWrapper\_ExponentSet.R'  
'SetWrapper\_PowersetSet.R' 'SetWrapper\_ProductSet.R'  
'SetWrapper\_UnionSet.R' 'Set\_Complex.R' 'Set\_ConditionalSet.R'  
'Set\_FuzzySet.R' 'Set\_FuzzySet\_FuzzyMultiset.R'  
'Set\_FuzzySet\_FuzzyTuple.R' 'Set\_Interval.R' 'setSymbol.R'  
'Set\_Interval\_SpecialSet.R' 'Set\_LogicalSet.R' 'Set\_Logicals.R'  
'Set\_Multiset.R' 'Set\_Tuple.R' 'Set\_Universal.R'  
'Set\_UniversalSet.R' 'asFuzzySet.R' 'asInterval.R' 'asSet.R'  
'helpers.R' 'assertions.R' 'atomic\_coercions.R'  
'listSpecialSets.R' 'operation\_cleaner.R'  
'operation\_powerset.R' 'operation\_setcomplement.R'  
'operation\_setintersect.R' 'operation\_setpower.R'

'operation\_setproduct.R' 'operation\_setsymdiff.R'  
 'operation\_setunion.R' 'operators.R' 'set6-deprecated.R'  
 'set6-package.R' 'set6.news.R' 'useUnicode.R' 'zzz.R'

**NeedsCompilation** yes

**Author** Raphael Sonabend [aut, cre] (<<https://orcid.org/0000-0001-9225-4654>>),  
 Franz Kiraly [aut]

**Maintainer** Raphael Sonabend <[raphael.sonabend.15@uc1.ac.uk](mailto:raphael.sonabend.15@uc1.ac.uk)>

**Repository** CRAN

**Date/Publication** 2021-02-17 11:50:02 UTC

## R topics documented:

set6-package	3
as.FuzzySet	4
as.Interval	7
as.Set	8
ComplementSet	10
Complex	12
ConditionalSet	15
contains	19
equals	19
ExponentSet	20
ExtendedReals	21
FuzzyMultiset	22
FuzzySet	25
FuzzyTuple	30
Integers	33
Interval	34
isSubset	39
listSpecialSets	40
Logicals	40
LogicalSet	41
Multiset	42
Naturals	44
NegIntegers	45
NegRationals	46
NegReals	47
PosIntegers	48
PosNaturals	49
PosRationals	50
PosReals	51
powerset	52
PowersetSet	53
ProductSet	55
Properties	57
Rationals	58

Reals	59
Set	60
set6News	68
setcomplement	68
setintersect	70
setpower	72
setproduct	74
setsymdiff	75
setunion	76
SetWrapper	78
SpecialSet	80
testClosed	81
testClosedAbove	82
testClosedBelow	82
testConditionalSet	83
testContains	84
testCountablyFinite	85
testCrisp	85
testEmpty	86
testFinite	87
testFuzzy	87
testFuzzyMultiset	88
testFuzzySet	89
testFuzzyTuple	90
testInterval	90
testMultiset	91
testSet	92
testSetList	93
testSubset	93
testTuple	94
Tuple	95
UnionSet	98
Universal	99
UniversalSet	104
useUnicode	108

**Index****109**

set6-package

*set6: R6 Mathematical Sets Interface***Description**

set6 upgrades the `{sets}` package to R6. Many forms of mathematical sets are implemented, including (countably finite) sets, tuples, intervals (countably infinite or uncountable), and fuzzy variants. Wrappers extend functionality by allowing symbolic representations of complex operations on sets, including unions, (cartesian) products, exponentiation, and differences (asymmetric and symmetric).

## Details

The main features of set6 are:

- Object-oriented programming, which allows a clear inheritance structure for Sets, Intervals, Tuples, and other variants.
- Set operations and wrappers for both explicit and symbolic representations for algebra of sets.
- Methods for assertions and comparison checks, including subsets, equality, and containedness.

To learn more about set6, start with the set6 vignette:

```
vignette("set6", "set6")
```

And for more advanced usage see the complete tutorials at

<https://github.com/xoopR/set6>

## Author(s)

**Maintainer:** Raphael Sonabend <raphael.sonabend.15@ucl.ac.uk> ([ORCID](#))

Authors:

- Franz Kiraly <f.kiraly@ucl.ac.uk>

## See Also

Useful links:

- <https://xoops.github.io/set6/>
- <https://github.com/xoopR/set6>
- Report bugs at <https://github.com/xoopR/set6/issues>

---

as.FuzzySet

*Coercion to R6 FuzzySet/FuzzyTuple*

---

## Description

Coerces object to an R6 [FuzzySet/FuzzyTuple](#)

## Usage

```
as.FuzzySet(object)
```

```
## S3 method for class 'numeric'  
as.FuzzySet(object)
```

```
## S3 method for class 'list'  
as.FuzzySet(object)
```

```
## S3 method for class 'matrix'
as.FuzzySet(object)

## S3 method for class 'data.frame'
as.FuzzySet(object)

## S3 method for class 'Set'
as.FuzzySet(object)

## S3 method for class 'FuzzySet'
as.FuzzySet(object)

## S3 method for class 'Interval'
as.FuzzySet(object)

## S3 method for class 'ConditionalSet'
as.FuzzySet(object)

as.FuzzyTuple(object)

## S3 method for class 'numeric'
as.FuzzyTuple(object)

## S3 method for class 'list'
as.FuzzyTuple(object)

## S3 method for class 'matrix'
as.FuzzyTuple(object)

## S3 method for class 'data.frame'
as.FuzzyTuple(object)

## S3 method for class 'Set'
as.FuzzyTuple(object)

## S3 method for class 'FuzzySet'
as.FuzzyTuple(object)

## S3 method for class 'Interval'
as.FuzzyTuple(object)

## S3 method for class 'ConditionalSet'
as.FuzzyTuple(object)

as.FuzzyMultiset(object)

## S3 method for class 'numeric'
as.FuzzyMultiset(object)
```

```
## S3 method for class 'list'  
as.FuzzyMultiset(object)  
  
## S3 method for class 'matrix'  
as.FuzzyMultiset(object)  
  
## S3 method for class 'data.frame'  
as.FuzzyMultiset(object)  
  
## S3 method for class 'Set'  
as.FuzzyMultiset(object)  
  
## S3 method for class 'FuzzySet'  
as.FuzzyMultiset(object)  
  
## S3 method for class 'Interval'  
as.FuzzyMultiset(object)  
  
## S3 method for class 'ConditionalSet'  
as.FuzzyMultiset(object)
```

### Arguments

object            object to coerce

### Details

- `as.FuzzySet.list` - Assumes `list` has two items, named `elements` and `membership`, and that they are ordered to be corresponding.
- `as.FuzzySet.matrix` - Assumes first column corresponds to `elements` and second column corresponds to their respective `membership`.
- `as.FuzzySet.data.frame` - First checks to see if one column is called `elements` and the other is called `membership`. If not then uses `as.FuzzySet.matrix`.
- `as.FuzzySet.Set` - Creates a [FuzzySet](#) by assuming `Set` elements all have `membership` equal to 1.
- `as.FuzzySet.Interval` - First tries coercion via [as.Set.Interval](#) then uses [as.FuzzySet.Set](#).

### See Also

[FuzzySet](#) [FuzzyTuple](#)

Other coercions: [as.Interval\(\)](#), [as.Set\(\)](#)

---

as.Interval	Coercion to R6 Interval
-------------	-------------------------

---

### Description

Coerces object to an R6 [Interval](#).

### Usage

```
as.Interval(object)

## S3 method for class 'Set'
as.Interval(object)

## S3 method for class 'Interval'
as.Interval(object)

## S3 method for class 'list'
as.Interval(object)

## S3 method for class 'data.frame'
as.Interval(object)

## S3 method for class 'matrix'
as.Interval(object)

## S3 method for class 'numeric'
as.Interval(object)

## S3 method for class 'ConditionalSet'
as.Interval(object)
```

### Arguments

object            object to coerce

### Details

- `as.Interval.list/as.Interval.data.frame` - Assumes the `list/data.frame` has named items/columns: `lower`, `upper`, `type`, `class`.
- `as.Interval.numeric` - If the `numeric` vector is a continuous interval with no breaks then coerces to an [Interval](#) with: `lower = min(object)`, `upper = max(object)`, `class = "integer"`. Ordering is ignored.
- `as.Interval.matrix` - Tries coercion via [as.Interval.numeric](#) on the first column of the matrix.
- `as.Interval.Set` - First tries coercion via [as.Interval.numeric](#), if possible wraps result in a [Set](#).

- `as.Interval.FuzzySet` - Tries coercion via `as.Interval.Set` on the support of the `FuzzySet`.

### See Also

[Interval](#)

Other coercions: [as.FuzzySet\(\)](#), [as.Set\(\)](#)

---

as.Set

*Coercion to R6 Set/Tuple*

---

### Description

Coerces object to an R6 [Set/Tuple](#)

### Usage

```
as.Set(object)

## Default S3 method:
as.Set(object)

## S3 method for class 'numeric'
as.Set(object)

## S3 method for class 'list'
as.Set(object)

## S3 method for class 'matrix'
as.Set(object)

## S3 method for class 'data.frame'
as.Set(object)

## S3 method for class 'Set'
as.Set(object)

## S3 method for class 'FuzzySet'
as.Set(object)

## S3 method for class 'Interval'
as.Set(object)

## S3 method for class 'ConditionalSet'
as.Set(object)

as.Tuple(object)
```



```
## Default S3 method:
as.Tuple(object)

## S3 method for class 'numeric'
as.Tuple(object)

## S3 method for class 'list'
as.Tuple(object)

## S3 method for class 'matrix'
as.Tuple(object)

## S3 method for class 'data.frame'
as.Tuple(object)

## S3 method for class 'FuzzySet'
as.Tuple(object)

## S3 method for class 'Set'
as.Tuple(object)

## S3 method for class 'Interval'
as.Tuple(object)

## S3 method for class 'ConditionalSet'
as.Tuple(object)

as.Multiset(object)

## Default S3 method:
as.Multiset(object)

## S3 method for class 'numeric'
as.Multiset(object)

## S3 method for class 'list'
as.Multiset(object)

## S3 method for class 'matrix'
as.Multiset(object)

## S3 method for class 'data.frame'
as.Multiset(object)

## S3 method for class 'FuzzySet'
as.Multiset(object)

## S3 method for class 'Set'
```

```

as.Multiset(object)

## S3 method for class 'Interval'
as.Multiset(object)

## S3 method for class 'ConditionalSet'
as.Multiset(object)

```

### Arguments

object                    object to coerce

### Details

- `as.Set.default` - Creates a [Set](#) using the object as the elements.
- `as.Set.list` - Creates a [Set](#) for each element in list.
- `as.Set.matrix/as.Set.data.frame` - Creates a [Set](#) for each column in matrix/data.frame.
- `as.Set.FuzzySet` - Creates a [Set](#) from the support of the [FuzzySet](#).
- `as.Set.Interval` - If the interval has finite cardinality then creates a [Set](#) from the [Interval](#) elements.

### See Also

[Set Tuple](#)

Other coercions: [as.FuzzySet\(\)](#), [as.Interval\(\)](#)

---

ComplementSet

*Set of Complements*

---

### Description

ComplementSet class for symbolic complement of mathematical sets.

### Details

The purpose of this class is to provide a symbolic representation for the complement of sets that cannot be represented in a simpler class. Whilst this is not an abstract class, it is not recommended to construct this class directly but via the set operation methods.

### Super classes

`set6::Set` -> `set6::SetWrapper` -> `ComplementSet`

**Active bindings**

elements Returns the elements in the object.

length Returns the number of elements in the object.

addedSet For the ComplementSet wrapper, X-Y, returns the set X.

subtractedSet For the ComplementSet wrapper, X-Y, returns the set Y.

**Methods****Public methods:**

- [ComplementSet\\$new\(\)](#)
- [ComplementSet\\$strprint\(\)](#)
- [ComplementSet\\$contains\(\)](#)
- [ComplementSet\\$clone\(\)](#)

**Method new():** Create a new ComplementSet object. It is not recommended to construct this class directly.

*Usage:*

```
ComplementSet$new(addset, subtractset, lower = NULL, upper = NULL, type = NULL)
```

*Arguments:*

addset [Set](#) to be subtracted from.

subtractset [Set](#) to subtract.

lower lower bound of new object.

upper upper bound of new object.

type closure type of new object.

*Returns:* A new ComplementSet object.

**Method strprint():** Creates a printable representation of the object.

*Usage:*

```
ComplementSet$strprint(n = 2)
```

*Arguments:*

n numeric. Number of elements to display on either side of ellipsis when printing.

*Returns:* A character string representing the object.

**Method contains():** Tests if elements x are contained in self.

*Usage:*

```
ComplementSet$contains(x, all = FALSE, bound = FALSE)
```

*Arguments:*

x [Set](#) or vector of [Sets](#).

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

bound logical

*Returns:* If `all == TRUE` then returns `TRUE` if all `x` are contained in `self`, otherwise `FALSE`. If `all == FALSE` returns a vector of logicals corresponding to the length of `x`, representing if each is contained in `self`. If `bound == TRUE` then an element is contained in `self` if it is on or within the (possibly-open) bounds of `self`, otherwise `TRUE` only if the element is within `self` or the bounds are closed.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
ComplementSet$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

### See Also

Set operations: [setunion](#), [setproduct](#), [setpower](#), [setcomplement](#), [setsymdiff](#), [powerset](#), [setintersect](#)

Other wrappers: [ExponentSet](#), [PowersetSet](#), [ProductSet](#), [UnionSet](#)

---

Complex

*Set of Complex Numbers*

---

### Description

The mathematical set of complex numbers, defined as the the set of reals with possibly imaginary components. i.e.

$$a + bi : a, b \in R$$

where  $R$  is the set of reals.

### Details

There is no inherent ordering in the set of complex numbers, hence only the `contains` method is implemented here.

### Super class

`set6::Set` -> Complex

### Methods

#### Public methods:

- [Complex\\$new\(\)](#)
- [Complex\\$contains\(\)](#)
- [Complex\\$equals\(\)](#)
- [Complex\\$isSubset\(\)](#)
- [Complex\\$strprint\(\)](#)
- [Complex\\$clone\(\)](#)

**Method** `new()`: Create a new Complex object.

*Usage:*

```
Complex$new()
```

*Returns:* A new Complex object.

**Method** `contains()`: Tests to see if `x` is contained in the Set.

*Usage:*

```
Complex$contains(x, all = FALSE, bound = NULL)
```

*Arguments:*

`x` any. Object or vector of objects to test.

`all` logical. If FALSE tests each `x` separately. Otherwise returns TRUE only if all `x` pass test.

`bound` logical.

*Details:* `x` can be of any type, including a Set itself. `x` should be a tuple if checking to see if it lies within a set of dimension greater than one. To test for multiple `x` at the same time, then provide these as a list.

If `all = TRUE` then returns TRUE if all `x` are contained in the Set, otherwise returns a vector of logicals. For [Intervals](#), `bound` is used to specify if elements lying on the (possibly open) boundary of the interval are considered contained (`bound = TRUE`) or not (`bound = FALSE`).

*Returns:* If `all` is TRUE then returns TRUE if all elements of `x` are contained in the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

The infix operator `%inset%` is available to test if `x` is an element in the Set, see examples.

**Method** `equals()`: Tests if two sets are equal.

*Usage:*

```
Complex$equals(x, all = FALSE)
```

*Arguments:*

`x` [Set](#) or vector of [Sets](#).

`all` logical. If FALSE tests each `x` separately. Otherwise returns TRUE only if all `x` pass test.

*Returns:* If `all` is TRUE then returns TRUE if all `x` are equal to the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

Infix operators can be used for:

```
Equal      ==
Not equal  !=
```

*Examples:*

```
# Equals
Set$new(1,2)$equals(Set$new(5,6))
Set$new(1,2)$equals(Interval$new(1,2))
Set$new(1,2) == Interval$new(1,2, class = "integer")

# Not equal
```

```
!Set$new(1,2)$equals(Set$new(1,2))
Set$new(1,2) != Set$new(1,5)
```

**Method** `isSubset()`: Test if one set is a (proper) subset of another

*Usage:*

```
Complex$isSubset(x, proper = FALSE, all = FALSE)
```

*Arguments:*

`x` any. Object or vector of objects to test.

`proper` logical. If TRUE tests for proper subsets.

`all` logical. If FALSE tests each `x` separately. Otherwise returns TRUE only if all `x` pass test.

*Details:* If using the method directly, and not via one of the operators then the additional boolean argument `proper` can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set.

When calling `$isSubset` on objects inheriting from [Interval](#), the method treats the interval as if it is a [Set](#), i.e. ordering and class are ignored. Use `$isSubinterval` to test if one interval is a subinterval of another.

Infix operators can be used for:

Subset	<
Proper Subset	<=
Superset	>
Proper Superset	>=

Every Set is a subset of a Universal. No Set is a super set of a Universal, and only a Universal is not a proper subset of a Universal.

*Returns:* If `all` is TRUE then returns TRUE if all `x` are subsets of the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

*Examples:*

```
Set$new(1,2,3)$isSubset(Set$new(1,2), proper = TRUE)
Set$new(1,2) < Set$new(1,2,3) # proper subset
```

```
c(Set$new(1,2,3), Set$new(1)) < Set$new(1,2,3) # not proper
Set$new(1,2,3) <= Set$new(1,2,3) # proper
```

**Method** `strprint()`: Creates a printable representation of the object.

*Usage:*

```
Complex$strprint(n = 2)
```

*Arguments:*

`n` numeric. Number of elements to display on either side of ellipsis when printing.

*Returns:* A character string representing the object.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
Complex$clone(deep = FALSE)
```

*Arguments:*

deep Whether to make a deep clone.

### See Also

Other special sets: [ExtendedReals](#), [Integers](#), [Logicals](#), [Naturals](#), [NegIntegers](#), [NegRationals](#), [NegReals](#), [PosIntegers](#), [PosNaturals](#), [PosRationals](#), [PosReals](#), [Rationals](#), [Reals](#), [Universal](#)

### Examples

```
## -----
## Method `Complex$equals`
## -----

# Equals
Set$new(1,2)$equals(Set$new(5,6))
Set$new(1,2)$equals(Interval$new(1,2))
Set$new(1,2) == Interval$new(1,2, class = "integer")

# Not equal
!Set$new(1,2)$equals(Set$new(1,2))
Set$new(1,2) != Set$new(1,5)

## -----
## Method `Complex$isSubset`
## -----

Set$new(1,2,3)$isSubset(Set$new(1,2), proper = TRUE)
Set$new(1,2) < Set$new(1,2,3) # proper subset

c(Set$new(1,2,3), Set$new(1)) < Set$new(1,2,3) # not proper
Set$new(1,2,3) <= Set$new(1,2,3) # proper
```

---

ConditionalSet

*Mathematical Set of Conditions*

---

### Description

A mathematical set defined by one or more logical conditions.

### Details

Conditional sets are a useful tool for symbolically defining possibly infinite sets. They can be combined using standard 'and', &, and 'or', |, operators.

### Super class

`set6::Set` -> ConditionalSet

**Active bindings**

`condition` Returns the condition defining the ConditionalSet.

`class` Returns `argclass`, see `$new`.

`elements` Returns NA.

**Methods****Public methods:**

- [ConditionalSet\\$new\(\)](#)
- [ConditionalSet\\$contains\(\)](#)
- [ConditionalSet\\$equals\(\)](#)
- [ConditionalSet\\$strprint\(\)](#)
- [ConditionalSet\\$summary\(\)](#)
- [ConditionalSet\\$isSubset\(\)](#)
- [ConditionalSet\\$clone\(\)](#)

**Method** `new()`: Create a new ConditionalSet object.

*Usage:*

```
ConditionalSet$new(condition = function(x) TRUE, argclass = NULL)
```

*Arguments:*

`condition` function. Defines the set, see details.

`argclass` list. Optional list of sets that the function arguments live in, see details.

*Details:* The condition should be given as a function that when evaluated returns either TRUE or FALSE. Further constraints can be given by providing the universe of the function arguments as [Sets](#), if these are not given then [Universal](#) is assumed. See examples. Defaults construct the Universal set.

*Returns:* A new ConditionalSet object.

**Method** `contains()`: Tests to see if x is contained in the Set.

*Usage:*

```
ConditionalSet$contains(x, all = FALSE, bound = NULL)
```

*Arguments:*

`x` any. Object or vector of objects to test.

`all` logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

`bound` ignored, added for consistency.

*Details:* x can be of any type, including a Set itself. x should be a tuple if checking to see if it lies within a set of dimension greater than one. To test for multiple x at the same time, then provide these as a list.

If `all = TRUE` then returns TRUE if all x are contained in the Set, otherwise returns a vector of logicals.

An element is contained in a ConditionalSet if it returns TRUE as an argument in the defining function. For sets that are defined with a function that takes multiple arguments, a [Tuple](#) should be passed to x.



*Returns:* If `all` is TRUE then returns TRUE if all elements of `x` are contained in the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

The infix operator `%inset%` is available to test if `x` is an element in the Set, see examples.

*Examples:*

```
# Set of positives
s = ConditionalSet$new(function(x) x > 0)
s$contains(list(1,-1))

# Set via equality
s = ConditionalSet$new(function(x, y) x + y == 2)
s$contains(list(Set$new(2, 0), Set$new(0, 2)))

# Tuples are recommended when using contains as they allow non-unique elements
s = ConditionalSet$new(function(x, y) x + y == 4)
\dontrun{
s$contains(Set$new(2, 2)) # Errors as Set$new(2,2) == Set$new(2)
}

# Set of Positive Naturals
s = ConditionalSet$new(function(x) TRUE, argclass = list(x = PosNaturals$new()))
s$contains(list(-2, 2))
```

**Method** `equals()`: Tests if two sets are equal.

*Usage:*

```
ConditionalSet$equals(x, all = FALSE)
```

*Arguments:*

`x` [Set](#) or vector of [Sets](#).

`all` logical. If FALSE tests each `x` separately. Otherwise returns TRUE only if all `x` pass test.

*Details:* Two sets are equal if they contain the same elements. Infix operators can be used for:

```
Equal      ==
Not equal  !=
```

*Returns:* If `all` is TRUE then returns TRUE if all `x` are equal to the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

**Method** `strprint()`: Creates a printable representation of the object.

*Usage:*

```
ConditionalSet$strprint(n = NULL)
```

*Arguments:*

`n` ignored, added for consistency.

*Returns:* A character string representing the object.

**Method** `summary()`: See `strprint`.

*Usage:*

ConditionalSet\$summary(n = NULL)

*Arguments:*

n ignored, added for consistency.

**Method** isSubset(): Currently undefined for ConditionalSets.

*Usage:*

ConditionalSet\$isSubset(x, proper = FALSE, all = FALSE)

*Arguments:*

x ignored, added for consistency.

proper ignored, added for consistency.

all ignored, added for consistency.

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

ConditionalSet\$clone(deep = FALSE)

*Arguments:*

deep Whether to make a deep clone.

## See Also

Other sets: [FuzzyMultiset](#), [FuzzySet](#), [FuzzyTuple](#), [Interval](#), [Multiset](#), [Set](#), [Tuple](#)

## Examples

```
# Set of Positive Naturals
s <- ConditionalSet$new(function(x) TRUE, argclass = list(x = PosNaturals$new()))

## -----
## Method `ConditionalSet$contains`
## -----

# Set of positives
s = ConditionalSet$new(function(x) x > 0)
s$contains(list(1,-1))

# Set via equality
s = ConditionalSet$new(function(x, y) x + y == 2)
s$contains(list(Set$new(2, 0), Set$new(0, 2)))

# Tuples are recommended when using contains as they allow non-unique elements
s = ConditionalSet$new(function(x, y) x + y == 4)
## Not run:
s$contains(Set$new(2, 2)) # Errors as Set$new(2,2) == Set$new(2)

## End(Not run)

# Set of Positive Naturals
s = ConditionalSet$new(function(x) TRUE, argclass = list(x = PosNaturals$new()))
s$contains(list(-2, 2))
```

---

contains                      *contains Operator*

---

### Description

Operator for \$contains methods. See [Set](#)\$contains for full details. Operators can be used for:

Name	Description	Operator
Contains	x contains y	y \$inset\$ x

### Usage

```
x %inset% y
```

### Arguments

x, y                      [Set](#)

---

equals                      *equals Operator*

---

### Description

Operator for \$equals methods. See [Set](#)\$equals for full details. Operators can be used for:

Name	Description	Operator
Equal	x equals y	==
Not Equal	x does not equal y	!=

### Usage

```
## S3 method for class 'Set'
x == y

## S3 method for class 'Set'
x != y
```

### Arguments

x, y                      [Set](#)

---

 ExponentSet

*Set of Exponentiations*


---

## Description

ExponentSet class for symbolic exponentiation of mathematical sets.

## Details

The purpose of this class is to provide a symbolic representation for the exponentiation of sets that cannot be represented in a simpler class. Whilst this is not an abstract class, it is not recommended to construct this class directly but via the set operation methods.

## Super classes

`set6::Set` -> `set6::SetWrapper` -> `set6::ProductSet` -> ExponentSet

## Active bindings

`power` Returns the power that the wrapped set is raised to.

## Methods

### Public methods:

- `ExponentSet$new()`
- `ExponentSet$toString()`
- `ExponentSet$contains()`
- `ExponentSet$clone()`

**Method** `new()`: Create a new ExponentSet object. It is not recommended to construct this class directly.

*Usage:*

`ExponentSet$new(set, power)`

*Arguments:*

`set` `Set` to wrap.

`power` numeric. Power to raise Set to.

*Returns:* A new ExponentSet object.

**Method** `toString()`: Creates a printable representation of the object.

*Usage:*

`ExponentSet$toString(n = 2)`

*Arguments:*

`n` numeric. Number of elements to display on either side of ellipsis when printing.

*Returns:* A character string representing the object.

**Method** `contains()`: Tests if elements `x` are contained in `self`.

*Usage:*

`ExponentSet$contains(x, all = FALSE, bound = FALSE)`

*Arguments:*

`x` [Set](#) or vector of [Sets](#).

`all` logical. If `FALSE` tests each `x` separately. Otherwise returns `TRUE` only if all `x` pass test.

`bound` logical

*Returns:* If `all == TRUE` then returns `TRUE` if all `x` are contained in `self`, otherwise `FALSE`. If `all == FALSE` returns a vector of logicals corresponding to the length of `x`, representing if each is contained in `self`. If `bound == TRUE` then an element is contained in `self` if it is on or within the (possibly-open) bounds of `self`, otherwise `TRUE` only if the element is within `self` or the bounds are closed.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

`ExponentSet$clone(deep = FALSE)`

*Arguments:*

`deep` Whether to make a deep clone.

## See Also

Set operations: [setunion](#), [setproduct](#), [setpower](#), [setcomplement](#), [setsymdiff](#), [powerset](#), [setintersect](#)

Other wrappers: [ComplementSet](#), [PowersetSet](#), [ProductSet](#), [UnionSet](#)

---

ExtendedReals

*Set of Extended Real Numbers*

---

## Description

The mathematical set of extended real numbers, defined as the union of the set of reals with  $\pm\infty$ .  
i.e.

$$R \cup -\infty, \infty$$

where  $R$  is the set of reals.

## Super classes

[set6::Set](#) -> [set6::Interval](#) -> [set6::SpecialSet](#) -> [set6::Reals](#) -> ExtendedReals

**Methods****Public methods:**

- [ExtendedReals\\$new\(\)](#)
- [ExtendedReals\\$clone\(\)](#)

**Method** `new()`: Create a new `ExtendedReals` object.

*Usage:*

```
ExtendedReals$new()
```

*Returns:* A new `ExtendedReals` object.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
ExtendedReals$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

**See Also**

Other special sets: [Complex](#), [Integers](#), [Logicals](#), [Naturals](#), [NegIntegers](#), [NegRationals](#), [NegReals](#), [PosIntegers](#), [PosNaturals](#), [PosRationals](#), [PosReals](#), [Rationals](#), [Reals](#), [Universal](#)

---

FuzzyMultiset

*Mathematical Fuzzy Multiset*

---

**Description**

A general `FuzzyMultiset` object for mathematical fuzzy multisets, inheriting from `FuzzySet`.

**Details**

Fuzzy multisets generalise standard mathematical multisets to allow for fuzzy relationships. Whereas a standard, or crisp, multiset assumes that an element is either in a multiset or not, a fuzzy multiset allows an element to be in a multiset to a particular degree, known as the membership function, which quantifies the inclusion of an element by a number in  $[0, 1]$ . Thus a (crisp) multiset is a fuzzy multiset where all elements have a membership equal to 1. Similarly to [Multisets](#), elements do not need to be unique.

**Super classes**

```
set6::Set -> set6::FuzzySet -> FuzzyMultiset
```

**Methods****Public methods:**

- `FuzzyMultiset$equals()`
- `FuzzyMultiset$isSubset()`
- `FuzzyMultiset$alphaCut()`
- `FuzzyMultiset$clone()`

**Method** `equals()`: Tests if two sets are equal.

*Usage:*

`FuzzyMultiset$equals(x, all = FALSE)`

*Arguments:*

x `Set` or vector of `Sets`.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Details:* Two fuzzy sets are equal if they contain the same elements with the same memberships and in the same order. Infix operators can be used for:

Equal	==
Not equal	!=

*Returns:* If all is TRUE then returns TRUE if all x are equal to the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

**Method** `isSubset()`: Test if one set is a (proper) subset of another

*Usage:*

`FuzzyMultiset$isSubset(x, proper = FALSE, all = FALSE)`

*Arguments:*

x any. Object or vector of objects to test.

proper logical. If TRUE tests for proper subsets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Details:* If using the method directly, and not via one of the operators then the additional boolean argument proper can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set.

Infix operators can be used for:

Subset	<
Proper Subset	<=
Superset	>
Proper Superset	>=

*Returns:* If all is TRUE then returns TRUE if all x are subsets of the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

**Method** `alphaCut()`: The alpha-cut of a fuzzy set is defined as the set

$$A_\alpha = \{x \in F \mid m \geq \alpha\}$$

where  $x$  is an element in the fuzzy set,  $F$ , and  $m$  is the corresponding membership.

*Usage:*

```
FuzzyMultiset$alphaCut(alpha, strong = FALSE, create = FALSE)
```

*Arguments:*

alpha numeric in [0, 1] to determine which elements to return

strong logical, if FALSE (default) then includes elements greater than or equal to alpha, otherwise only strictly greater than

create logical, if FALSE (default) returns the elements in the alpha cut, otherwise returns a crisp set of the elements

*Returns:* Elements in [FuzzyMultiset](#) or a [Set](#) of the elements.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
FuzzyMultiset$clone(deep = FALSE)
```

*Arguments:*

deep Whether to make a deep clone.

## See Also

Other sets: [ConditionalSet](#), [FuzzySet](#), [FuzzyTuple](#), [Interval](#), [Multiset](#), [Set](#), [Tuple](#)

## Examples

```
# Different constructors
FuzzyMultiset$new(1, 0.5, 2, 1, 3, 0)
FuzzyMultiset$new(elements = 1:3, membership = c(0.5, 1, 0))

# Crisp sets are a special case FuzzyMultiset
# Note membership defaults to full membership
FuzzyMultiset$new(elements = 1:5) == Multiset$new(1:5)

f <- FuzzyMultiset$new(1, 0.2, 2, 1, 3, 0)
f$membership()
f$alphaCut(0.3)
f$core()
f$inclusion(0)
f$membership(0)
f$membership(1)

# Elements can be duplicated, and with different memberships,
# although this is not necessarily sensible.
FuzzyMultiset$new(1, 0.1, 1, 1)

# Like FuzzySets, ordering does not matter.
FuzzyMultiset$new(1, 0.1, 2, 0.2) == FuzzyMultiset$new(2, 0.2, 1, 0.1)
```



---

FuzzySet

*Mathematical Fuzzy Set*

---

## Description

A general FuzzySet object for mathematical fuzzy sets, inheriting from Set.

## Details

Fuzzy sets generalise standard mathematical sets to allow for fuzzy relationships. Whereas a standard, or crisp, set assumes that an element is either in a set or not, a fuzzy set allows an element to be in a set to a particular degree, known as the membership function, which quantifies the inclusion of an element by a number in  $[0, 1]$ . Thus a (crisp) set is a fuzzy set where all elements have a membership equal to 1. Similarly to [Sets](#), elements must be unique and the ordering does not matter, to establish order and non-unique elements, [FuzzyTuples](#) can be used.

## Super class

[set6::Set](#) -> FuzzySet

## Methods

### Public methods:

- [FuzzySet\\$new\(\)](#)
- [FuzzySet\\$strprint\(\)](#)
- [FuzzySet\\$membership\(\)](#)
- [FuzzySet\\$alphaCut\(\)](#)
- [FuzzySet\\$support\(\)](#)
- [FuzzySet\\$core\(\)](#)
- [FuzzySet\\$inclusion\(\)](#)
- [FuzzySet\\$equals\(\)](#)
- [FuzzySet\\$isSubset\(\)](#)
- [FuzzySet\\$clone\(\)](#)

**Method** [new\(\)](#): Create a new FuzzySet object.

*Usage:*

```
FuzzySet$new(  
  ...,  
  elements = NULL,  
  membership = rep(1, length(elements)),  
  class = NULL  
)
```

*Arguments:*

... Alternating elements and membership, see details.

`elements` Elements in the set, see details.

`membership` Corresponding membership of the elements, see details.

`class` Optional string naming a class that if supplied gives the set the typed property.

*Details:* FuzzySets can be constructed in one of two ways, either by supplying the elements and their membership in alternate order, or by providing a list of elements to `elements` and a list of respective memberships to `membership`, see examples. If the `class` argument is non-NULL, then all elements will be coerced to the given class in construction, and if elements of a different class are added these will either be rejected or coerced.

*Returns:* A new FuzzySet object.

**Method** `strprint()`: Creates a printable representation of the object.

*Usage:*

```
FuzzySet$strprint(n = 2)
```

*Arguments:*

`n` numeric. Number of elements to display on either side of ellipsis when printing.

*Returns:* A character string representing the object.

**Method** `membership()`: Returns the membership, i.e. value in [0, 1], of either the given element(s) or all elements in the fuzzy set.

*Usage:*

```
FuzzySet$membership(element = NULL)
```

*Arguments:*

`element` element or list of element in the set, if NULL returns membership of all elements

*Details:* For FuzzySets this is straightforward and returns the membership of the given element(s), however in FuzzyTuples and FuzzyMultisets when an element may be duplicated, the function returns the membership of all instances of the element.

*Returns:* Value, or list of values, in [0, 1].

*Examples:*

```
f = FuzzySet$new(1, 0.1, 2, 0.5, 3, 1)
f$membership()
f$membership(2)
f$membership(list(1, 2))
```

**Method** `alphaCut()`: The alpha-cut of a fuzzy set is defined as the set

$$A_\alpha = \{x \in F | m \geq \alpha\}$$

where  $x$  is an element in the fuzzy set,  $F$ , and  $m$  is the corresponding membership.

*Usage:*

```
FuzzySet$alphaCut(alpha, strong = FALSE, create = FALSE)
```

*Arguments:*

`alpha` numeric in [0, 1] to determine which elements to return

`strong` logical, if FALSE (default) then includes elements greater than or equal to alpha, otherwise only strictly greater than

create logical, if FALSE (default) returns the elements in the alpha cut, otherwise returns a crisp set of the elements

*Returns:* Elements in [FuzzySet](#) or a [Set](#) of the elements.

*Examples:*

```
f = FuzzySet$new(1, 0.1, 2, 0.5, 3, 1)
# Alpha-cut
f$alphaCut(0.5)
```

```
# Strong alpha-cut
f$alphaCut(0.5, strong = TRUE)
```

```
# Create a set from the alpha-cut
f$alphaCut(0.5, create = TRUE)
```

**Method** support(): The support of a fuzzy set is defined as the set of elements whose membership is greater than zero, or the strong alpha-cut with  $\alpha = 0$ ,

$$A_\alpha = \{x \in F \mid m > 0\}$$

where  $x$  is an element in the fuzzy set,  $F$ , and  $m$  is the corresponding membership.

*Usage:*

```
FuzzySet$support(create = FALSE)
```

*Arguments:*

create logical, if FALSE (default) returns the support elements, otherwise returns a [Set](#) of the support elements

*Returns:* Support elements in fuzzy set or a [Set](#) of the support elements.

*Examples:*

```
f = FuzzySet$new(0.1, 0, 1, 0.1, 2, 0.5, 3, 1)
f$support()
f$support(TRUE)
```

**Method** core(): The core of a fuzzy set is defined as the set of elements whose membership is equal to one, or the alpha-cut with  $\alpha = 1$ ,

$$A_\alpha = \{x \in F : m \geq 1\}$$

where  $x$  is an element in the fuzzy set,  $F$ , and  $m$  is the corresponding membership.

*Usage:*

```
FuzzySet$core(create = FALSE)
```

*Arguments:*

create logical, if FALSE (default) returns the core elements, otherwise returns a [Set](#) of the core elements

*Returns:* Core elements in [FuzzySet](#) or a [Set](#) of the core elements.

*Examples:*

```
f = FuzzySet$new(0.1, 0, 1, 0.1, 2, 0.5, 3, 1)
f$core()
f$core(TRUE)
```

**Method** `inclusion()`: An element in a fuzzy set, with corresponding membership  $m$ , is:

- Included - If  $m = 1$
- Partially Included - If  $0 < m < 1$
- Not Included - If  $m = 0$

*Usage:*

```
FuzzySet$inclusion(element)
```

*Arguments:*

`element` element or list of elements in fuzzy set for which to get the inclusion level

*Details:* For [FuzzySets](#) this is straightforward and returns the inclusion level of the given element(s), however in [FuzzyTuples](#) and [FuzzyMultisets](#) when an element may be duplicated, the function returns the inclusion level of all instances of the element.

*Returns:* One of: "Included", "Partially Included", "Not Included"

*Examples:*

```
f = FuzzySet$new(0.1, 0, 1, 0.1, 2, 0.5, 3, 1)
f$inclusion(0.1)
f$inclusion(1)
f$inclusion(3)
```

**Method** `equals()`: Tests if two sets are equal.

*Usage:*

```
FuzzySet$equals(x, all = FALSE)
```

*Arguments:*

`x` [Set](#) or vector of [Sets](#).

`all` logical. If FALSE tests each `x` separately. Otherwise returns TRUE only if all `x` pass test.

*Details:* Two fuzzy sets are equal if they contain the same elements with the same memberships. Infix operators can be used for:

```
Equal      ==
Not equal  !=
```

*Returns:* If `all` is TRUE then returns TRUE if all `x` are equal to the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

**Method** `isSubset()`: Test if one set is a (proper) subset of another

*Usage:*

```
FuzzySet$isSubset(x, proper = FALSE, all = FALSE)
```

*Arguments:*

`x` any. Object or vector of objects to test.

proper logical. If TRUE tests for proper subsets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Details:* If using the method directly, and not via one of the operators then the additional boolean argument proper can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set.

Infix operators can be used for:

Subset	<
Proper Subset	<=
Superset	>
Proper Superset	>=

*Returns:* If all is TRUE then returns TRUE if all x are subsets of the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

```
FuzzySet$clone(deep = FALSE)
```

*Arguments:*

deep Whether to make a deep clone.

## See Also

Other sets: [ConditionalSet](#), [FuzzyMultiset](#), [FuzzyTuple](#), [Interval](#), [Multiset](#), [Set](#), [Tuple](#)

## Examples

```
# Different constructors
FuzzySet$new(1, 0.5, 2, 1, 3, 0)
FuzzySet$new(elements = 1:3, membership = c(0.5, 1, 0))

# Crisp sets are a special case FuzzySet
# Note membership defaults to full membership
FuzzySet$new(elements = 1:5) == Set$new(1:5)

f <- FuzzySet$new(1, 0.2, 2, 1, 3, 0)
f$membership()
f$alphaCut(0.3)
f$core()
f$inclusion(0)
f$membership(0)
f$membership(1)

## -----
## Method `FuzzySet$membership`
## -----

f = FuzzySet$new(1, 0.1, 2, 0.5, 3, 1)
f$membership()
```

```

f$membership(2)
f$membership(list(1, 2))

## -----
## Method `FuzzySet$alphaCut`
## -----

f = FuzzySet$new(1, 0.1, 2, 0.5, 3, 1)
# Alpha-cut
f$alphaCut(0.5)

# Strong alpha-cut
f$alphaCut(0.5, strong = TRUE)

# Create a set from the alpha-cut
f$alphaCut(0.5, create = TRUE)

## -----
## Method `FuzzySet$support`
## -----

f = FuzzySet$new(0.1, 0, 1, 0.1, 2, 0.5, 3, 1)
f$support()
f$support(TRUE)

## -----
## Method `FuzzySet$core`
## -----

f = FuzzySet$new(0.1, 0, 1, 0.1, 2, 0.5, 3, 1)
f$core()
f$core(TRUE)

## -----
## Method `FuzzySet$inclusion`
## -----

f = FuzzySet$new(0.1, 0, 1, 0.1, 2, 0.5, 3, 1)
f$inclusion(0.1)
f$inclusion(1)
f$inclusion(3)

```

---

FuzzyTuple

---

*Mathematical Fuzzy Tuple*


---

### Description

A general FuzzyTuple object for mathematical fuzzy tuples, inheriting from FuzzySet.

**Details**

Fuzzy tuples generalise standard mathematical tuples to allow for fuzzy relationships. Whereas a standard, or crisp, tuple assumes that an element is either in a tuple or not, a fuzzy tuple allows an element to be in a tuple to a particular degree, known as the membership function, which quantifies the inclusion of an element by a number in  $[0, 1]$ . Thus a (crisp) tuple is a fuzzy tuple where all elements have a membership equal to 1. Similarly to [Tuples](#), elements do not need to be unique and the ordering does matter, [FuzzySets](#) are special cases where the ordering does not matter and elements must be unique.

**Super classes**

```
set6::Set -> set6::FuzzySet -> FuzzyTuple
```

**Methods****Public methods:**

- [FuzzyTuple\\$equals\(\)](#)
- [FuzzyTuple\\$isSubset\(\)](#)
- [FuzzyTuple\\$alphaCut\(\)](#)
- [FuzzyTuple\\$clone\(\)](#)

**Method** `equals()`: Tests if two sets are equal.

*Usage:*

```
FuzzyTuple$equals(x, all = FALSE)
```

*Arguments:*

x [Set](#) or vector of [Sets](#).

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Details:* Two fuzzy sets are equal if they contain the same elements with the same memberships and in the same order. Infix operators can be used for:

```
Equal      ==
Not equal  !=
```

*Returns:* If all is TRUE then returns TRUE if all x are equal to the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

**Method** `isSubset()`: Test if one set is a (proper) subset of another

*Usage:*

```
FuzzyTuple$isSubset(x, proper = FALSE, all = FALSE)
```

*Arguments:*

x any. Object or vector of objects to test.

proper logical. If TRUE tests for proper subsets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Details:* If using the method directly, and not via one of the operators then the additional boolean argument `proper` can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set.

Infix operators can be used for:

Subset	<
Proper Subset	<=
Superset	>
Proper Superset	>=

*Returns:* If `all` is TRUE then returns TRUE if all `x` are subsets of the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

**Method** `alphaCut()`: The alpha-cut of a fuzzy set is defined as the set

$$A_\alpha = \{x \in F \mid m \geq \alpha\}$$

where `x` is an element in the fuzzy set, `F`, and `m` is the corresponding membership.

*Usage:*

```
FuzzyTuple$alphaCut(alpha, strong = FALSE, create = FALSE)
```

*Arguments:*

`alpha` numeric in [0, 1] to determine which elements to return

`strong` logical, if FALSE (default) then includes elements greater than or equal to alpha, otherwise only strictly greater than

`create` logical, if FALSE (default) returns the elements in the alpha cut, otherwise returns a crisp set of the elements

*Returns:* Elements in [FuzzyTuple](#) or a [Set](#) of the elements.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
FuzzyTuple$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

## See Also

Other sets: [ConditionalSet](#), [FuzzyMultiset](#), [FuzzySet](#), [Interval](#), [Multiset](#), [Set](#), [Tuple](#)

## Examples

```
# Different constructors
FuzzyTuple$new(1, 0.5, 2, 1, 3, 0)
FuzzyTuple$new(elements = 1:3, membership = c(0.5, 1, 0))

# Crisp sets are a special case FuzzyTuple
# Note membership defaults to full membership
FuzzyTuple$new(elements = 1:5) == Tuple$new(1:5)
```



```

f <- FuzzyTuple$new(1, 0.2, 2, 1, 3, 0)
f$membership()
f$alphaCut(0.3)
f$core()
f$inclusion(0)
f$membership(0)
f$membership(1)

# Elements can be duplicated, and with different memberships,
# although this is not necessarily sensible.
FuzzyTuple$new(1, 0.1, 1, 1)

# More important is ordering.
FuzzyTuple$new(1, 0.1, 2, 0.2) != FuzzyTuple$new(2, 0.2, 1, 0.1)
FuzzySet$new(1, 0.1, 2, 0.2) == FuzzySet$new(2, 0.2, 1, 0.1)

```

---

Integers

*Set of Integers*


---

### Description

The mathematical set of integers, defined as the set of whole numbers. i.e.

$$\dots, -3, -2, -1, 0, 1, 2, 3, \dots$$

### Super classes

`set6::Set` -> `set6::Interval` -> `set6::SpecialSet` -> Integers

### Methods

#### Public methods:

- `Integers$new()`
- `Integers$clone()`

**Method** `new()`: Create a new Integers object.

*Usage:*

```
Integers$new(lower = -Inf, upper = Inf, type = "()")
```

*Arguments:*

`lower` numeric. Where to start the set. Advised to ignore, used by child-classes.

`upper` numeric. Where to end the set. Advised to ignore, used by child-classes.

`type` character Set closure type. Advised to ignore, used by child-classes.

*Returns:* A new Integers object.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
Integers$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

**See Also**

Other special sets: [Complex](#), [ExtendedReals](#), [Logicals](#), [Naturals](#), [NegIntegers](#), [NegRationals](#), [NegReals](#), [PosIntegers](#), [PosNaturals](#), [PosRationals](#), [PosReals](#), [Rationals](#), [Reals](#), [Universal](#)

Interval

*Mathematical Finite or Infinite Interval***Description**

A general Interval object for mathematical intervals, inheriting from [Set](#). Intervals may be open, closed, or half-open; as well as bounded above, below, or not at all.

**Details**

The Interval class can be used for finite or infinite intervals, but often Sets will be preferred for integer intervals over a finite continuous range.

**Super class**

```
set6::Set -> Interval
```

**Active bindings**

`length` If the Interval is countably finite then returns the number of elements in the Interval, otherwise Inf. See the cardinality property for the type of infinity.

`elements` If the Interval is finite then returns all elements in the Interval, otherwise NA.

**Methods****Public methods:**

- [Interval\\$new\(\)](#)
- [Interval\\$strprint\(\)](#)
- [Interval\\$equals\(\)](#)
- [Interval\\$contains\(\)](#)
- [Interval\\$isSubset\(\)](#)
- [Interval\\$isSubinterval\(\)](#)
- [Interval\\$clone\(\)](#)

**Method** `new()`: Create a new Interval object.

*Usage:*

```
Interval$new(
  lower = -Inf,
  upper = Inf,
  type = c("[", "]", "[)", "()"),
  class = "numeric",
  universe = ExtendedReals$new()
)
```

*Arguments:*

lower numeric. Lower limit of the interval.

upper numeric. Upper limit of the interval.

type character. One of: '()', '()', '[]', '[]', which specifies if interval is open, left-open, right-open, or closed.

class character. One of: 'numeric', 'integer', which specifies if interval is over the Reals or Integers.

universe Set. Universe that the interval lives in, default [Reals](#).

*Details:* Intervals are constructed by specifying the Interval limits, the boundary type, the class, and the possible universe. The universe differs from class as it is primarily used for the [setcomplement](#) method. Whereas class specifies if the interval takes integers or numerics, the universe specifies what range the interval could take.

*Returns:* A new Interval object.

**Method** `strprint()`: Creates a printable representation of the object.

*Usage:*

```
Interval$strprint(...)
```

*Arguments:*

... ignored, added for consistency.

*Returns:* A character string representing the object.

**Method** `equals()`: Tests if two sets are equal.

*Usage:*

```
Interval$equals(x, all = FALSE)
```

*Arguments:*

x [Set](#) or vector of [Sets](#).

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Details:* Two Intervals are equal if they have the same: class, type, and bounds. Infix operators can be used for:

```
Equal      ==
Not equal  !=
```

*Returns:* If all is TRUE then returns TRUE if all x are equal to the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

*Examples:*

```
Interval$new(1,5) == Interval$new(1,5)
```

```
Interval$new(1,5, class = "integer") != Interval$new(1,5,class="numeric")
```

**Method** `contains()`: Tests to see if x is contained in the Set.

*Usage:*

```
Interval$contains(x, all = FALSE, bound = FALSE)
```

*Arguments:*

*x* any. Object or vector of objects to test.

*all* logical. If FALSE tests each *x* separately. Otherwise returns TRUE only if all *x* pass test.

*bound* logical.

*Details:* *x* can be of any type, including a Set itself. *x* should be a tuple if checking to see if it lies within a set of dimension greater than one. To test for multiple *x* at the same time, then provide these as a list.

If *all* = TRUE then returns TRUE if all *x* are contained in the Set, otherwise returns a vector of logicals. For [Intervals](#), *bound* is used to specify if elements lying on the (possibly open) boundary of the interval are considered contained (*bound* = TRUE) or not (*bound* = FALSE).

*Returns:* If *all* is TRUE then returns TRUE if all elements of *x* are contained in the Set, otherwise FALSE. If *all* is FALSE then returns a vector of logicals corresponding to each individual element of *x*.

The infix operator `%inset%` is available to test if *x* is an element in the Set, see examples.

*Examples:*

```
s = Set$new(1:5)

# Simplest case
s$contains(4)
8 %inset% s

# Test if multiple elements lie in the set
s$contains(4:6, all = FALSE)
s$contains(4:6, all = TRUE)

# Check if a tuple lies in a Set of higher dimension
s2 = s * s
s2$contains(Tuple$new(2,1))
c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2
```

**Method** `isSubset()`: Test if one set is a (proper) subset of another

*Usage:*

```
Interval$isSubset(x, proper = FALSE, all = FALSE)
```

*Arguments:*

*x* any. Object or vector of objects to test.

*proper* logical. If TRUE tests for proper subsets.

*all* logical. If FALSE tests each *x* separately. Otherwise returns TRUE only if all *x* pass test.

*Details:* If using the method directly, and not via one of the operators then the additional boolean argument *proper* can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set.

When calling `isSubset` on objects inheriting from `Interval`, the method treats the interval as if it is a [Set](#), i.e. ordering and class are ignored. Use `isSubinterval` to test if one interval is a subinterval of another.

Infix operators can be used for:

Subset	<
Proper Subset	<=
Superset	>
Proper Superset	>=

*Returns:* If `all` is TRUE then returns TRUE if all `x` are subsets of the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

*Examples:*

```
Interval$new(1,3) < Interval$new(1,5)
Set$new(1,3) < Interval$new(0,5)
```

**Method** `isSubinterval()`: Test if one interval is a (proper) subinterval of another

*Usage:*

```
Interval$isSubinterval(x, proper = FALSE, all = FALSE)
```

*Arguments:*

`x` Set or list

`proper` If TRUE then tests if `x` is a proper subinterval (i.e. subinterval and not equal to) of `self`, otherwise FALSE tests if `x` is a (non-proper) subinterval.

`all` If TRUE then returns TRUE if all `x` are subintervals, otherwise returns a vector of logicals.

*Details:* If `x` is a [Set](#) then will be coerced to an [Interval](#) if possible. `$isSubinterval` differs from `$isSubset` in that ordering and class are respected in `$isSubinterval`. See examples for a clearer illustration of the difference.

*Returns:* If `all` is TRUE then returns TRUE if all `x` are subsets of the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

*Examples:*

```
Interval$new(1,3)$isSubset(Set$new(1,2)) # TRUE
Interval$new(1,3)$isSubset(Set$new(2, 1)) # TRUE
Interval$new(1,3, class = "integer")$isSubinterval(Set$new(1, 2)) # TRUE
Interval$new(1,3)$isSubinterval(Set$new(1, 2)) # FALSE
Interval$new(1,3)$isSubinterval(Set$new(2, 1)) # FALSE
```

```
Reals$new()$isSubset(Integers$new()) # TRUE
Reals$new()$isSubinterval(Integers$new()) # FALSE
```

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
Interval$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

## See Also

Other sets: [ConditionalSet](#), [FuzzyMultiset](#), [FuzzySet](#), [FuzzyTuple](#), [Multiset](#), [Set](#), [Tuple](#)

**Examples**

```

# Set of Reals
Interval$new()

# Set of Integers
Interval$new(class = "integer")

# Half-open interval
i <- Interval$new(1, 10, "[)")
i$contains(c(1, 10))
i$contains(c(1, 10), bound = TRUE)

# Equivalent Set and Interval
Set$new(1:5) == Interval$new(1, 5, class = "integer")

# SpecialSets can provide more efficient implementation
Interval$new() == ExtendedReals$new()
Interval$new(class = "integer", type = "()") == Integers$new()

## -----
## Method `Interval$equals`
## -----

Interval$new(1,5) == Interval$new(1,5)
Interval$new(1,5, class = "integer") != Interval$new(1,5,class="numeric")

## -----
## Method `Interval$contains`
## -----

s = Set$new(1:5)

# Simplest case
s$contains(4)
8 %inset% s

# Test if multiple elements lie in the set
s$contains(4:6, all = FALSE)
s$contains(4:6, all = TRUE)

# Check if a tuple lies in a Set of higher dimension
s2 = s * s
s2$contains(Tuple$new(2,1))
c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2

## -----
## Method `Interval$isSubset`
## -----

Interval$new(1,3) < Interval$new(1,5)
Set$new(1,3) < Interval$new(0,5)

```

```
## -----
## Method `Interval$isSubinterval`
## -----

Interval$new(1,3)$isSubset(Set$new(1,2)) # TRUE
Interval$new(1,3)$isSubset(Set$new(2, 1)) # TRUE
Interval$new(1,3, class = "integer")$isSubinterval(Set$new(1, 2)) # TRUE
Interval$new(1,3)$isSubinterval(Set$new(1, 2)) # FALSE
Interval$new(1,3)$isSubinterval(Set$new(2, 1)) # FALSE

Reals$new()$isSubset(Integers$new()) # TRUE
Reals$new()$isSubinterval(Integers$new()) # FALSE
```

isSubset

*isSubset Operator***Description**

Operator for `$isSubset` methods. See [Set\\$isSubset](#) for full details. Operators can be used for:

<b>Name</b>	<b>Description</b>	<b>Operator</b>
Subset	x is a subset of y	$x \leq y$
Proper Subset	x is a proper subset of y	$x < y$
Superset	x is a superset of y	$x \geq y$
Proper Superset	x is a proper superset of y	$x > y$

**Usage**

```
## S3 method for class 'Set'
x < y

## S3 method for class 'Set'
x <= y

## S3 method for class 'Set'
x > y

## S3 method for class 'Set'
x >= y
```

**Arguments**

x, y                    [Set](#)

---

listSpecialSets	<i>Lists Implemented R6 Special Sets</i>
-----------------	--

---

**Description**

Lists special sets that can be used in Set.

**Usage**

```
listSpecialSets(simplify = FALSE)
```

**Arguments**

simplify	logical. If FALSE (default) returns data.frame of set name and symbol, otherwise set names as characters.
----------	---

**Value**

Either a list of characters (if simplify is TRUE) or a data.frame of SpecialSets and their traits.

**Examples**

```
listSpecialSets()  
listSpecialSets(TRUE)
```

---

Logicals	<i>Set of Logicals</i>
----------	------------------------

---

**Description**

The Logicals is defined as the [Set](#) containing the elements TRUE and FALSE.

**Super class**

```
set6::Set -> Logicals
```

**Methods****Public methods:**

- [Logicals\\$new\(\)](#)
- [Logicals\\$clone\(\)](#)

**Method new():** Create a new Logicals object.

*Usage:*

```
Logicals$new()
```



*Details:* The Logical set is the set containing TRUE and FALSE.

*Returns:* A new Logicals object.

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

```
Logicals$clone(deep = FALSE)
```

*Arguments:*

deep Whether to make a deep clone.

### See Also

Other special sets: [Complex](#), [ExtendedReals](#), [Integers](#), [Naturals](#), [NegIntegers](#), [NegRationals](#), [NegReals](#), [PosIntegers](#), [PosNaturals](#), [PosRationals](#), [PosReals](#), [Rationals](#), [Reals](#), [Universal](#)

### Examples

```
l <- Logicals$new()
print(l)
l$contains(list(TRUE, 1, FALSE))
```

---

LogicalSet

*Set of Logicals*

---

### Description

The LogicalSet is defined as the [Set](#) containing the elements TRUE and FALSE.

### Super class

```
set6::Set -> LogicalSet
```

### Methods

#### Public methods:

- [LogicalSet\\$new\(\)](#)
- [LogicalSet\\$clone\(\)](#)

**Method** new(): Create a new LogicalSet object.

*Usage:*

```
LogicalSet$new()
```

*Details:* The Logical set is the set containing TRUE and FALSE.

*Returns:* A new LogicalSet object.

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

```
LogicalSet$clone(deep = FALSE)
```

*Arguments:*

deep Whether to make a deep clone.

**Examples**

```
l <- LogicalSet$new()
print(l)
l$contains(list(TRUE, 1, FALSE))
```

---

Multiset

*Mathematical Multiset*


---

**Description**

A general Multiset object for mathematical Multisets, inheriting from Set.

**Details**

Multisets are generalisations of sets that allow an element to be repeated. They can be thought of as [Tuples](#) without ordering.

**Super class**

```
set6::Set -> Multiset
```

**Methods****Public methods:**

- [Multiset\\$equals\(\)](#)
- [Multiset\\$isSubset\(\)](#)
- [Multiset\\$clone\(\)](#)

**Method** `equals()`: Tests if two sets are equal.

*Usage:*

```
Multiset$equals(x, all = FALSE)
```

*Arguments:*

x [Set](#) or vector of [Sets](#).

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Details:* An object is equal to a Multiset if it contains all the same elements, and in the same order. Infix operators can be used for:

```
Equal      ==
Not equal  !=
```

*Returns:* If all is TRUE then returns TRUE if all x are equal to the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

*Examples:*

```
Multiset$new(1,2) == Multiset$new(1,2)
```

```
Multiset$new(1,2) != Multiset$new(1,2)
Multiset$new(1,1) != Set$new(1,1)
```

**Method** `isSubset()`: Test if one set is a (proper) subset of another

*Usage:*

```
Multiset$isSubset(x, proper = FALSE, all = FALSE)
```

*Arguments:*

`x` any. Object or vector of objects to test.

`proper` logical. If TRUE tests for proper subsets.

`all` logical. If FALSE tests each `x` separately. Otherwise returns TRUE only if all `x` pass test.

*Details:* If using the method directly, and not via one of the operators then the additional boolean argument `proper` can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set.

When calling `$isSubset` on objects inheriting from [Interval](#), the method treats the interval as if it is a [Set](#), i.e. ordering and class are ignored. Use `$isSubinterval` to test if one interval is a subinterval of another.

Infix operators can be used for:

Subset	<
Proper Subset	<=
Superset	>
Proper Superset	>=

An object is a (proper) subset of a Multiset if it contains all (some) of the same elements, and in the same order.

*Returns:* If `all` is TRUE then returns TRUE if all `x` are subsets of the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

*Examples:*

```
Multiset$new(1,2,3) < Multiset$new(1,2,3,4)
Multiset$new(1,3,2) < Multiset$new(1,2,3,4)
Multiset$new(1,3,2,4) <= Multiset$new(1,2,3,4)
```

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
Multiset$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

## See Also

Other sets: [ConditionalSet](#), [FuzzyMultiset](#), [FuzzySet](#), [FuzzyTuple](#), [Interval](#), [Set](#), [Tuple](#)

**Examples**

```

# Multiset of integers
Multiset$new(1:5)

# Multiset of multiple types
Multiset$new("a", 5, Set$new(1), Multiset$new(2))

# Each Multiset has properties and traits
t <- Multiset$new(1, 2, 3)
t$traits
t$properties

# Elements can be duplicated
Multiset$new(2, 2) != Multiset$new(2)

# Ordering does not matter
Multiset$new(1, 2) == Multiset$new(2, 1)

## -----
## Method `Multiset$equals`
## -----

Multiset$new(1,2) == Multiset$new(1,2)
Multiset$new(1,2) != Multiset$new(1,2)
Multiset$new(1,1) != Set$new(1,1)

## -----
## Method `Multiset$isSubset`
## -----

Multiset$new(1,2,3) < Multiset$new(1,2,3,4)
Multiset$new(1,3,2) < Multiset$new(1,2,3,4)
Multiset$new(1,3,2,4) <= Multiset$new(1,2,3,4)

```

---

Naturals

*Set of Natural Numbers*

---

**Description**

The mathematical set of natural numbers, defined as the counting numbers. i.e.

0, 1, 2, ...

**Super classes**

`set6::Set` -> `set6::Interval` -> `set6::SpecialSet` -> `Naturals`

**Methods****Public methods:**

- [Naturals\\$new\(\)](#)
- [Naturals\\$clone\(\)](#)

**Method** `new()`: Create a new `Naturals` object.

*Usage:*

```
Naturals$new(lower = 0)
```

*Arguments:*

`lower` numeric. Where to start the set. Advised to ignore, used by child-classes.

*Returns:* A new `Naturals` object.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
Naturals$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

**See Also**

Other special sets: [Complex](#), [ExtendedReals](#), [Integers](#), [Logicals](#), [NegIntegers](#), [NegRationals](#), [NegReals](#), [PosIntegers](#), [PosNaturals](#), [PosRationals](#), [PosReals](#), [Rationals](#), [Reals](#), [Universal](#)

---

NegIntegers

*Set of Negative Integers*

---

**Description**

The mathematical set of negative integers, defined as the set of negative whole numbers. i.e.

$$\dots, -3, -2, -1, 0$$
**Super classes**

[set6::Set](#) -> [set6::Interval](#) -> [set6::SpecialSet](#) -> [set6::Integers](#) -> NegIntegers

**Methods****Public methods:**

- [NegIntegers\\$new\(\)](#)
- [NegIntegers\\$clone\(\)](#)

**Method** `new()`: Create a new `NegIntegers` object.

*Usage:*

NegIntegers\$new(zero = FALSE)

*Arguments:*

zero logical. If TRUE, zero is included in the set.

*Returns:* A new NegIntegers object.

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

NegIntegers\$clone(deep = FALSE)

*Arguments:*

deep Whether to make a deep clone.

### See Also

Other special sets: [Complex](#), [ExtendedReals](#), [Integers](#), [Logicals](#), [Naturals](#), [NegRationals](#), [NegReals](#), [PosIntegers](#), [PosNaturals](#), [PosRationals](#), [PosReals](#), [Rationals](#), [Reals](#), [Universal](#)

---

NegRationals

*Set of Negative Rational Numbers*

---

### Description

The mathematical set of negative rational numbers, defined as the set of numbers that can be written as a fraction of two integers and are non-positive. i.e.

$$\frac{p}{q} : p, q \in \mathbb{Z}, p/q \leq 0, q \neq 0$$

where  $\mathbb{Z}$  is the set of integers.

### Details

The \$contains method does not work for the set of Rationals as it is notoriously difficult/impossible to find an algorithm for determining if any given number is rational or not. Furthermore, computers must truncate all irrational numbers to rational numbers.

### Super classes

[set6::Set](#) -> [set6::Interval](#) -> [set6::SpecialSet](#) -> [set6::Rationals](#) -> NegRationals

### Methods

#### Public methods:

- [NegRationals\\$new\(\)](#)
- [NegRationals\\$clone\(\)](#)

**Method** new(): Create a new NegRationals object.

*Usage:*

```
NegRationals$new(zero = FALSE)
```

*Arguments:*

zero logical. If TRUE, zero is included in the set.

*Returns:* A new NegRationals object.

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

```
NegRationals$clone(deep = FALSE)
```

*Arguments:*

deep Whether to make a deep clone.

**See Also**

Other special sets: [Complex](#), [ExtendedReals](#), [Integers](#), [Logicals](#), [Naturals](#), [NegIntegers](#), [NegReals](#), [PosIntegers](#), [PosNaturals](#), [PosRationals](#), [PosReals](#), [Rationals](#), [Reals](#), [Universal](#)

---

 NegReals

*Set of Negative Real Numbers*


---

**Description**

The mathematical set of negative real numbers, defined as the union of the set of negative rationals and negative irrationals. i.e.

$$I^- \cup Q^-$$

where  $I^-$  is the set of negative irrationals and  $Q^-$  is the set of negative rationals.

**Super classes**

```
set6::Set -> set6::Interval -> set6::SpecialSet -> set6::Reals -> NegReals
```

**Methods****Public methods:**

- [NegReals\\$new\(\)](#)
- [NegReals\\$clone\(\)](#)

**Method** new(): Create a new NegReals object.

*Usage:*

```
NegReals$new(zero = FALSE)
```

*Arguments:*

zero logical. If TRUE, zero is included in the set.

*Returns:* A new NegReals object.

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

NegReals\$clone(deep = FALSE)

*Arguments:*

deep Whether to make a deep clone.

### See Also

Other special sets: [Complex](#), [ExtendedReals](#), [Integers](#), [Logicals](#), [Naturals](#), [NegIntegers](#), [NegRationals](#), [PosIntegers](#), [PosNaturals](#), [PosRationals](#), [PosReals](#), [Rationals](#), [Reals](#), [Universal](#)

---

PosIntegers

*Set of Positive Integers*

---

### Description

The mathematical set of positive integers, defined as the set of positive whole numbers. i.e.

0, 1, 2, 3, ...

### Super classes

[set6::Set](#) -> [set6::Interval](#) -> [set6::SpecialSet](#) -> [set6::Integers](#) -> PosIntegers

### Methods

#### Public methods:

- [PosIntegers\\$new\(\)](#)
- [PosIntegers\\$clone\(\)](#)

**Method** new(): Create a new PosIntegers object.

*Usage:*

PosIntegers\$new(zero = FALSE)

*Arguments:*

zero logical. If TRUE, zero is included in the set.

*Returns:* A new PosIntegers object.

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

PosIntegers\$clone(deep = FALSE)

*Arguments:*

deep Whether to make a deep clone.

### See Also

Other special sets: [Complex](#), [ExtendedReals](#), [Integers](#), [Logicals](#), [Naturals](#), [NegIntegers](#), [NegRationals](#), [NegReals](#), [PosNaturals](#), [PosRationals](#), [PosReals](#), [Rationals](#), [Reals](#), [Universal](#)



---

PosNaturals	<i>Set of Positive Natural Numbers</i>
-------------	--

---

## Description

The mathematical set of positive natural numbers, defined as the positive counting numbers. i.e.

1, 2, 3, ...

## Super classes

`set6::Set` -> `set6::Interval` -> `set6::SpecialSet` -> `set6::Naturals` -> PosNaturals

## Methods

### Public methods:

- `PosNaturals$new()`
- `PosNaturals$clone()`

**Method** `new()`: Create a new PosNaturals object.

*Usage:*

`PosNaturals$new()`

*Returns:* A new PosNaturals object.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

`PosNaturals$clone(deep = FALSE)`

*Arguments:*

`deep` Whether to make a deep clone.

## See Also

Other special sets: [Complex](#), [ExtendedReals](#), [Integers](#), [Logicals](#), [Naturals](#), [NegIntegers](#), [NegRationals](#), [NegReals](#), [PosIntegers](#), [PosRationals](#), [PosReals](#), [Rationals](#), [Reals](#), [Universal](#)

---

PosRationals                      *Set of Positive Rational Numbers*

---

### Description

The mathematical set of positive rational numbers, defined as the set of numbers that can be written as a fraction of two integers and are non-negative. i.e.

$$\frac{p}{q} : p, q \in \mathbb{Z}, p/q \geq 0, q \neq 0$$

where  $\mathbb{Z}$  is the set of integers.

### Details

The `$contains` method does not work for the set of Rationals as it is notoriously difficult/impossible to find an algorithm for determining if any given number is rational or not. Furthermore, computers must truncate all irrational numbers to rational numbers.

### Super classes

`set6::Set` -> `set6::Interval` -> `set6::SpecialSet` -> `set6::Rationals` -> PosRationals

### Methods

#### Public methods:

- `PosRationals$new()`
- `PosRationals$clone()`

**Method** `new()`: Create a new PosRationals object.

*Usage:*

`PosRationals$new(zero = FALSE)`

*Arguments:*

`zero` logical. If TRUE, zero is included in the set.

*Returns:* A new PosRationals object.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

`PosRationals$clone(deep = FALSE)`

*Arguments:*

`deep` Whether to make a deep clone.

### See Also

Other special sets: [Complex](#), [ExtendedReals](#), [Integers](#), [Logicals](#), [Naturals](#), [NegIntegers](#), [NegRationals](#), [NegReals](#), [PosIntegers](#), [PosNaturals](#), [PosReals](#), [Rationals](#), [Reals](#), [Universal](#)

---

PosReals

*Set of Positive Real Numbers*

---

## Description

The mathematical set of positive real numbers, defined as the union of the set of positive rationals and positive irrationals. i.e.

$$I^+ \cup Q^+$$

where  $I^+$  is the set of positive irrationals and  $Q^+$  is the set of positive rationals.

## Super classes

[set6::Set](#) -> [set6::Interval](#) -> [set6::SpecialSet](#) -> [set6::Reals](#) -> PosReals

## Methods

### Public methods:

- [PosReals\\$new\(\)](#)
- [PosReals\\$clone\(\)](#)

**Method** `new()`: Create a new PosReals object.

*Usage:*

```
PosReals$new(zero = FALSE)
```

*Arguments:*

`zero` logical. If TRUE, zero is included in the set.

*Returns:* A new PosReals object.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
PosReals$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

## See Also

Other special sets: [Complex](#), [ExtendedReals](#), [Integers](#), [Logicals](#), [Naturals](#), [NegIntegers](#), [NegRationals](#), [NegReals](#), [PosIntegers](#), [PosNaturals](#), [PosRationals](#), [Rationals](#), [Reals](#), [Universal](#)

---

powerset

*Calculate a Set's Powerset*

---

### Description

Calculates and returns the powerset of a Set.

### Usage

```
powerset(x, simplify = FALSE)
```

### Arguments

x	<a href="#">Set</a>
simplify	logical, if TRUE then tries to simplify the result to a Set otherwise creates an object of class <a href="#">PowersetSet</a> .

### Details

A powerset of a set, S, is defined as the set of all subsets of S, including S itself and the empty set.

### Value

[Set](#)

### See Also

Other operators: [setcomplement\(\)](#), [setintersect\(\)](#), [setpower\(\)](#), [setproduct\(\)](#), [setsymdiff\(\)](#), [setunion\(\)](#)

### Examples

```
# simplify = FALSE is default
powerset(Set$new(1, 2))
powerset(Set$new(1, 2), simplify = TRUE)

# powerset of intervals
powerset(Interval$new())

# powerset of powersets
powerset(powerset(Reals$new()))
powerset(powerset(Reals$new()))$properties$cardinality
```

---

PowersetSet

*Set of Powersets*


---

## Description

PowersetSet class for symbolic powerset of mathematical sets.

## Details

The purpose of this class is to provide a symbolic representation for the powerset of sets that cannot be represented in a simpler class. Whilst this is not an abstract class, it is not recommended to construct this class directly but via the set operation methods.

## Super classes

`set6::Set` -> `set6::SetWrapper` -> `set6::ProductSet` -> PowersetSet

## Methods

### Public methods:

- `PowersetSet$new()`
- `PowersetSet$strprint()`
- `PowersetSet$contains()`
- `PowersetSet$isSubset()`
- `PowersetSet$clone()`

**Method** `new()`: Create a new PowersetSet object. It is not recommended to construct this class directly.

*Usage:*

`PowersetSet$new(set)`

*Arguments:*

set `Set` to wrap.

*Returns:* A new PowersetSet object.

**Method** `strprint()`: Creates a printable representation of the object.

*Usage:*

`PowersetSet$strprint(n = 2)`

*Arguments:*

n numeric. Number of elements to display on either side of ellipsis when printing.

*Returns:* A character string representing the object.

**Method** `contains()`: Tests if elements x are contained in self.

*Usage:*

`PowersetSet$contains(x, all = FALSE, bound = NULL)`

*Arguments:*

x [Set](#) or vector of [Sets](#).

x [Set](#) or vector of [Sets](#).

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

bound logical

*Returns:* If all == TRUE then returns TRUE if all x are contained in self, otherwise FALSE. If all == FALSE returns a vector of logicals corresponding to the length of x, representing if each is contained in self. If bound == TRUE then an element is contained in self if it is on or within the (possibly-open) bounds of self, otherwise TRUE only if the element is within self or the bounds are closed.

**Method** `isSubset()`: Tests if x is a (proper) subset of self.

*Usage:*

`PowersetSet$isSubset(x, proper = FALSE, all = FALSE)`

*Arguments:*

x [Set](#) or vector of [Sets](#).

x [Set](#) or vector of [Sets](#).

proper logical. If TRUE tests for proper subsets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Returns:* If all == TRUE then returns TRUE if all x are (proper) subsets of self, otherwise FALSE. If all == FALSE returns a vector of logicals corresponding to the length of x, representing if each is a (proper) subset of self.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

`PowersetSet$clone(deep = FALSE)`

*Arguments:*

deep Whether to make a deep clone.

## See Also

Set operations: [setunion](#), [setproduct](#), [setpower](#), [setcomplement](#), [setsymdiff](#), [powerset](#), [setintersect](#)

Other wrappers: [ComplementSet](#), [ExponentSet](#), [ProductSet](#), [UnionSet](#)

---

ProductSet	<i>Set of Products</i>
------------	------------------------

---

## Description

ProductSet class for symbolic product of mathematical sets.

## Details

The purpose of this class is to provide a symbolic representation for the product of sets that cannot be represented in a simpler class. Whilst this is not an abstract class, it is not recommended to construct this class directly but via the set operation methods.

## Super classes

`set6::Set` -> `set6::SetWrapper` -> ProductSet

## Active bindings

`length` Returns the number of elements in the object.

## Methods

### Public methods:

- `ProductSet$new()`
- `ProductSet$strprint()`
- `ProductSet$contains()`
- `ProductSet$clone()`

**Method** `new()`: Create a new ProductSet object. It is not recommended to construct this class directly.

*Usage:*

```
ProductSet$new(  
  setlist,  
  lower = NULL,  
  upper = NULL,  
  type = NULL,  
  cardinality = NULL  
)
```

*Arguments:*

`setlist` list of [Sets](#) to wrap.

`lower` lower bound of new object.

`upper` upper bound of new object.

`type` closure type of new object.

*cardinality* Either an integer, "Aleph0", or a beth number. If NULL then calculated automatically (recommended).

*Returns:* A new ProductSet object.

**Method** `strprint()`: Creates a printable representation of the object.

*Usage:*

```
ProductSet$strprint(n = 2)
```

*Arguments:*

*n* numeric. Number of elements to display on either side of ellipsis when printing.

*Returns:* A character string representing the object.

**Method** `contains()`: Tests if elements *x* are contained in *self*.

*Usage:*

```
ProductSet$contains(x, all = FALSE, bound = FALSE)
```

*Arguments:*

*x* [Set](#) or vector of [Sets](#).

*all* logical. If FALSE tests each *x* separately. Otherwise returns TRUE only if all *x* pass test.

*bound* logical

*Returns:* If *all* == TRUE then returns TRUE if all *x* are contained in *self*, otherwise FALSE. If *all* == FALSE returns a vector of logicals corresponding to the length of *x*, representing if each is contained in *self*. If *bound* == TRUE then an element is contained in *self* if it is on or within the (possibly-open) bounds of *self*, otherwise TRUE only if the element is within *self* or the bounds are closed.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
ProductSet$clone(deep = FALSE)
```

*Arguments:*

*deep* Whether to make a deep clone.

### See Also

Set operations: [setunion](#), [setproduct](#), [setpower](#), [setcomplement](#), [setsymdiff](#), [powerset](#), [setintersect](#)

Other wrappers: [ComplementSet](#), [ExponentSet](#), [PowersetSet](#), [UnionSet](#)



---

 Properties

 Set Properties Class
 

---

### Description

Used to store the properties of a [Set](#). Though this is not an abstract class, it should never be constructed outside of the [Set](#) constructor.

### Active bindings

`closure` Returns the closure of the Set. One of "open", "half-open", or "closed."

`countability` Returns the countability of the Set. One of "countably finite", "countably infinite", or "uncountable".

`cardinality` Returns the cardinality of the Set. Either an integer if the Set is countably finite,  $\aleph_0$  if countably infinite, or a Beth number.

`empty` Returns if the Set is empty or not. TRUE if the Set cardinality is  $\emptyset$ , FALSE otherwise.

`singleton` Returns if the Set is a singleton or not. TRUE if the Set cardinality is 1, FALSE otherwise.

### Methods

#### Public methods:

- [Properties\\$new\(\)](#)
- [Properties#print\(\)](#)
- [Properties\\$strprint\(\)](#)
- [Properties\\$clone\(\)](#)

**Method** `new()`: Creates a new Properties object.

*Usage:*

```
Properties$new(closure = character(0), cardinality = NULL)
```

*Arguments:*

`closure` One of "open", "half-open", or "closed."

`cardinality` If non-NULL then either an integer, " $\aleph_0$ ", or a Beth number.

*Returns:* A new Properties object.

**Method** `print()`: Prints the Properties list.

*Usage:*

```
Properties#print()
```

*Returns:* Prints Properties list to console.

**Method** `strprint()`: Creates a printable representation of the Properties.

*Usage:*

```
Properties$strprint()
```

*Returns:* A list of properties.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
Properties$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

Rationals

*Set of Rational Numbers*

### Description

The mathematical set of rational numbers, defined as the set of numbers that can be written as a fraction of two integers. i.e.

$$\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0$$

where  $\mathbb{Z}$  is the set of integers.

### Details

The `$contains` method does not work for the set of Rationals as it is notoriously difficult/impossible to find an algorithm for determining if any given number is rational or not. Furthermore, computers must truncate all irrational numbers to rational numbers.

### Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> Rationals
```

### Methods

#### Public methods:

- `Rationals$new()`
- `Rationals$contains()`
- `Rationals$isSubset()`
- `Rationals$equals()`
- `Rationals$clone()`

**Method** `new()`: Create a new Rationals object.

*Usage:*

```
Rationals$new(lower = -Inf, upper = Inf, type = "()")
```

*Arguments:*

`lower` numeric. Where to start the set. Advised to ignore, used by child-classes.

`upper` numeric. Where to end the set. Advised to ignore, used by child-classes.

type character Set closure type. Advised to ignore, used by child-classes.

*Returns:* A new Rationals object.

**Method** contains(): Method not possible for Rationals.

*Usage:*

Rationals\$contains(...)

*Arguments:*

... Ignored

**Method** isSubset(): Method not possible for Rationals.

*Usage:*

Rationals\$isSubset(...)

*Arguments:*

... Ignored

**Method** equals(): Method not possible for Rationals.

*Usage:*

Rationals\$equals(...)

*Arguments:*

... Ignored

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

Rationals\$clone(deep = FALSE)

*Arguments:*

deep Whether to make a deep clone.

### See Also

Other special sets: [Complex](#), [ExtendedReals](#), [Integers](#), [Logicals](#), [Naturals](#), [NegIntegers](#), [NegRationals](#), [NegReals](#), [PosIntegers](#), [PosNaturals](#), [PosRationals](#), [PosReals](#), [Reals](#), [Universal](#)

---

Reals

*Set of Real Numbers*

---

### Description

The mathematical set of real numbers, defined as the union of the set of rationals and irrationals. i.e.

$$I \cup Q$$

where  $I$  is the set of irrationals and  $Q$  is the set of rationals.

**Super classes**

`set6::Set` -> `set6::Interval` -> `set6::SpecialSet` -> `Reals`

**Methods****Public methods:**

- `Reals$new()`
- `Reals$clone()`

**Method** `new()`: Create a new `Reals` object.

*Usage:*

```
Reals$new(lower = -Inf, upper = Inf, type = "()")
```

*Arguments:*

`lower` numeric. Where to start the set. Advised to ignore, used by child-classes.

`upper` numeric. Where to end the set. Advised to ignore, used by child-classes.

`type` character Set closure type. Advised to ignore, used by child-classes.

*Returns:* A new `Reals` object.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
Reals$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

**See Also**

Other special sets: [Complex](#), [ExtendedReals](#), [Integers](#), [Logicals](#), [Naturals](#), [NegIntegers](#), [NegRationals](#), [NegReals](#), [PosIntegers](#), [PosNaturals](#), [PosRationals](#), [PosReals](#), [Rationals](#), [Universal](#)

---

Set

*Mathematical Set*

---

**Description**

A general `Set` object for mathematical sets. This also serves as the parent class to intervals, tuples, and fuzzy variants.

**Details**

Mathematical sets can loosely be thought of as a collection of objects of any kind. The `Set` class is used for sets of finite elements, for infinite sets use [Interval](#). These can be expanded for fuzzy logic by using [FuzzySets](#). Elements in a set cannot be duplicated and ordering of elements does not matter, [Tuples](#) can be used if duplicates or ordering are required.

**Active bindings**

**properties** Returns an object of class `Properties`, which lists the properties of the Set. Set properties include:

- `empty` - is the Set empty or does it contain elements?
- `singleton` - is the Set a singleton? i.e. Does it contain only one element?
- `cardinality` - number of elements in the Set
- `countability` - One of: countably finite, countably infinite, uncountable
- `closure` - One of: closed, open, half-open

**traits** List the traits of the Set. Set traits include:

- `crisp` - is the Set crisp or fuzzy?

**type** Returns the type of the Set. One of: `()`, `(I)`, `(D)`, `([])`, `{}`

**max** If the Set consists of numerics only then returns the maximum element in the Set. For open or half-open sets, then the maximum is defined by

$$upper - 1e - 15$$

**min** If the Set consists of numerics only then returns the minimum element in the Set. For open or half-open sets, then the minimum is defined by

$$lower + 1e - 15$$

**upper** If the Set consists of numerics only then returns the upper bound of the Set.

**lower** If the Set consists of numerics only then returns the lower bound of the Set.

**class** If all elements in the Set are the same class then returns that class, otherwise "ANY".

**elements** If the Set is finite then returns all elements in the Set as a list, otherwise "NA".

**universe** Returns the universe of the Set, i.e. the set of values that can be added to the Set.

**range** If the Set consists of numerics only then returns the range of the Set defined by

$$upper - lower$$

**length** If the Set is finite then returns the number of elements in the Set, otherwise Inf. See the cardinality property for the type of infinity.

**Methods****Public methods:**

- `Set$new()`
- `Set$print()`
- `Set$strprint()`
- `Set$summary()`
- `Set$contains()`
- `Set$equals()`
- `Set$isSubset()`
- `Set$add()`

- [Set\\$remove\(\)](#)
- [Set\\$multiplicity\(\)](#)
- [Set\\$clone\(\)](#)

**Method new():** Create a new Set object.

*Usage:*

```
Set$new(..., universe = Universal$new(), elements = NULL, class = NULL)
```

*Arguments:*

... ANY Elements can be of any class except list, as long as there is a unique as.character coercion method available.

universe Set. Universe that the Set lives in, i.e. elements that could be added to the Set. Default is [Universal](#).

elements list. Alternative constructor that may be more efficient if passing objects of multiple classes.

class character. Optional string naming a class that if supplied gives the set the typed property.

All elements will be coerced to this class and therefore there must be a coercion method to this class available.

*Returns:* A new Set object.

**Method print():** Prints a symbolic representation of the Set.

*Usage:*

```
Set$print(n = 2)
```

*Arguments:*

n numeric. Number of elements to display on either side of ellipsis when printing.

*Details:* The function [useUnicode\(\)](#) can be used to determine if unicode should be used when printing the Set. Internally print first calls `strprint` to create a printable representation of the Set.

**Method strprint():** Creates a printable representation of the object.

*Usage:*

```
Set$strprint(n = 2)
```

*Arguments:*

n numeric. Number of elements to display on either side of ellipsis when printing.

*Returns:* A character string representing the object.

**Method summary():** Summarises the Set.

*Usage:*

```
Set$summary(n = 2)
```

*Arguments:*

n numeric. Number of elements to display on either side of ellipsis when printing.

*Details:* The function [useUnicode\(\)](#) can be used to determine if unicode should be used when printing the Set. Summarised details include the Set class, properties, and traits.

**Method** `contains()`: Tests to see if `x` is contained in the Set.

*Usage:*

```
Set$contains(x, all = FALSE, bound = NULL)
```

*Arguments:*

`x` any. Object or vector of objects to test.

`all` logical. If FALSE tests each `x` separately. Otherwise returns TRUE only if all `x` pass test.

`bound` ignored, added for consistency.

*Details:* `x` can be of any type, including a Set itself. `x` should be a tuple if checking to see if it lies within a set of dimension greater than one. To test for multiple `x` at the same time, then provide these as a list.

If `all = TRUE` then returns TRUE if all `x` are contained in the Set, otherwise returns a vector of logicals.

*Returns:* If `all` is TRUE then returns TRUE if all elements of `x` are contained in the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

The infix operator `%inset%` is available to test if `x` is an element in the Set, see examples.

*Examples:*

```
s = Set$new(elements = 1:5)
```

```
# Simplest case
```

```
s$contains(4)
```

```
8 %inset% s
```

```
# Test if multiple elements lie in the set
```

```
s$contains(4:6, all = FALSE)
```

```
s$contains(4:6, all = TRUE)
```

```
# Check if a tuple lies in a Set of higher dimension
```

```
s2 = s * s
```

```
s2$contains(Tuple$new(2,1))
```

```
c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2
```

**Method** `equals()`: Tests if two sets are equal.

*Usage:*

```
Set$equals(x, all = FALSE)
```

*Arguments:*

`x` Set or vector of Sets.

`all` logical. If FALSE tests each `x` separately. Otherwise returns TRUE only if all `x` pass test.

*Details:* Two sets are equal if they contain the same elements. Infix operators can be used for:

```
Equal      ==
Not equal  !=
```

*Returns:* If `all` is TRUE then returns TRUE if all `x` are equal to the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

*Examples:*

```
# Equals
Set$new(1,2)$equals(Set$new(5,6))
Set$new(1,2)$equals(Interval$new(1,2))
Set$new(1,2) == Interval$new(1,2, class = "integer")

# Not equal
!Set$new(1,2)$equals(Set$new(1,2))
Set$new(1,2) != Set$new(1,5)
```

**Method** `isSubset()`: Test if one set is a (proper) subset of another

*Usage:*

```
Set$isSubset(x, proper = FALSE, all = FALSE)
```

*Arguments:*

`x` any. Object or vector of objects to test.

`proper` logical. If TRUE tests for proper subsets.

`all` logical. If FALSE tests each `x` separately. Otherwise returns TRUE only if all `x` pass test.

*Details:* If using the method directly, and not via one of the operators then the additional boolean argument `proper` can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set.

Infix operators can be used for:

Subset	<
Proper Subset	<=
Superset	>
Proper Superset	>=

*Returns:* If `all` is TRUE then returns TRUE if all `x` are subsets of the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

*Examples:*

```
Set$new(1,2,3)$isSubset(Set$new(1,2), proper = TRUE)
Set$new(1,2) < Set$new(1,2,3) # proper subset

c(Set$new(1,2,3), Set$new(1)) < Set$new(1,2,3) # not proper
Set$new(1,2,3) <= Set$new(1,2,3) # proper
```

**Method** `add()`: Add elements to a set.

*Usage:*

```
Set$add(...)
```

*Arguments:*

`...` elements to add



*Details:* \$add is a wrapper around the setunion method with setunion(self, Set\$new(...)). Note a key difference is that any elements passed to ... are first converted to a Set, this important difference is illustrated in the examples by adding an [Interval](#) to a Set.

Additionally, \$add first coerces ... to \$class if self is a typed-set (i.e. \$class != "ANY"), and \$add checks if elements in ... live in the universe of self.

*Returns:* An object inheriting from [Set](#).

*Examples:*

```
Set$new(1,2)$add(3)$print()
Set$new(1,2,universe = Interval$new(1,3))$add(3)$print()
\dontrun{
# errors as 4 is not in [1,3]
Set$new(1,2,universe = Interval$new(1,3))$add(4)$print()
}
# coerced to complex
Set$new(0+1i, 2i, class = "complex")$add(4)$print()

# setunion vs. add
Set$new(1,2)$add(Interval$new(5,6))$print()
Set$new(1,2) + Interval$new(5,6)
```

**Method** remove(): Remove elements from a set.

*Usage:*

```
Set$remove(...)
```

*Arguments:*

... elements to remove

*Details:* \$remove is a wrapper around the setcomplement method with setcomplement(self, Set\$new(...)). Note a key difference is that any elements passed to ... are first converted to a Set, this important difference is illustrated in the examples by removing an [Interval](#) from a Set.

*Returns:* If the complement cannot be simplified to a Set then a [ComplementSet](#) is returned otherwise an object inheriting from [Set](#) is returned.

*Examples:*

```
Set$new(1,2,3)$remove(1,2)$print()
Set$new(1,Set$new(1),2)$remove(Set$new(1))$print()
Interval$new(1,5)$remove(5)$print()
Interval$new(1,5)$remove(4)$print()

# setcomplement vs. remove
Set$new(1,2,3)$remove(Interval$new(5,7))$print()
Set$new(1,2,3) - Interval$new(5,7)
```

**Method** multiplicity(): Returns the number of times an element appears in a set,

*Usage:*

```
Set$multiplicity(element = NULL)
```

*Arguments:*

element element or list of elements in the set, if NULL returns multiplicity of all elements

*Returns:* Value, or list of values, in R+.

*Examples:*

```
Set$new(1, 1, 2)$multiplicity()
Set$new(1, 1, 2)$multiplicity(1)
Set$new(1, 1, 2)$multiplicity(list(1, 2))
Tuple$new(1, 1, 2)$multiplicity(1)
Tuple$new(1, 1, 2)$multiplicity(2)
```

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
Set$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

### See Also

Other sets: [ConditionalSet](#), [FuzzyMultiset](#), [FuzzySet](#), [FuzzyTuple](#), [Interval](#), [Multiset](#), [Tuple](#)

### Examples

```
# Set of integers
Set$new(1:5)

# Set of multiple types
Set$new("a", 5, Set$new(1))

# Each Set has properties and traits
s <- Set$new(1, 2, 3)
s$traits
s$properties

# Elements cannot be duplicated
Set$new(2, 2) == Set$new(2)

# Ordering does not matter
Set$new(1, 2) == Set$new(2, 1)

## -----
## Method `Set$contains`
## -----

s = Set$new(elements = 1:5)

# Simplest case
s$contains(4)
8 %inset% s

# Test if multiple elements lie in the set
s$contains(4:6, all = FALSE)
```

```

s$contains(4:6, all = TRUE)

# Check if a tuple lies in a Set of higher dimension
s2 = s * s
s2$contains(Tuple$new(2,1))
c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2

## -----
## Method `Set$equals`
## -----

# Equals
Set$new(1,2)$equals(Set$new(5,6))
Set$new(1,2)$equals(Interval$new(1,2))
Set$new(1,2) == Interval$new(1,2, class = "integer")

# Not equal
!Set$new(1,2)$equals(Set$new(1,2))
Set$new(1,2) != Set$new(1,5)

## -----
## Method `Set$isSubset`
## -----

Set$new(1,2,3)$isSubset(Set$new(1,2), proper = TRUE)
Set$new(1,2) < Set$new(1,2,3) # proper subset

c(Set$new(1,2,3), Set$new(1)) < Set$new(1,2,3) # not proper
Set$new(1,2,3) <= Set$new(1,2,3) # proper

## -----
## Method `Set$add`
## -----

Set$new(1,2)$add(3)$print()
Set$new(1,2,universe = Interval$new(1,3))$add(3)$print()
## Not run:
# errors as 4 is not in [1,3]
Set$new(1,2,universe = Interval$new(1,3))$add(4)$print()

## End(Not run)
# coerced to complex
Set$new(0+1i, 2i, class = "complex")$add(4)$print()

# setunion vs. add
Set$new(1,2)$add(Interval$new(5,6))$print()
Set$new(1,2) + Interval$new(5,6)

## -----
## Method `Set$remove`
## -----

Set$new(1,2,3)$remove(1,2)$print()

```

```

Set$new(1, Set$new(1), 2)$remove(Set$new(1))$print()
Interval$new(1, 5)$remove(5)$print()
Interval$new(1, 5)$remove(4)$print()

# setcomplement vs. remove
Set$new(1, 2, 3)$remove(Interval$new(5, 7))$print()
Set$new(1, 2, 3) - Interval$new(5, 7)

## -----
## Method `Set$multiplicity`
## -----

Set$new(1, 1, 2)$multiplicity()
Set$new(1, 1, 2)$multiplicity(1)
Set$new(1, 1, 2)$multiplicity(list(1, 2))
Tuple$new(1, 1, 2)$multiplicity(1)
Tuple$new(1, 1, 2)$multiplicity(2)

```

---

set6News

*Show set6 NEWS.md File*


---

### Description

Displays the contents of the NEWS.md file for viewing set6 release information.

### Usage

```
set6News()
```

### Value

NEWS.md in viewer.

### Examples

```
set6News()
```

---

setcomplement

*Complement of Two Sets*


---

### Description

Returns the set difference of two objects inheriting from class Set. If y is missing then the complement of x from its universe is returned.

**Usage**

```

setcomplement(x, y, simplify = TRUE)

## S3 method for class 'Set'
setcomplement(x, y, simplify = TRUE)

## S3 method for class 'Interval'
setcomplement(x, y, simplify = TRUE)

## S3 method for class 'FuzzySet'
setcomplement(x, y, simplify = TRUE)

## S3 method for class 'ConditionalSet'
setcomplement(x, y, simplify = TRUE)

## S3 method for class 'Reals'
setcomplement(x, y, simplify = TRUE)

## S3 method for class 'Rationals'
setcomplement(x, y, simplify = TRUE)

## S3 method for class 'Integers'
setcomplement(x, y, simplify = TRUE)

## S3 method for class 'ComplementSet'
setcomplement(x, y, simplify = TRUE)

## S3 method for class 'Set'
x - y

```

**Arguments**

x, y	Set
simplify	logical, if TRUE (default) returns the result in its simplest form, usually a Set or <a href="#">UnionSet</a> , otherwise a ComplementSet.

**Details**

The difference of two sets,  $X, Y$ , is defined as the set of elements that exist in set  $X$  and not  $Y$ ,

$$X - Y = \{z : z \in X \text{ and } \neg(z \in Y)\}$$

The set difference of two [ConditionalSets](#) is defined by combining their defining functions by a negated 'and', !&, operator. See examples.

The complement of fuzzy and crisp sets first coerces fuzzy sets to crisp sets by finding their support.

**Value**

An object inheriting from Set containing the set difference of elements in x and y.

**See Also**

Other operators: [powerset\(\)](#), [setintersect\(\)](#), [setpower\(\)](#), [setproduct\(\)](#), [setsymdiff\(\)](#), [setunion\(\)](#)

**Examples**

```
# absolute complement
setcomplement(Set$new(1, 2, 3, universe = Reals$new()))
setcomplement(Set$new(1, 2, universe = Set$new(1, 2, 3, 4, 5)))

# complement of two sets

Set$new(-2:4) - Set$new(2:5)
setcomplement(Set$new(1, 4, "a"), Set$new("a", 6))

# complement of two intervals

Interval$new(1, 10) - Interval$new(5, 15)
Interval$new(1, 10) - Interval$new(-15, 15)
Interval$new(1, 10) - Interval$new(-1, 2)

# complement of mixed set types

Set$new(1:10) - Interval$new(5, 15)
Set$new(5, 7) - Tuple$new(6, 8, 7)

# FuzzySet-Set returns a FuzzySet
FuzzySet$new(1, 0.1, 2, 0.5) - Set$new(2:5)
# Set-FuzzySet returns a Set
Set$new(2:5) - FuzzySet$new(1, 0.1, 2, 0.5)

# complement of conditional sets

ConditionalSet$new(function(x, y, simplify = TRUE) x >= y) -
  ConditionalSet$new(function(x, y, simplify = TRUE) x == y)

# complement of special sets
Reals$new() - NegReals$new()
Rationals$new() - PosRationals$new()
Integers$new() - PosIntegers$new()
```

---

setintersect

*Intersection of Two Sets*

---

**Description**

Returns the intersection of two objects inheriting from class Set.

**Usage**

```

setintersect(x, y)

## S3 method for class 'Interval'
setintersect(x, y)

## S3 method for class 'ConditionalSet'
setintersect(x, y)

## S3 method for class 'UnionSet'
setintersect(x, y)

## S3 method for class 'ComplementSet'
setintersect(x, y)

## S3 method for class 'ProductSet'
setintersect(x, y)

## S3 method for class 'Set'
x & y

```

**Arguments**

x, y                    Set

**Details**

The intersection of two sets,  $X, Y$ , is defined as the set of elements that exist in both sets,

$$X \cap Y = \{z : z \in X \text{ and } z \in Y\}$$

In the case where no elements are common to either set, then the empty set is returned.

The intersection of two [ConditionalSets](#) is defined by combining their defining functions by an 'and', &, operator. See examples.

The intersection of fuzzy and crisp sets first coerces fuzzy sets to crisp sets by finding their support.

**Value**

A Set consisting of elements in both x and y.

**See Also**

Other operators: [powerset\(\)](#), [setcomplement\(\)](#), [setpower\(\)](#), [setproduct\(\)](#), [setsymdiff\(\)](#), [setunion\(\)](#)

**Examples**

```
# intersection of two sets
```

```

Set$new(-2:4) & Set$new(2:5)
setintersect(Set$new(1, 4, "a"), Set$new("a", 6))
Set$new(1:4) & Set$new(5:7)

# intersection of two intervals

Interval$new(1, 10) & Interval$new(5, 15)
Interval$new(1, 2) & Interval$new(2, 3)
Interval$new(1, 5, class = "integer") &
  Interval$new(2, 7, class = "integer")

# intersection of mixed set types

Set$new(1:10) & Interval$new(5, 15)
Set$new(5, 7) & Tuple$new(6, 8, 7)

# Ignores membership of FuzzySet

FuzzySet$new(1, 0.1, 2, 0.5) & Set$new(2:5)

# intersection of conditional sets

ConditionalSet$new(function(x, y) x >= y) &
  ConditionalSet$new(function(x, y) x == y)
ConditionalSet$new(function(x) x == 2) &
  ConditionalSet$new(function(y) y == 3)

# But be careful not to make an empty set

ConditionalSet$new(function(x) x == 2) &
  ConditionalSet$new(function(x) x == 3)

```

---

 setpower

*Power of a Set*


---

### Description

A convenience wrapper for the n-ary cartesian product of a Set by itself, possibly multiple times.

### Usage

```
setpower(x, power, simplify = FALSE, nest = FALSE)
```

```
## S3 method for class 'Set'
x ^ power
```

### Arguments

x                    Set



power	power to raise set to, if "n" then a variable dimension set is created, see examples.'
simplify	logical, if TRUE returns the result in its simplest (unwrapped) form, usually a Set, otherwise an ExponentSet.
nest	logical, if FALSE (default) returns the n-ary cartesian product, otherwise returns the cartesian product applied n times. <a href="#">Sets</a> . See details and examples.

### Details

See the details of [setproduct](#) for a longer discussion on the use of the nest argument, in particular with regards to n-ary cartesian products vs. 'standard' cartesian products.

### Value

An R6 object of class Set or ExponentSet inheriting from ProductSet.

### See Also

Other operators: [powerset\(\)](#), [setcomplement\(\)](#), [setintersect\(\)](#), [setproduct\(\)](#), [setsymdiff\(\)](#), [setunion\(\)](#)

### Examples

```
# Power of a Set
setpower(Set$new(1, 2), 3, simplify = FALSE)
setpower(Set$new(1, 2), 3, simplify = TRUE)
Set$new(1, 2)^3

# Power of an interval
Interval$new(2, 5)^5
Reals$new()^3

# Use tuples for contains
(PosNaturals$new()^3)$contains(Tuple$new(1, 2, 3))

# Power of ConditionalSet is meaningless
ConditionalSet$new(function(x) TRUE)^2

# Power of FuzzySet
FuzzySet$new(1, 0.1, 2, 0.5)^2

# Variable length
x <- Interval$new(0, 1)^"n"
x$contains(Tuple$new(0))
x$contains(Tuple$new(0, 1))
x$contains(Tuple$new(0, 1, 0, 0, 1, 1, 0))
x$contains(list(Tuple$new(0, 2), Tuple$new(1, 1)))
```

setproduct

*Cartesian Product of Sets***Description**

Returns the cartesian product of objects inheriting from class Set.

**Usage**

```
setproduct(..., simplify = FALSE, nest = FALSE)
```

```
## S3 method for class 'Set'
x * y
```

**Arguments**

...	<a href="#">Sets</a>
simplify	logical, if TRUE returns the result in its simplest (unwrapped) form, usually a Set otherwise a ProductSet.
nest	logical, if FALSE (default) then will treat any <a href="#">ProductSets</a> passed to ... as unwrapped <a href="#">Sets</a> . See details and examples.
x, y	<a href="#">Set</a>

**Details**

The cartesian product of multiple sets, the 'n-ary Cartesian product', is often implemented in programming languages as being identical to the cartesian product of two sets applied recursively. However, for sets  $X, Y, Z$ ,

$$XYZ \neq (XY)Z$$

This is accommodated with the nest argument. If nest == TRUE then  $X * Y * Z == (XY)Z$ , i.e. the cartesian product for two sets is applied recursively. If nest == FALSE then  $X * Y * Z == (XYZ)$  and the n-ary cartesian product is computed. As it appears the latter (n-ary product) is more common, nest = FALSE is the default. The N-ary cartesian product of  $N$  sets,  $X_1, \dots, X_N$ , is defined as

$$X_1 \dots X_N = (x_1, \dots, x_N) : x_1 \in X_1 \cap \dots \cap x_N \in X_N$$

where  $(x_1, \dots, x_N)$  is a tuple.

The product of fuzzy and crisp sets first coerces fuzzy sets to crisp sets by finding their support.

**Value**

Either an object of class ProductSet or an unwrapped object inheriting from Set.

**See Also**

Other operators: [powerset\(\)](#), [setcomplement\(\)](#), [setintersect\(\)](#), [setpower\(\)](#), [setsymdiff\(\)](#), [setunion\(\)](#)

**Examples**

```

# difference between nesting
Set$new(1, 2) * Set$new(2, 3) * Set$new(4, 5)
setproduct(Set$new(1, 2) * Set$new(2, 3), Set$new(4, 5), nest = FALSE) # same as above
setproduct(Set$new(1, 2) * Set$new(2, 3), Set$new(4, 5), nest = TRUE)
unnest_set <- setproduct(Set$new(1, 2) * Set$new(2, 3), Set$new(4, 5), nest = FALSE)
nest_set <- setproduct(Set$new(1, 2) * Set$new(2, 3), Set$new(4, 5), nest = TRUE)
# note the difference when using contains
unnest_set$contains(Tuple$new(1, 3, 5))
nest_set$contains(Tuple$new(Tuple$new(1, 3), 5))

# product of two sets
Set$new(-2:4) * Set$new(2:5)
setproduct(Set$new(1, 4, "a"), Set$new("a", 6))
setproduct(Set$new(1, 4, "a"), Set$new("a", 6), simplify = TRUE)

# product of two intervals
Interval$new(1, 10) * Interval$new(5, 15)
Interval$new(1, 2, type = "()") * Interval$new(2, 3, type = "[]")
Interval$new(1, 5, class = "integer") *
  Interval$new(2, 7, class = "integer")

# product of mixed set types
Set$new(1:10) * Interval$new(5, 15)
Set$new(5, 7) * Tuple$new(6, 8, 7)
FuzzySet$new(1, 0.1) * Set$new(2)

# product of FuzzySet
FuzzySet$new(1, 0.1, 2, 0.5) * Set$new(2:5)

# product of conditional sets
ConditionalSet$new(function(x, y) x >= y) *
  ConditionalSet$new(function(x, y) x == y)

# product of special sets
PosReals$new() * NegReals$new()

```

---

setsymdiff

*Symmetric Difference of Two Sets*


---

**Description**

Returns the symmetric difference of two objects inheriting from class Set.

**Usage**

```
setsymdiff(x, y, simplify = TRUE)
```

```
x %-% y
```

**Arguments**

x, y	Set
simplify	logical, if TRUE (default) returns the result in its simplest form, usually a Set or <a href="#">UnionSet</a> , otherwise a ComplementSet.

**Details**

The symmetric difference, aka disjunctive union, of two sets,  $X, Y$ , is defined as the set of elements that exist in set  $X$  or in  $Y$  but not both,

$$\{z : (z \in X \cup z \in Y) \cap \neg(z \in X \cap z \in Y)\}$$

The symmetric difference can also be expressed as the union of two sets minus the intersection. Therefore `setsymdiff` is written as a thin wrapper over these operations, so for two sets, A,B:  
 $A \% \% B = (A \mid B) - (A \& B)$ .

The symmetric difference of fuzzy and crisp sets first coerces fuzzy sets to crisp sets by finding their support.

**Value**

An object inheriting from Set containing the symmetric difference of elements in both x and y.

**See Also**

Other operators: [powerset\(\)](#), [setcomplement\(\)](#), [setintersect\(\)](#), [setpower\(\)](#), [setproduct\(\)](#), [setunion\(\)](#)

**Examples**

```
# symmetrical difference compared to union and intersection
Set$new(1, 2, 3) \% \% Set$new(3, 4)
(Set$new(1, 2, 3) | Set$new(3, 4)) - (Set$new(1, 2, 3) & Set$new(3, 4))

# ConditionalSets demonstrate the internal logic
ConditionalSet$new(function(x) x > 0) \% \%
  ConditionalSet$new(function(y) y == 0)
```

---

 setunion

*Union of Sets*


---

**Description**

Returns the union of objects inheriting from class [Set](#).

**Usage**

```
setunion(..., simplify = TRUE)

## S3 method for class 'Set'
x + y

## S3 method for class 'Set'
x | y
```

**Arguments**

...	<a href="#">Sets</a>
simplify	logical, if TRUE (default) returns the result in its simplest (unwrapped) form, usually a <a href="#">Set</a> , otherwise a <a href="#">UnionSet</a> .
x, y	<a href="#">Set</a>

**Details**

The union of  $N$  sets,  $X_1, \dots, X_N$ , is defined as the set of elements that exist in one or more sets,

$$U = \{x : x \in X_1 \text{ or } x \in X_2 \text{ or } \dots \text{ or } x \in X_N\}$$

The union of multiple [ConditionalSets](#) is given by combining their defining functions by an 'or', |, operator. See examples.

The union of fuzzy and crisp sets first coerces fuzzy sets to crisp sets by finding their support.

**Value**

An object inheriting from [Set](#) containing the union of supplied sets.

**See Also**

Other operators: [powerset\(\)](#), [setcomplement\(\)](#), [setintersect\(\)](#), [setpower\(\)](#), [setproduct\(\)](#), [setsymdiff\(\)](#)

**Examples**

```
# union of Sets

Set$new(-2:4) + Set$new(2:5)
setunion(Set$new(1, 4, "a"), Set$new("a", 6))
Set$new(1, 2) + Set$new("a", 1i) + Set$new(9)

# union of intervals

Interval$new(1, 10) + Interval$new(5, 15) + Interval$new(20, 30)
Interval$new(1, 2, type = "()") + Interval$new(2, 3, type = "[]")
Interval$new(1, 5, class = "integer") +
  Interval$new(2, 7, class = "integer")
```

```

# union of mixed types

Set$new(1:10) + Interval$new(5, 15)
Set$new(1:10) + Interval$new(5, 15, class = "integer")
Set$new(5, 7) | Tuple$new(6, 8, 7)

# union of FuzzySet
FuzzySet$new(1, 0.1, 2, 0.5) + Set$new(2:5)

# union of conditional sets

ConditionalSet$new(function(x, y) x >= y) +
  ConditionalSet$new(function(x, y) x == y) +
  ConditionalSet$new(function(x) x == 2)

# union of special sets
PosReals$new() + NegReals$new()
Set$new(-Inf, Inf) + Reals$new()

```

---

SetWrapper

*Abstract SetWrapper Class*


---

## Description

This class should not be constructed directly. Parent class to SetWrappers.

## Details

Wrappers in set6 are utilised to facilitate lazy evaluation and symbolic representation. Each operation has an associated wrapper that will be returned if `simplify = FALSE` or if the result would be too complex to return as a simple `Set`. Wrappers have an identical interface to `Set`. Their primary advantage lies in a neat representation of any set composition (the result of an operation) and the ability to query the set contents without ever directly evaluating the set elements.

## Super class

```
set6::Set -> SetWrapper
```

## Active bindings

`wrappedSets` Returns the list of Sets that are wrapped in the given wrapper.

## Methods

### Public methods:

- `SetWrapper$new()`
- `SetWrapper$equals()`

- [SetWrapper\\$isSubset\(\)](#)
- [SetWrapper\\$clone\(\)](#)

**Method new():** Create a new SetWrapper object. It is not recommended to construct this class directly.

*Usage:*

```
SetWrapper$new(
  setlist,
  lower = NULL,
  upper = NULL,
  type = NULL,
  class = NULL,
  cardinality
)
```

*Arguments:*

setlist List of [Sets](#) to wrap.

lower [Set](#). Lower bound of wrapper.

upper [Set](#). Upper bound of wrapper.

type character. Closure type of wrapper.

class character. Ignored.

cardinality character or integer. Cardinality of wrapper.

*Returns:* A new SetWrapper object.

**Method equals():** Tests if x is equal to self.

*Usage:*

```
SetWrapper$equals(x, all = FALSE)
```

*Arguments:*

x [Set](#) or vector of [Sets](#).

x any. Object or vector of objects to test.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Returns:* If all == TRUE then returns TRUE if all x are equal to self, otherwise FALSE. If all == FALSE returns a vector of logicals corresponding to the length of x, representing if each is equal to self.

**Method isSubset():** Tests if x is a (proper) subset of self.

*Usage:*

```
SetWrapper$isSubset(x, proper = FALSE, all = FALSE)
```

*Arguments:*

x [Set](#) or vector of [Sets](#).

x any. Object or vector of objects to test.

proper logical. If TRUE tests for proper subsets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Returns:* If all == TRUE then returns TRUE if all x are (proper) subsets of self, otherwise FALSE. If all == FALSE returns a vector of logicals corresponding to the length of x, representing if each is a (proper) subset of self.

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

```
SetWrapper$clone(deep = FALSE)
```

*Arguments:*

deep Whether to make a deep clone.

---

SpecialSet

*Abstract Class for Special Sets*

---

## Description

The 'special sets' are the group of sets that are commonly used in mathematics and are thus given their own names.

## Details

This is an abstract class and should not be constructed directly. Use [listSpecialSets](#) to see the list of implemented special sets.

## Super classes

```
set6::Set -> set6::Interval -> SpecialSet
```

## Methods

### Public methods:

- [SpecialSet\\$new\(\)](#)
- [SpecialSet\\$strprint\(\)](#)
- [SpecialSet\\$clone\(\)](#)

**Method** new(): SpecialSet is an abstract class, the constructor cannot be used directly.

*Usage:*

```
SpecialSet$new(lower = -Inf, upper = Inf, type = "()", class = "numeric")
```

*Arguments:*

lower defines the lower bound of the interval.

upper defines the upper bound of the interval.

type defines the interval closure type.

class defines the interval class.



**Method** `strprint()`: Creates a printable representation of the object.

*Usage:*

```
SpecialSet$strprint(n = NULL)
```

*Arguments:*

`n` ignored, added for consistency.

*Returns:* A character string representing the object.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
SpecialSet$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

---

testClosed

*assert/check/test/Closed*

---

## Description

Validation checks to test if a given object is closed.

## Usage

```
testClosed(object, errmsg = "This is not a closed set")
```

```
checkClosed(object, errmsg = "This is not a closed set")
```

```
assertClosed(object, errmsg = "This is not a closed set")
```

## Arguments

`object` object to test

`errmsg` error message to overwrite default if check fails

## Value

If check passes then `assert` returns object invisibly and `test/check` return TRUE. If check fails, `assert` stops code with error, `check` returns an error message as string, and `test` returns FALSE.

## Examples

```
testClosed(Interval$new(1, 10, type = "[)")
```

```
testClosed(Interval$new(1, 10, type = "(]"))
```

---

testClosedAbove	<i>assert/check/test/ClosedAbove</i>
-----------------	--------------------------------------

---

**Description**

Validation checks to test if a given object is closedabove.

**Usage**

```
testClosedAbove(object, errmsg = "This is not a set closed above")
checkClosedAbove(object, errmsg = "This is not a set closed above")
assertClosedAbove(object, errmsg = "This is not a set closed above")
```

**Arguments**

object	object to test
errmsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testClosedAbove(Interval$new(1, 10, type = "[ ]"))
testClosedAbove(Interval$new(1, 10, type = "[ ]"))
```

---

testClosedBelow	<i>assert/check/test/ClosedBelow</i>
-----------------	--------------------------------------

---

**Description**

Validation checks to test if a given object is closedbelow.

**Usage**

```
testClosedBelow(object, errmsg = "This is not a set closed below")
checkClosedBelow(object, errmsg = "This is not a set closed below")
assertClosedBelow(object, errmsg = "This is not a set closed below")
```

**Arguments**

object            object to test  
 errmsg           error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testClosedBelow(Interval$new(1, 10, type = "[ ]"))
testClosedBelow(Interval$new(1, 10, type = "( ]"))
```

---

testConditionalSet    *assert/check/test/ConditionalSet*

---

**Description**

Validation checks to test if a given object is an R6 ConditionalSet.

**Usage**

```
testConditionalSet(
  object,
  errmsg = "This is not an R6 ConditionalSet object"
)

checkConditionalSet(
  object,
  errmsg = "This is not an R6 ConditionalSet object"
)

assertConditionalSet(
  object,
  errmsg = "This is not an R6 ConditionalSet object"
)
```

**Arguments**

object            object to test  
 errmsg           error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```

testConditionalSet(Set$new(2, 3))
testConditionalSet(list(Set$new(2), Set$new(3)))
testConditionalSet(Tuple$new(2, 3))
testConditionalSet(Interval$new())
testConditionalSet(FuzzySet$new(2, 0.1))
testConditionalSet(FuzzyTuple$new(2, 0.1))
testConditionalSet(ConditionalSet$new(function(x) x == 0))

```

---

testContains

*assert/check/test/Contains*


---

**Description**

Validation checks to test if given elements are contained in a set.

**Usage**

```

testContains(
  object,
  elements,
  errorMsg = "elements are not contained in the set"
)

```

```

checkContains(
  object,
  elements,
  errorMsg = "elements are not contained in the set"
)

```

```

assertContains(
  object,
  elements,
  errorMsg = "elements are not contained in the set"
)

```

**Arguments**

object	object to test
elements	elements to check
errorMsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testContains(Set$new(1,2,3), c(1,2))
testContains(Set$new(1,2,3), c(3,4))
```

---

testCountablyFinite     *assert/check/test/CountablyFinite*

---

**Description**

Validation checks to test if a given object is countablyfinite.

**Usage**

```
testCountablyFinite(object, errmsg = "This is not a countably finite set")
checkCountablyFinite(object, errmsg = "This is not a countably finite set")
assertCountablyFinite(object, errmsg = "This is not a countably finite set")
```

**Arguments**

object	object to test
errmsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testCountablyFinite(Set$new(1,2,3))
testCountablyFinite(Interval$new(1,10))
```

---

testCrisp                     *assert/check/test/Crisp*

---

**Description**

Validation checks to test if a given object is crisp.

**Usage**

```
testCrisp(object, errmsg = "This is not crisp.")
checkCrisp(object, errmsg = "This is not crisp.")
assertCrisp(object, errmsg = "This is not crisp.")
```

**Arguments**

object	object to test
errmsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testCrisp(Set$new(1))
testCrisp(FuzzySet$new(1, 0.5))
```

---

testEmpty	<i>assert/check/test/Empty</i>
-----------	--------------------------------

---

**Description**

Validation checks to test if a given object is empty.

**Usage**

```
testEmpty(object, errmsg = "This is not an empty set")
checkEmpty(object, errmsg = "This is not an empty set")
assertEmpty(object, errmsg = "This is not an empty set")
```

**Arguments**

object	object to test
errmsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testEmpty(Set$new())  
testEmpty(Set$new(1))
```

---

testFinite	<i>assert/check/test/Finite</i>
------------	---------------------------------

---

**Description**

Validation checks to test if a given object is finite.

**Usage**

```
testFinite(object, errorMsg = "This is not finite")  
checkFinite(object, errorMsg = "This is not finite")  
assertFinite(object, errorMsg = "This is not finite")
```

**Arguments**

object	object to test
errorMsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testFinite(Interval$new(1, 10, class = "integer"))  
testFinite(Interval$new(1, 10, class = "numeric"))
```

---

testFuzzy	<i>assert/check/test/Fuzzy</i>
-----------	--------------------------------

---

**Description**

Validation checks to test if a given object is fuzzy.

**Usage**

```
testFuzzy(object, errorMsg = "This is not fuzzy.")  
checkFuzzy(object, errorMsg = "This is not fuzzy.")  
assertFuzzy(object, errorMsg = "This is not fuzzy.")
```

**Arguments**

object	object to test
errmsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testFuzzy(FuzzySet$new(1, 0.5))
testFuzzy(Set$new(1))
```

---

testFuzzyMultiset	<i>assert/check/test/FuzzyMultiset</i>
-------------------	--

---

**Description**

Validation checks to test if a given object is an R6 FuzzyMultiset.

**Usage**

```
testFuzzyMultiset(object, errmsg = "This is not an R6 FuzzyMultiset object")
checkFuzzyMultiset(object, errmsg = "This is not an R6 FuzzyMultiset object")
assertFuzzyMultiset(
  object,
  errmsg = "This is not an R6 FuzzyMultiset object"
)
```

**Arguments**

object	object to test
errmsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.



**Examples**

```

testFuzzyMultiset(Set$new(2, 3))
testFuzzyMultiset(list(Set$new(2), Set$new(3)))
testFuzzyMultiset(Tuple$new(2, 3))
testFuzzyMultiset(Interval$new())
testFuzzyMultiset(FuzzySet$new(2, 0.1))
testFuzzyMultiset(FuzzyTuple$new(2, 0.1))
testFuzzyMultiset(ConditionalSet$new(function(x) x == 0))

```

---

testFuzzySet	<i>assert/check/test/FuzzySet</i>
--------------	-----------------------------------

---

**Description**

Validation checks to test if a given object is an R6 FuzzySet.

**Usage**

```

testFuzzySet(object, errmsg = "This is not an R6 FuzzySet object")

checkFuzzySet(object, errmsg = "This is not an R6 FuzzySet object")

assertFuzzySet(object, errmsg = "This is not an R6 FuzzySet object")

```

**Arguments**

object	object to test
errmsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```

testFuzzySet(Set$new(2, 3))
testFuzzySet(list(Set$new(2), Set$new(3)))
testFuzzySet(Tuple$new(2, 3))
testFuzzySet(Interval$new())
testFuzzySet(FuzzySet$new(2, 0.1))
testFuzzySet(FuzzyTuple$new(2, 0.1))
testFuzzySet(ConditionalSet$new(function(x) x == 0))

```

---

testFuzzyTuple	<i>assert/check/test/FuzzyTuple</i>
----------------	-------------------------------------

---

**Description**

Validation checks to test if a given object is an R6 FuzzyTuple.

**Usage**

```
testFuzzyTuple(object, errmsg = "This is not an R6 FuzzyTuple object")
checkFuzzyTuple(object, errmsg = "This is not an R6 FuzzyTuple object")
assertFuzzyTuple(object, errmsg = "This is not an R6 FuzzyTuple object")
```

**Arguments**

object	object to test
errmsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testFuzzyTuple(Set$new(2, 3))
testFuzzyTuple(list(Set$new(2), Set$new(3)))
testFuzzyTuple(Tuple$new(2, 3))
testFuzzyTuple(Interval$new())
testFuzzyTuple(FuzzySet$new(2, 0.1))
testFuzzyTuple(FuzzyTuple$new(2, 0.1))
testFuzzyTuple(ConditionalSet$new(function(x) x == 0))
```

---

testInterval	<i>assert/check/test/Interval</i>
--------------	-----------------------------------

---

**Description**

Validation checks to test if a given object is an R6 Interval.

**Usage**

```
testInterval(object, errmsg = "This is not an R6 Interval object")
checkInterval(object, errmsg = "This is not an R6 Interval object")
assertInterval(object, errmsg = "This is not an R6 Interval object")
```

**Arguments**

object	object to test
errmsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testInterval(Set$new(2, 3))
testInterval(list(Set$new(2), Set$new(3)))
testInterval(Tuple$new(2, 3))
testInterval(Interval$new())
testInterval(FuzzySet$new(2, 0.1))
testInterval(FuzzyTuple$new(2, 0.1))
testInterval(ConditionalSet$new(function(x) x == 0))
```

---

testMultiset

*assert/check/test/Multiset*


---

**Description**

Validation checks to test if a given object is an R6 Multiset.

**Usage**

```
testMultiset(object, errmsg = "This is not an R6 Multiset object")
checkMultiset(object, errmsg = "This is not an R6 Multiset object")
assertMultiset(object, errmsg = "This is not an R6 Multiset object")
```

**Arguments**

object	object to test
errmsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testMultiset(Set$new(2, 3))
testMultiset(list(Set$new(2), Set$new(3)))
testMultiset(Tuple$new(2, 3))
testMultiset(Interval$new())
testMultiset(FuzzySet$new(2, 0.1))
testMultiset(FuzzyTuple$new(2, 0.1))
testMultiset(ConditionalSet$new(function(x) x == 0))
```

---

testSet

*assert/check/test/Set*


---

**Description**

Validation checks to test if a given object is an R6 Set.

**Usage**

```
testSet(object, errormsg = "This is not an R6 Set object")
checkSet(object, errormsg = "This is not an R6 Set object")
assertSet(object, errormsg = "This is not an R6 Set object")
```

**Arguments**

object	object to test
errormsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testSet(Set$new(2, 3))
testSet(list(Set$new(2), Set$new(3)))
testSet(Tuple$new(2, 3))
testSet(Interval$new())
testSet(FuzzySet$new(2, 0.1))
testSet(FuzzyTuple$new(2, 0.1))
testSet(ConditionalSet$new(function(x) x == 0))
```

---

testSetList	<i>assert/check/test/SetList</i>
-------------	----------------------------------

---

**Description**

Validation checks to test if a given object is an R6 SetList.

**Usage**

```
testSetList(object, errmsg = "One or more items in the list are not Sets")
```

```
checkSetList(object, errmsg = "One or more items in the list are not Sets")
```

```
assertSetList(object, errmsg = "One or more items in the list are not Sets")
```

**Arguments**

object            object to test

errmsg           error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testSetList(Set$new(2, 3))
testSetList(list(Set$new(2), Set$new(3)))
testSetList(Tuple$new(2, 3))
testSetList(Interval$new())
testSetList(FuzzySet$new(2, 0.1))
testSetList(FuzzyTuple$new(2, 0.1))
testSetList(ConditionalSet$new(function(x) x == 0))
```

---

testSubset	<i>assert/check/test/Subset</i>
------------	---------------------------------

---

**Description**

Validation checks to test if given sets are subsets of a set.

**Usage**

```
testSubset(
  object,
  sets,
  proper = FALSE,
  errmsg = "sets are not subsets of the object"
)
```

```
checkSubset(
  object,
  sets,
  proper = FALSE,
  errmsg = "sets are not subsets of the object"
)
```

```
assertSubset(
  object,
  sets,
  proper = FALSE,
  errmsg = "sets are not subsets of the object"
)
```

**Arguments**

object	object to test
sets	sets to check
proper	logical. If TRUE tests for proper subsets.
errmsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testSubset(Set$new(1,2,3), Set$new(1,2))
testSubset(Set$new(1,2,3), Set$new(3,4))
```

---

testTuple

*assert/check/test/Tuple*


---

**Description**

Validation checks to test if a given object is an R6 Tuple.

**Usage**

```
testTuple(object, errorMsg = "This is not an R6 Tuple object")  
  
checkTuple(object, errorMsg = "This is not an R6 Tuple object")  
  
assertTuple(object, errorMsg = "This is not an R6 Tuple object")
```

**Arguments**

object	object to test
errorMsg	error message to overwrite default if check fails

**Value**

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

**Examples**

```
testTuple(Set$new(2, 3))  
testTuple(list(Set$new(2), Set$new(3)))  
testTuple(Tuple$new(2, 3))  
testTuple(Interval$new())  
testTuple(FuzzySet$new(2, 0.1))  
testTuple(FuzzyTuple$new(2, 0.1))  
testTuple(ConditionalSet$new(function(x) x == 0))
```

---

Tuple

*Mathematical Tuple*

---

**Description**

A general Tuple object for mathematical tuples, inheriting from Set.

**Details**

Tuples are similar to sets, except that they drop the constraint for elements to be unique, and ordering in a tuple does matter. Tuples are useful for methods including \$contains that may require non-unique elements. They are also the return type of the product of sets. See examples.

**Super class**

`set6::Set` -> Tuple

**Methods****Public methods:**

- `Tuple$equals()`
- `Tuple$isSubset()`
- `Tuple$clone()`

**Method** `equals()`: Tests if two sets are equal.

*Usage:*

```
Tuple$equals(x, all = FALSE)
```

*Arguments:*

x `Set` or vector of `Sets`.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Details:* An object is equal to a `Tuple` if it contains all the same elements, and in the same order. Infix operators can be used for:

```
Equal      ==
Not equal  !=
```

*Returns:* If all is TRUE then returns TRUE if all x are equal to the `Set`, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

*Examples:*

```
Tuple$new(1,2) == Tuple$new(1,2)
Tuple$new(1,2) != Tuple$new(1,2)
Tuple$new(1,1) != Set$new(1,1)
```

**Method** `isSubset()`: Test if one set is a (proper) subset of another

*Usage:*

```
Tuple$isSubset(x, proper = FALSE, all = FALSE)
```

*Arguments:*

x any. Object or vector of objects to test.

proper logical. If TRUE tests for proper subsets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Details:* If using the method directly, and not via one of the operators then the additional boolean argument `proper` can be used to specify testing of subsets or proper subsets. A `Set` is a proper subset of another if it is fully contained by the other `Set` (i.e. not equal to) whereas a `Set` is a (non-proper) subset if it is fully contained by, or equal to, the other `Set`.

When calling `$isSubset` on objects inheriting from `Interval`, the method treats the interval as if it is a `Set`, i.e. ordering and class are ignored. Use `$isSubinterval` to test if one interval is a subinterval of another.

Infix operators can be used for:

```
Subset      <
Proper Subset <=
Superset    >
Proper Superset >=
```



An object is a (proper) subset of a Tuple if it contains all (some) of the same elements, and in the same order.

*Returns:* If `all` is TRUE then returns TRUE if all `x` are subsets of the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

*Examples:*

```
Tuple$new(1,2,3) < Tuple$new(1,2,3,4)
Tuple$new(1,3,2) < Tuple$new(1,2,3,4)
```

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
Tuple$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

## See Also

Other sets: [ConditionalSet](#), [FuzzyMultiset](#), [FuzzySet](#), [FuzzyTuple](#), [Interval](#), [Multiset](#), [Set](#)

## Examples

```
# Tuple of integers
Tuple$new(1:5)

# Tuple of multiple types
Tuple$new("a", 5, Set$new(1), Tuple$new(2))

# Each Tuple has properties and traits
t <- Tuple$new(1, 2, 3)
t$traits
t$properties

# Elements can be duplicated
Tuple$new(2, 2) != Tuple$new(2)

# Ordering does matter
Tuple$new(1, 2) != Tuple$new(2, 1)

## -----
## Method `Tuple$equals`
## -----

Tuple$new(1,2) == Tuple$new(1,2)
Tuple$new(1,2) != Tuple$new(1,2)
Tuple$new(1,1) != Set$new(1,1)

## -----
## Method `Tuple$isSubset`
## -----

Tuple$new(1,2,3) < Tuple$new(1,2,3,4)
Tuple$new(1,3,2) < Tuple$new(1,2,3,4)
```

---

UnionSet

*Set of Unions*

---

## Description

UnionSet class for symbolic union of mathematical sets.

## Details

The purpose of this class is to provide a symbolic representation for the union of sets that cannot be represented in a simpler class. Whilst this is not an abstract class, it is not recommended to construct this class directly but via the set operation methods.

## Super classes

`set6::Set -> set6::SetWrapper -> UnionSet`

## Active bindings

`elements` Returns the elements in the object.

`length` Returns the number of elements in the object.

## Methods

### Public methods:

- `UnionSet$new()`
- `UnionSet$strprint()`
- `UnionSet$contains()`
- `UnionSet$clone()`

**Method** `new()`: Create a new UnionSet object. It is not recommended to construct this class directly.

*Usage:*

```
UnionSet$new(setlist, lower = NULL, upper = NULL, type = NULL)
```

*Arguments:*

`setlist` list of [Sets](#) to wrap.

`lower` lower bound of new object.

`upper` upper bound of new object.

`type` closure type of new object.

*Returns:* A new UnionSet object.

**Method** `strprint()`: Creates a printable representation of the object.

*Usage:*

```
UnionSet$strprint(n = 2)
```

*Arguments:*

*n* numeric. Number of elements to display on either side of ellipsis when printing.

*Returns:* A character string representing the object.

**Method** `contains()`: Tests if elements *x* are contained in `self`.

*Usage:*

```
UnionSet$contains(x, all = FALSE, bound = FALSE)
```

*Arguments:*

*x* any. Object or vector of objects to test.

*all* logical. If FALSE tests each *x* separately. Otherwise returns TRUE only if all *x* pass test.

*bound* logical.

*Returns:* If *all* == TRUE then returns TRUE if all *x* are contained in `self`, otherwise FALSE. If *all* == FALSE returns a vector of logicals corresponding to the length of *x*, representing if each is contained in `self`. If *bound* == TRUE then an element is contained in `self` if it is on or within the (possibly-open) bounds of `self`, otherwise TRUE only if the element is within `self` or the bounds are closed.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
UnionSet$clone(deep = FALSE)
```

*Arguments:*

*deep* Whether to make a deep clone.

**See Also**

Set operations: [setunion](#), [setproduct](#), [setpower](#), [setcomplement](#), [setsymdiff](#), [powerset](#), [setintersect](#)

Other wrappers: [ComplementSet](#), [ExponentSet](#), [PowersetSet](#), [ProductSet](#)

Universal

*Mathematical Universal Set***Description**

The `Universal` is defined as the [Set](#) containing all possible elements.

**Details**

The `Universal` set is the default universe to all sets, and is the largest possible set. The `Universal` set contains every single possible element. We denote the `Universal` set with  $V$  instead of  $U$  to avoid confusion with the union symbol. The `Universal` set cardinality is set to  $\text{Inf}$  where we assume  $\text{Inf}$  is greater than any Aleph or Beth numbers. The `Universal` set is also responsible for a few set paradoxes, to resolve these we use the following results:

Let  $V$  be the universal set,  $S$  be any non-universal set, and  $0$  the empty set, then

$$V \cup S = V$$

$$V \cap S = S$$

$$S - V = 0$$

$$V^n = V$$

$$P(V) = V$$

### Super class

`set6::Set` -> Universal

### Methods

#### Public methods:

- `Universal$new()`
- `Universal$equals()`
- `Universal$isSubset()`
- `Universal$contains()`
- `Universal$strprint()`
- `Universal$clone()`

**Method** `new()`: Create a new Universal object.

*Usage:*

`Universal$new()`

*Details:* The Universal set is the set containing every possible element.

*Returns:* A new Universal object.

**Method** `equals()`: Tests if two sets are equal.

*Usage:*

`Universal$equals(x, all = FALSE)`

*Arguments:*

x `Set` or vector of `Sets`.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Returns:* If all is TRUE then returns TRUE if all x are equal to the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

Infix operators can be used for:

Equal	==
Not equal	!=

*Examples:*

```
# Equals
Set$new(1,2)$equals(Set$new(5,6))
Set$new(1,2)$equals(Interval$new(1,2))
Set$new(1,2) == Interval$new(1,2, class = "integer")

# Not equal
!Set$new(1,2)$equals(Set$new(1,2))
Set$new(1,2) != Set$new(1,5)
```

**Method** `isSubset()`: Test if one set is a (proper) subset of another

*Usage:*

```
Universal$isSubset(x, proper = FALSE, all = FALSE)
```

*Arguments:*

`x` any. Object or vector of objects to test.

`proper` logical. If TRUE tests for proper subsets.

`all` logical. If FALSE tests each `x` separately. Otherwise returns TRUE only if all `x` pass test.

*Details:* If using the method directly, and not via one of the operators then the additional boolean argument `proper` can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set.

When calling `$isSubset` on objects inheriting from [Interval](#), the method treats the interval as if it is a [Set](#), i.e. ordering and class are ignored. Use `$isSubinterval` to test if one interval is a subinterval of another.

Infix operators can be used for:

Subset	<
Proper Subset	<=
Superset	>
Proper Superset	>=

Every Set is a subset of a Universal. No Set is a super set of a Universal, and only a Universal is not a proper subset of a Universal.

*Returns:* If `all` is TRUE then returns TRUE if all `x` are subsets of the Set, otherwise FALSE. If `all` is FALSE then returns a vector of logicals corresponding to each individual element of `x`.

*Examples:*

```
Set$new(1,2,3)$isSubset(Set$new(1,2), proper = TRUE)
Set$new(1,2) < Set$new(1,2,3) # proper subset
```

```
c(Set$new(1,2,3), Set$new(1)) < Set$new(1,2,3) # not proper
Set$new(1,2,3) <= Set$new(1,2,3) # proper
```

**Method** `contains()`: Tests to see if `x` is contained in the Set.

*Usage:*

```
Universal$contains(x, all = FALSE, bound = NULL)
```

*Arguments:*

*x* any. Object or vector of objects to test.

*all* logical. If FALSE tests each *x* separately. Otherwise returns TRUE only if all *x* pass test.

*bound* ignored.

*Details:* *x* can be of any type, including a Set itself. *x* should be a tuple if checking to see if it lies within a set of dimension greater than one. To test for multiple *x* at the same time, then provide these as a list.

If using the method directly, and not via one of the operators then the additional boolean arguments *all* and *bound*. If *all* = TRUE then returns TRUE if all *x* are contained in the Set, otherwise returns a vector of logicals. For [Intervals](#), *bound* is used to specify if elements lying on the (possibly open) boundary of the interval are considered contained (*bound* = TRUE) or not (*bound* = FALSE).

*Returns:* If *all* is TRUE then returns TRUE if all elements of *x* are contained in the Set, otherwise FALSE. If *all* is FALSE then returns a vector of logicals corresponding to each individual element of *x*.

The infix operator `%inset%` is available to test if *x* is an element in the Set, see examples.

Every element is contained within the Universal set.

*Examples:*

```
s = Set$new(1:5)
```

```
# Simplest case
```

```
s$contains(4)
```

```
8 %inset% s
```

```
# Test if multiple elements lie in the set
```

```
s$contains(4:6, all = FALSE)
```

```
s$contains(4:6, all = TRUE)
```

```
# Check if a tuple lies in a Set of higher dimension
```

```
s2 = s * s
```

```
s2$contains(Tuple$new(2,1))
```

```
c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2
```

**Method** `strprint()`: Creates a printable representation of the object.

*Usage:*

```
Universal$strprint(n = NULL)
```

*Arguments:*

*n* numeric. Number of elements to display on either side of ellipsis when printing.

*Returns:* A character string representing the object.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
Universal$clone(deep = FALSE)
```

*Arguments:*

*deep* Whether to make a deep clone.

**See Also**

Other special sets: [Complex](#), [ExtendedReals](#), [Integers](#), [Logicals](#), [Naturals](#), [NegIntegers](#), [NegRationals](#), [NegReals](#), [PosIntegers](#), [PosNaturals](#), [PosRationals](#), [PosReals](#), [Rationals](#), [Reals](#)

**Examples**

```

u <- Universal$new()
print(u)
u$contains(c(1, letters, TRUE, Set$new()), all = TRUE)

## -----
## Method `Universal$equals`
## -----

# Equals
Set$new(1,2)$equals(Set$new(5,6))
Set$new(1,2)$equals(Interval$new(1,2))
Set$new(1,2) == Interval$new(1,2, class = "integer")

# Not equal
!Set$new(1,2)$equals(Set$new(1,2))
Set$new(1,2) != Set$new(1,5)

## -----
## Method `Universal$isSubset`
## -----

Set$new(1,2,3)$isSubset(Set$new(1,2), proper = TRUE)
Set$new(1,2) < Set$new(1,2,3) # proper subset

c(Set$new(1,2,3), Set$new(1)) < Set$new(1,2,3) # not proper
Set$new(1,2,3) <= Set$new(1,2,3) # proper

## -----
## Method `Universal$contains`
## -----

s = Set$new(1:5)

# Simplest case
s$contains(4)
8 %inset% s

# Test if multiple elements lie in the set
s$contains(4:6, all = FALSE)
s$contains(4:6, all = TRUE)

# Check if a tuple lies in a Set of higher dimension
s2 = s * s
s2$contains(Tuple$new(2,1))
c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2

```

---

 UniversalSet

 Mathematical Universal Set
 

---

### Description

The UniversalSet is defined as the [Set](#) containing all possible elements.

### Details

The Universal set is the default universe to all sets, and is the largest possible set. The Universal set contains every single possible element. We denote the Universal set with  $V$  instead of  $U$  to avoid confusion with the union symbol. The Universal set cardinality is set to  $\text{Inf}$  where we assume  $\text{Inf}$  is greater than any Aleph or Beth numbers. The Universal set is also responsible for a few set paradoxes, to resolve these we use the following results:

Let  $V$  be the universal set,  $S$  be any non-universal set, and  $0$  the empty set, then

$$V \cup S = V$$

$$V \cap S = S$$

$$S - V = 0$$

$$V^n = V$$

$$P(V) = V$$

### Super class

`set6::Set` -> UniversalSet

### Methods

#### Public methods:

- `UniversalSet$new()`
- `UniversalSet$equals()`
- `UniversalSet$isSubset()`
- `UniversalSet$contains()`
- `UniversalSet$strprint()`
- `UniversalSet$clone()`

**Method** `new()`: Create a new UniversalSet object.

*Usage:*

`UniversalSet$new()`

*Details:* The Universal set is the set containing every possible element.

*Returns:* A new UniversalSet object.

**Method** `equals()`: Tests if two sets are equal.



*Usage:*

```
UniversalSet$equals(x, all = FALSE)
```

*Arguments:*

x [Set](#) or vector of [Sets](#).

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Returns:* If all is TRUE then returns TRUE if all x are equal to the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

Infix operators can be used for:

```
Equal      ==
Not equal  !=
```

*Examples:*

```
# Equals
Set$new(1,2)$equals(Set$new(5,6))
Set$new(1,2)$equals(Interval$new(1,2))
Set$new(1,2) == Interval$new(1,2, class = "integer")

# Not equal
!Set$new(1,2)$equals(Set$new(1,2))
Set$new(1,2) != Set$new(1,5)
```

**Method** `isSubset()`: Test if one set is a (proper) subset of another

*Usage:*

```
UniversalSet$isSubset(x, proper = FALSE, all = FALSE)
```

*Arguments:*

x any. Object or vector of objects to test.

proper logical. If TRUE tests for proper subsets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

*Details:* If using the method directly, and not via one of the operators then the additional boolean argument `proper` can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set.

When calling `$isSubset` on objects inheriting from [Interval](#), the method treats the interval as if it is a [Set](#), i.e. ordering and class are ignored. Use `$isSubinterval` to test if one interval is a subinterval of another.

Infix operators can be used for:

```
Subset      <
Proper Subset <=
Superset    >
Proper Superset >=
```

Every Set is a subset of a UniversalSet. No Set is a super set of a UniversalSet, and only a

UniversalSet is not a proper subset of a UniversalSet.

*Returns:* If all is TRUE then returns TRUE if all x are subsets of the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

*Examples:*

```
Set$new(1,2,3)$isSubset(Set$new(1,2), proper = TRUE)
Set$new(1,2) < Set$new(1,2,3) # proper subset
```

```
c(Set$new(1,2,3), Set$new(1)) < Set$new(1,2,3) # not proper
Set$new(1,2,3) <= Set$new(1,2,3) # proper
```

**Method** contains(): Tests to see if x is contained in the Set.

*Usage:*

```
UniversalSet$contains(x, all = FALSE, bound = NULL)
```

*Arguments:*

x any. Object or vector of objects to test.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

bound ignored.

*Details:* x can be of any type, including a Set itself. x should be a tuple if checking to see if it lies within a set of dimension greater than one. To test for multiple x at the same time, then provide these as a list.

If using the method directly, and not via one of the operators then the additional boolean arguments all and bound. If all = TRUE then returns TRUE if all x are contained in the Set, otherwise returns a vector of logicals. For [Intervals](#), bound is used to specify if elements lying on the (possibly open) boundary of the interval are considered contained (bound = TRUE) or not (bound = FALSE).

*Returns:* If all is TRUE then returns TRUE if all elements of x are contained in the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

The infix operator %inset% is available to test if x is an element in the Set, see examples. Every element is contained within the Universal set.

*Examples:*

```
s = Set$new(1:5)
```

```
# Simplest case
s$contains(4)
8 %inset% s
```

```
# Test if multiple elements lie in the set
s$contains(4:6, all = FALSE)
s$contains(4:6, all = TRUE)
```

```
# Check if a tuple lies in a Set of higher dimension
s2 = s * s
s2$contains(Tuple$new(2,1))
c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2
```

**Method** `strprint()`: Creates a printable representation of the object.

*Usage:*

```
UniversalSet$strprint(n = NULL)
```

*Arguments:*

`n` numeric. Number of elements to display on either side of ellipsis when printing.

*Returns:* A character string representing the object.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
UniversalSet$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

## Examples

```
u <- UniversalSet$new()
print(u)
u$contains(c(1, letters, TRUE, Set$new()), all = TRUE)

## -----
## Method `UniversalSet$equals`
## -----

# Equals
Set$new(1,2)$equals(Set$new(5,6))
Set$new(1,2)$equals(Interval$new(1,2))
Set$new(1,2) == Interval$new(1,2, class = "integer")

# Not equal
!Set$new(1,2)$equals(Set$new(1,2))
Set$new(1,2) != Set$new(1,5)

## -----
## Method `UniversalSet$isSubset`
## -----

Set$new(1,2,3)$isSubset(Set$new(1,2), proper = TRUE)
Set$new(1,2) < Set$new(1,2,3) # proper subset

c(Set$new(1,2,3), Set$new(1)) < Set$new(1,2,3) # not proper
Set$new(1,2,3) <= Set$new(1,2,3) # proper

## -----
## Method `UniversalSet$contains`
## -----

s = Set$new(1:5)

# Simplest case
```

```
s$contains(4)
8 %inset% s

# Test if multiple elements lie in the set
s$contains(4:6, all = FALSE)
s$contains(4:6, all = TRUE)

# Check if a tuple lies in a Set of higher dimension
s2 = s * s
s2$contains(Tuple$new(2,1))
c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2
```

---

useUnicode

*Get/Set Unicode Printing Method*

---

### Description

Change whether unicode symbols should be used when printing sets.

### Usage

```
useUnicode(use)
```

### Arguments

use                    logical, if TRUE unicode will be used in printing, otherwise simpler character strings. If missing the current setting is returned.

### Details

Using unicode symbols makes the printing of sets and properties 'prettier', however may not work on all machines or versions of R. Therefore this function is used to decide whether unicode representations should be used, or standard alpha-numeric and special characters.

By default set6 starts with unicode printing turned on.

### Examples

```
current <- useUnicode()
useUnicode(TRUE)
useUnicode()
useUnicode(current)
```

# Index

- !=.Set (equals), 19
- \* **coercions**
  - as.FuzzySet, 4
  - as.Interval, 7
  - as.Set, 8
- \* **operators**
  - powerset, 52
  - setcomplement, 68
  - setintersect, 70
  - setpower, 72
  - setproduct, 74
  - setsymdiff, 75
  - setunion, 76
- \* **sets**
  - ConditionalSet, 15
  - FuzzyMultiset, 22
  - FuzzySet, 25
  - FuzzyTuple, 30
  - Interval, 34
  - Multiset, 42
  - Set, 60
  - Tuple, 95
- \* **special sets**
  - Complex, 12
  - ExtendedReals, 21
  - Integers, 33
  - Logicals, 40
  - Naturals, 44
  - NegIntegers, 45
  - NegRationals, 46
  - NegReals, 47
  - PosIntegers, 48
  - PosNaturals, 49
  - PosRationals, 50
  - PosReals, 51
  - Rationals, 58
  - Reals, 59
  - Universal, 99
- \* **wrappers**
  - ComplementSet, 10
  - ExponentSet, 20
  - PowersetSet, 53
  - ProductSet, 55
  - UnionSet, 98
- \*.Set (setproduct), 74
- +.Set (setunion), 76
- .Set (setcomplement), 68
- <.Set (isSubset), 39
- <=.Set (isSubset), 39
- ==.Set (equals), 19
- >.Set (isSubset), 39
- >=.Set (isSubset), 39
- %-% (setsymdiff), 75
- %inset% (contains), 19
- &.Set (setintersect), 70
- ^.Set (setpower), 72
- as.FuzzyMultiset (as.FuzzySet), 4
- as.FuzzySet, 4, 8, 10
- as.FuzzySet.Set, 6
- as.FuzzyTuple (as.FuzzySet), 4
- as.Interval, 6, 7, 10
- as.Interval.numeric, 7
- as.Interval.Set, 8
- as.Multiset (as.Set), 8
- as.Set, 6, 8, 8
- as.Set.Interval, 6
- as.Tuple (as.Set), 8
- assertClosed (testClosed), 81
- assertClosedAbove (testClosedAbove), 82
- assertClosedBelow (testClosedBelow), 82
- assertConditionalSet (testConditionalSet), 83
- assertContains (testContains), 84
- assertCountablyFinite (testCountablyFinite), 85
- assertCrisp (testCrisp), 85
- assertEmpty (testEmpty), 86
- assertFinite (testFinite), 87

- assertFuzzy (testFuzzy), 87
- assertFuzzyMultiset (testFuzzyMultiset), 88
- assertFuzzySet (testFuzzySet), 89
- assertFuzzyTuple (testFuzzyTuple), 90
- assertInterval (testInterval), 90
- assertMultiset (testMultiset), 91
- assertSet (testSet), 92
- assertSetList (testSetList), 93
- assertSubset (testSubset), 93
- assertTuple (testTuple), 94
  
- checkClosed (testClosed), 81
- checkClosedAbove (testClosedAbove), 82
- checkClosedBelow (testClosedBelow), 82
- checkConditionalSet (testConditionalSet), 83
- checkContains (testContains), 84
- checkCountablyFinite (testCountablyFinite), 85
- checkCrisp (testCrisp), 85
- checkEmpty (testEmpty), 86
- checkFinite (testFinite), 87
- checkFuzzy (testFuzzy), 87
- checkFuzzyMultiset (testFuzzyMultiset), 88
- checkFuzzySet (testFuzzySet), 89
- checkFuzzyTuple (testFuzzyTuple), 90
- checkInterval (testInterval), 90
- checkMultiset (testMultiset), 91
- checkSet (testSet), 92
- checkSetList (testSetList), 93
- checkSubset (testSubset), 93
- checkTuple (testTuple), 94
- ComplementSet, 10, 21, 54, 56, 65, 99
- Complex, 12, 22, 34, 41, 45–51, 59, 60, 103
- ConditionalSet, 15, 24, 29, 32, 37, 43, 66, 69, 71, 77, 97
- contains, 19
  
- equals, 19
- ExponentSet, 12, 20, 54, 56, 99
- ExtendedReals, 15, 21, 34, 41, 45–51, 59, 60, 103
  
- FuzzyMultiset, 18, 22, 24, 28, 29, 32, 37, 43, 66, 97
- FuzzySet, 4, 6, 8, 10, 18, 24, 25, 27, 28, 31, 32, 37, 43, 60, 66, 97
- FuzzyTuple, 4, 6, 18, 24, 25, 28, 29, 30, 32, 37, 43, 66, 97
  
- Integers, 15, 22, 33, 41, 45–51, 59, 60, 103
- Interval, 7, 8, 10, 13, 14, 18, 24, 29, 32, 34, 36, 37, 43, 60, 65, 66, 96, 97, 101, 102, 105, 106
- isSubset, 39
  
- listSpecialSets, 40, 80
- Logicals, 15, 22, 34, 40, 45–51, 59, 60, 103
- LogicalSet, 41
  
- Multiset, 18, 22, 24, 29, 32, 37, 42, 66, 97
  
- Naturals, 15, 22, 34, 41, 44, 46–51, 59, 60, 103
- NegIntegers, 15, 22, 34, 41, 45, 45, 47–51, 59, 60, 103
- NegRationals, 15, 22, 34, 41, 45, 46, 46, 48–51, 59, 60, 103
- NegReals, 15, 22, 34, 41, 45–47, 47, 48–51, 59, 60, 103
  
- PosIntegers, 15, 22, 34, 41, 45–48, 48, 49–51, 59, 60, 103
- PosNaturals, 15, 22, 34, 41, 45–48, 49, 50, 51, 59, 60, 103
- PosRationals, 15, 22, 34, 41, 45–49, 50, 51, 59, 60, 103
- PosReals, 15, 22, 34, 41, 45–50, 51, 59, 60, 103
- powerset, 12, 21, 52, 54, 56, 70, 71, 73, 74, 76, 77, 99
- PowersetSet, 12, 21, 52, 53, 56, 99
- ProductSet, 12, 21, 54, 55, 74, 99
- Properties, 57
  
- Rationals, 15, 22, 34, 41, 45–51, 58, 60, 103
- Reals, 15, 22, 34, 35, 41, 45–51, 59, 59, 103
  
- Set, 6–8, 10, 11, 13, 14, 16–21, 23–25, 27–29, 31, 32, 34–37, 39–43, 52–57, 60, 63, 65, 73, 74, 76–79, 96–101, 104, 105
- set6 (set6-package), 3
- set6-package, 3
- set6::FuzzySet, 22, 31
- set6::Integers, 45, 48
- set6::Interval, 21, 33, 44–51, 58, 60, 80
- set6::Naturals, 49

- set6::ProductSet, [20, 53](#)
- set6::Rationals, [46, 50](#)
- set6::Reals, [21, 47, 51](#)
- set6::Set, [10, 12, 15, 20–22, 25, 31, 33, 34, 40–42, 44–51, 53, 55, 58, 60, 78, 80, 95, 98, 100, 104](#)
- set6::SetWrapper, [10, 20, 53, 55, 98](#)
- set6::SpecialSet, [21, 33, 44–51, 58, 60](#)
- set6News, [68](#)
- setcomplement, [12, 21, 35, 52, 54, 56, 68, 71, 73, 74, 76, 77, 99](#)
- setintersect, [12, 21, 52, 54, 56, 70, 70, 73, 74, 76, 77, 99](#)
- setpower, [12, 21, 52, 54, 56, 70, 71, 72, 74, 76, 77, 99](#)
- setproduct, [12, 21, 52, 54, 56, 70, 71, 73, 74, 76, 77, 99](#)
- setsymdiff, [12, 21, 52, 54, 56, 70, 71, 73, 74, 75, 77, 99](#)
- setunion, [12, 21, 52, 54, 56, 70, 71, 73, 74, 76, 76, 99](#)
- SetWrapper, [78](#)
- SpecialSet, [80](#)
  
- testClosed, [81](#)
- testClosedAbove, [82](#)
- testClosedBelow, [82](#)
- testConditionalSet, [83](#)
- testContains, [84](#)
- testCountablyFinite, [85](#)
- testCrisp, [85](#)
- testEmpty, [86](#)
- testFinite, [87](#)
- testFuzzy, [87](#)
- testFuzzyMultiset, [88](#)
- testFuzzySet, [89](#)
- testFuzzyTuple, [90](#)
- testInterval, [90](#)
- testMultiset, [91](#)
- testSet, [92](#)
- testSetList, [93](#)
- testSubset, [93](#)
- testTuple, [94](#)
- Tuple, [8, 10, 16, 18, 24, 29, 31, 32, 37, 42, 43, 60, 66, 95](#)
  
- UnionSet, [12, 21, 54, 56, 69, 76, 98](#)
- Universal, [15, 16, 22, 34, 41, 45–51, 59, 60, 62, 99](#)
- UniversalSet, [104](#)
- useUnicode, [108](#)
- useUnicode(), [62](#)