Package ‘skewMLRM’

June 11, 2021

Type Package

Title Estimation for Scale-Shape Mixtures of Skew-Normal Distributions

Version 1.0

Date 2021-06-10

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Description Provide data generation and estimation tools for the multivariate scale mixtures of normal presented in Lange and Sinsheimer (1993) <doi:10.2307/1390698>, the multivariate scale mixtures of skew-normal presented in Zeller, Lachos and Vilca (2011) <doi:10.1080/02664760903406504>, the multivariate skew scale mixtures of normal presented in Louredo, Zeller and Ferreira (2021) <doi:10.1007/s13571-021-00257-y> and the multivariate scale mixtures of skew-normal-Cauchy presented in Kahrari et al. (2020) <doi:10.1080/03610918.2020.1804582>.

Depends stats, foreach

Imports moments, clusterGeneration, doParallel, parallel, MASS, mvtnorm

Suggests sn

License GPL (>= 2)

NeedsCompilation no

Repository CRAN

Date/Publication 2021-06-11 09:50:02 UTC

R topics documented:

`choose2` *Select a distribution in the MSMN, MSSMN, MSMSN or/and MSMSNC classes and perform covariates selection.*

Description

choose2 select a model inside the multivariate scale mixtures of normal (MSMN), the multivariate scale mixtures of skew-normal (MSMSN), the multivariate skew scale mixtures of normal (MSSMN) or/and the multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for supported distributions within each class. Then, implement the covariates selection based on the significance, the Akaike's information criteria (AIC) or Schwartz's information criteria (BIC).

Usage

```
choose2(y, X = NULL, max.iter = 1000, prec = 1e-04, class = "MSMN",
   est.var = TRUE, criteria = "AIC", criteria.cov = "significance",
   significance = 0.05, cluster = FALSE)
```

Arguments

`y`	The multivariate vector of responses. The univariate case also is supported.
`X`	The regressor matrix.
`max.iter`	The maximum number of iterations.
`prec`	The convergence tolerance for parameters.
`class`	class in which will be performed a distribution: MSMN (default), MSSMN, MSMSN, MSMSNC or ALL (which consider all the mentioned classes). See details.
`est.var`	Logical. If TRUE the standard errors are estimated.
`criteria`	criteria to perform the selection model: AIC (default) or BIC.
`criteria.cov`	criteria to perform the covariates selection: significance (default), AIC or BIC.
`significance`	the level of significance to perform the covariate selection. By default is 0.05.
`cluster`	logical. If TRUE, parallel computing is used. FALSE is the default value.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

`fitted.models`	A vector with the fitted models
`selected.model`	Selected model based on the specified criteria (AIC or BIC)
`comment`	A comment indicating how many coefficients were eliminated
`estimate`	A matrix with the estimates (and standard errors, if est.var=TRUE) for the selected model
`logLik`	The log-likelihood function evaluated in the estimated parameters for the selected model
`AIC`	Akaike's Information Criterion for the selected model
`BIC`	Bayesian's Information Criterion for the selected model
`conv`	An integer code for the selected model. 0 indicates successful completion. 1 otherwise.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the covariate matrix X
##X

##Select a distribution within the MSMN class. Then, perform covariate
##selection based on the significance
choose2(y, X, class="MSMN")
##Identical process within the MSSMN class.
##may take some time on some systems
choose2(y, X, class="MSSMN")
```

---

chooseM *Choose a distribution in the MSMN, MSMSN, MSSMN and MSMSNC classes*

Description

choose.xxx select a model inside the xxx class, where xxx is the multivariate scale mixtures of normal (MSMN), the multivariate scale mixtures of skew-normal (MSMSN), the multivariate skew scale mixtures of normal (MSSMN) or the multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for supported distributions within each class. choose.models select a model among the MSMN, MSMSN, MSSMN and MSMSNC classes.

Usage

```
choose.MSMN(y, X = NULL, max.iter = 1000, prec = 1e-4,
        est.var = TRUE, criteria = "AIC", cluster = FALSE)
choose.MSMSN(y, X = NULL, max.iter = 1000, prec = 1e-4,
        est.var = TRUE, criteria = "AIC", cluster = FALSE)
choose.MSSMN(y, X = NULL, max.iter = 1000, prec = 1e-4,
        est.var = TRUE, criteria = "AIC", cluster = FALSE)
choose.MSMSNC(y, X = NULL, max.iter = 1000, prec = 1e-4,
        est.var = TRUE, criteria = "AIC", cluster = FALSE)
choose.models(y, X = NULL, max.iter = 1000, prec = 1e-4,
        est.var = TRUE, criteria = "AIC", cluster = FALSE)
```

Arguments

`y`	The multivariate vector of responses. The univariate case also is supported.
`X`	The regressor matrix.
`max.iter`	The maximum number of iterations.
`prec`	The convergence tolerance for parameters.
`est.var`	Logical. If TRUE the standard errors are estimated.
`criteria`	criteria to perform the selection model: AIC (default) or BIC.
`cluster`	logical. If TRUE, parallel computing is used. FALSE is the default value.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

A list containing the following components:

`fitted.models`	A vector with the fitted models
`selected.model`	Selected model based on the specified criteria (AIC or BIC)
`estimate`	A matrix with the estimates (and standard errors, if est.var=TRUE) for the selected model
`logLik`	The log-likelihood function evaluated in the estimated parameters for the selected model
`AIC`	Akaike's Information Criterion for the selected model
`BIC`	Bayesian's Information Criterion for the selected model
`iterations`	the number of iterations until convergence (if attached) for the selected model
`conv`	An integer code for the selected model. 0 indicates successful completion. 1 otherwise.

Note

This function does not consider selection of covariates.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the covariate matrix X
##X
##Select a distribution within the MSMN class.
choose.MSMN(y,X)

##Identical process within the MSSMN class.
##may take some time on some systems
choose.MSSMN(y,X)
```

`estimateM` *Fitting a model in the MSMN, MSMSN, MSSMN and MSMSNC classes*

Description

estimate.Mxxx computes the maximum likelihood estimates for the distribution xxx, where xxx is any supported model in the multivariate scale mixtures of normal (MSMN), multivariate scale mixtures of skew-normal (MSMSN), multivariate skew scale mixtures of normal (MSSMN) or multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for supported distributions.

Usage

```
estimate.MN(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MT(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSL(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MCN(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSN(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSTN(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSSL(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSCN(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSTT(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSSL2(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSCN2(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSNC(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSTEC(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSSLEC(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
estimate.MSCEC(y, X, max.iter = 1000, prec = 1e-04, est.var = TRUE)
```

Arguments

`y`	The multivariate vector of responses. The univariate case also is supported.
`X`	The regressor matrix.
`max.iter`	The maximum number of iterations.
`prec`	The convergence tolerance for parameters.
`est.var`	Logical. If TRUE the standard errors are estimated.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

A list containing the following components:

`estimate`	A matrix with the estimates (and standard errors, if est.var=TRUE)
`nu`	The estimated nu (only for MSTT, MSSL2, MSCN2, MSTEC, MSSLEC and MSCEC models)
`gamma`	The estimated gamma (only for MSCN2 and MSCEC models)
`logLik`	The log-likelihood function evaluated in the estimated parameters
`AIC`	Akaike's Information Criterion
`BIC`	Bayesian's Information Criterion
`iterations`	the number of iterations until convergence (if attached)
`time`	execution time in seconds
`conv`	An integer code. 0 indicates successful completion. 1 otherwise.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
```

```
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the covariate matrix X
##X
estimate.MN(y, X) ##Estimate the parameters for the MN regression model
estimate.MT(y, X) ##Estimate the parameters for the MT regression model

##may take some time on some systems
estimate.MSSL(y, X) ##Estimate the parameters for the MSSL regression model
estimate.MSTT(y, X) ##Estimate the parameters for the MSTT regression model
estimate.MSNC(y, X) ##Estimate the parameters for the MSNC regression model
estimate.MSCEC(y, X) ##Estimate the parameters for the MSCEC regression model
```

FIM *Observed Fisher information matrix for distributions in the MSMN, MSMSN, MSSMN and MSMSNC classes.*

Description

FI.xxx computes the observed Fisher information (FI) matrix for the distribution xxx, where xxx is any supported model in the multivariate scale mixtures of normal (MSMN), multivariate scale mixtures of skew-normal (MSMSN), multivariate skew scale mixtures of normal (MSSMN) or multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for supported distributions.

Usage

```
FI.MN(P, y, X)
FI.MT(P, y, X)
FI.MSL(P, y, X)
FI.MCN(P, y, X)
FI.MSN(P, y, X)
FI.MSTN(P, y, X)
FI.MSSL(P, y, X)
FI.MSCN(P, y, X)
FI.MSTT(P, y, X, nu)
FI.MSSL2(P, y, X, nu)
FI.MSCN2(P, y, X, nu, gamma)
FI.MSNC(P, y, X)
FI.MSTEC(P, y, X, nu)
FI.MSSLEC(P, y, X, nu)
FI.MSCEC(P, y, X, nu, gamma)
```

Arguments

P	the estimated parameters returned by a function of the form estimate.xxx, where xxx is a supported distribution.

y	The multivariate vector of responses. The univariate case also is supported.
X	The regressor matrix.
nu	nu parameter. Only for MSTT, MSSL2, MSTEC, MSSLEC and MSCEC distributions.
gamma	gamma parameter. Only for MSCN2 and MSCEC distributions.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

A matrix with the observed FI matrix for the specified model.

Note

For MSTEC and MSSLEC and distributions, nu>0 is considered as fixed. For MSCEC distribution, 0<nu<1 and 0<gamma<1 are considered as fixed.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
set.seed(2020)
n=200   # length of the sample
nv<-3   # number of explanatory variables
p<-nv+1 # nv + intercept
m<-4    # dimension of Y
q0=p*m
X<-array(0,c(q0,m,n))
for(i in 1:n) {
    aux=rep(1,p)
    aux[2:p]<-rMN(1,mu=rnorm(nv),Sigma=diag(nv))
    mi=matrix(0,q0,m)
    for (j in 1:m) mi[((j-1)*p+1):(j*p),j]=aux
    X[,,i]<-mi
} #Simulated matrix covariates
betas<-matrix(rnorm(q0),ncol=1) ## True betas
Sigmas <- clusterGeneration::genPositiveDefMat(m,rangeVar=c(1,3),
lambdaLow=1, ratioLambda=3)$Sigma ##True Sigma
lambda<-rnorm(m) ##True lambda
y=matrix(0,n,m)
for(i in 1:n) {
     mui<-t(X[,,i])%*%betas
     y[i,]<-rMSN(n=1,c(mui),Sigmas,lambda)}

P=estimate.MSN(y,X) ##Estimate parameters for MSN model
P  ## Output of estimate.MSN
P$estimate[,2] ##Estimated standard errors by the estimate.MSN function
##Estimated standard errors by minus the square root of
##the diagonal from the observed FI matrix of the MSN model
sqrt(diag(solve(-FI.MSN(P$estimate[,1], y, X))))
```

`matrix.sqrt` *Square root of a matrix*

Description

Compute the square root of a matrix

Usage

```
matrix.sqrt(A)
```

Arguments

A	a symmetric semi-definite positive matrix

Value

A symmetric matrix, say B, such as B^t*B=A

Note

For internal use.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

Examples

```
A<-matrix(c(1,2,2,5),nrow=2)
B<-matrix.sqrt(A)
##Recovering A
t(B)%*%B
A
```

---

mbackcrit *Multivariate backward based on the AIC or BIC criteria*

Description

mbackcrit implements the covariates selection based on backward and the Akaike's information criteria (AIC) or Schwartz's information criteria (BIC) in a specified multivariate model in the multivariate scale mixtures of normal (MSMN), multivariate scale mixtures of skew-normal (MSMSN), multivariate skew scale mixtures of normal (MSSMN) or multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for avaliable distributions.

Usage

```
mbackcrit(y, X = NULL, max.iter = 1000, prec = 1e-04, dist = "MN",
criteria = "AIC", cluster = FALSE)
```

Arguments

y	The multivariate vector of responses. The univariate case also is supported.
X	The regressor matrix.
max.iter	The maximum number of iterations.
prec	The convergence tolerance for parameters.
dist	the multivariate distribution in which the covariates selection will be implemented.
criteria	criteria used to perform the covariates selection. AIC (default) and BIC available.
cluster	logical. If TRUE, parallel computing is used. FALSE is the default value.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

A list with the following components:

`comment`	A comment indicating how many coefficients were eliminated.
`estimate`	A matrix with the estimates and standard errors for the final model.
`logLik`	The log-likelihood function evaluated in the estimated parameters
`AIC`	Akaike's Information Criterion
`BIC`	Bayesian's Information Criterion
`conv`	An integer code. 0 indicates successful completion. 1 otherwise.
`eliminated`	An string vector with the eliminated betas (in order of elimination).

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the regressor matrix X
##X

##Perform covariates selection in the MN distribution
##based on the AIC criteria
##may take some time on some systems
mbackcrit(y, X, dist="MN")
##Identical process for MT distribution
mbackcrit(y, X, dist="MT")
##and for MSN distribution
mbackcrit(y, X, dist="MSN")
```

---

mbacksign *Multivariate Backward Based on Significance*

Description

mbacksign implements the covariates selection based on the significance of the covariates in a specified multivariate model in the multivariate scale mixtures of normal (MSMN), multivariate scale mixtures of skew-normal (MSMSN), multivariate skew scale mixtures of normal (MSSMN) or multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for avaliable distributions.

Usage

```
mbacksign(y, X = NULL, max.iter = 1000, prec = 1e-04, dist = "MN",
     significance = 0.05)
```

Arguments

`y`	The multivariate vector of responses. The univariate case also is supported.
`X`	The regressor matrix.
`max.iter`	The maximum number of iterations.

`prec`	The convergence tolerance for parameters.
`dist`	the multivariate distribution in which the covariates selection will be implemented.
`significance`	the level of significance to perform the covariate selection. By default is 0.05.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

A list with the following components:

`comment`	A comment indicating how many coefficients were eliminated.
`estimate`	A matrix with the estimates and standard errors for the final model.
`logLik`	The log-likelihood function evaluated in the estimated parameters
`AIC`	Akaike's Information Criterion
`BIC`	Bayesian's Information Criterion
`conv`	An integer code. 0 indicates successful completion. 1 otherwise.
`eliminated`	An string vector with the eliminated betas (in order of elimination).

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the regressor matrix X
##X
##Perform covariates selection in the MN distribution
##based on a significance level of 1%, 5% and 10%
mbacksign(y, X, dist="MN", sign=0.01)
mbacksign(y, X, dist="MN", sign=0.05)
mbacksign(y, X, dist="MN", sign=0.10)

##may take some time on some systems
##identical process in the MCN model with
##significance level of 5%
mbacksign(y, X, dist="MCN")
##for MSSL model
mbacksign(y, X, dist="MSSL")
##for MSNC model
mbacksign(y, X, dist="MSNC")
```

`plotMahal` *Mahalanobis distance for fitted models in the MSMN, MSMSN, MSSMN and MSMSNC classes*

Description

Compute and plot the Mahalanobis distance for any supported model in the multivariate scale mixtures of normal (MSMN), multivariate scale mixtures of skew-normal (MSMSN), multivariate skew scale mixtures of normal (MSSMN) or multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for supported distributions.

Usage

```
plotMahal(fit, y, X, dist = "MN", alpha = 0.95, ...)
```

Arguments

`fit`	an object returned by the function estimate.xxx, where xxx is any supported model. See details for supported distributions.
`y`	The multivariate vector of responses. The univariate case also is supported.
`X`	the regressor matrix.
`dist`	the distribution used to obtain the fit object. See details for supported distributions.
`alpha`	significance level (0.05 by default).
`...`	... can include any of the other arguments of plots

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

A plot with the Mahalanobis distance for all the observations and a cut-off to detect possible influent observations based on the specified significance (0.05 by default).

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
set.seed(2020)
n=200   # length of the sample
nv<-3   # number of explanatory variables
p<-nv+1 # nv + intercept
m<-4    # dimension of Y
q0=p*m
X<-array(0,c(q0,m,n))
for(i in 1:n) {
    aux=rep(1,p)
    aux[2:p]<-rMN(1,mu=rnorm(nv),Sigma=diag(nv)) ##simulating covariates
    mi=matrix(0,q0,m)
    for (j in 1:m) mi[((j-1)*p+1):(j*p),j]=aux
    X[,,i]<-mi
} ##X is the simulated regressor matrix
betas<-matrix(rnorm(q0),ncol=1) ##True betas
Sigmas <- clusterGeneration::genPositiveDefMat(m,rangeVar=c(1,3),
lambdaLow=1, ratioLambda=3)$Sigma ##True Sigma
y=matrix(0,n,m)
for(i in 1:n) {
     mui<-t(X[,,i])%*%betas
     y[i,]<-rMN(n=1,c(mui),Sigmas) ## simulating the response vector
}
theta.MN=estimate.MN(y,X) #fit the MN model
plotMahal(theta.MN, y, X) #Plot the Mahalanobis distance for MN model
theta.MT=estimate.MT(y,X) #fit the MT model
plotMahal(theta.MT, y, X) #Plot the Mahalanobis distance for MT model
```

`rM` *Random generation for models in the MSMN, MSMSN, MSSMN and MSMSNC classes*

Description

rxxx generates random values for the distribution xxx, where xxx is any supported model in the multivariate scale mixtures of normal (MSMN), multivariate scale mixtures of skew-normal (MSMSN), multivariate skew scale mixtures of normal (MSSMN) or multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for supported distributions.

Usage

```
rMN(n, mu, Sigma)
rMT(n, mu, Sigma, nu = 1)
rMSL(n, mu, Sigma, nu = 1)
rMCN(n, mu, Sigma, nu = 0.5, gamma = 0.5)
rMSN(n, mu, Sigma, lambda)
rMSTN(n, mu, Sigma, lambda, nu = 1)
rMSSL(n, mu, Sigma, lambda, nu = 1)
rMSCN(n, mu, Sigma, lambda, nu = 0.5, gamma = 0.5)
rMSTT(n, mu, Sigma, lambda, nu = 1)
rMSSL2(n, mu, Sigma, lambda, nu = 1)
rMSCN2(n, mu, Sigma, lambda, nu = 0.5, gamma = 0.5)
rMSNC(n, mu, Sigma, lambda)
rMSTEC(n, mu, Sigma, lambda, nu = 1)
rMSSLEC(n, mu, Sigma, lambda, nu = 1)
rMSCEC(n, mu, Sigma, lambda, nu = 0.5, gamma = 0.5)
```

Arguments

`n`	number of observations to be generated.
`mu`	vector of location parameters.
`Sigma`	covariance matrix (a positive definite matrix).
`lambda`	vector of shape parameters.
`nu`	nu parameter. A positive scalar for MT, MSL, MSTN, MSSL, MSTT, MSSL2, MSTEC and MSSLEC models. A value in the interval (0,1) for MCN, MSCN, MSCN2 and MSCEC models.
`gamma`	gamma parameter. A value in the interval (0,1) for MCN, MSCN, MSCN2 and MSCEC models.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

MN used `mvrnorm`. For MT, MSL and MCN, the generation is based on the MSMN class. See Lange and Sinsheimer (1993) for details. For MSTN, MSSL and MSCN, the generation is based on the MSSMN class. See Ferreira, Lachos and Bolfarine (2016) for details. For MSTT, MSSL2 and MSCN2, the generation is based on the multivariate scale mixtures of skew-normal (MSMSN) class. See Branco and Dey (2001) for details. For MSNC, the generation is based on the stochastic representation in Proposition 2.1 of Kahrari et al. (2016). For the MSTEC, MSSLEC and MSCEC models, the generation is based on the MSMSNC class. See Kahrari et al. (2017) for details.

Value

A matrix with the generated data.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

References

Branco, M.D., Dey, D.K. (2001). A general class of multivariate skew-elliptical distributions. Journal of Multivariate Analysis 79, 99-113.

Ferreira, C.S., Lachos, V.H., Bolfarine, H. (2016). Likelihood-based inference for multivariate skew scale mixtures of normal distributions. AStA Advances in Statistical Analysis 100, 421-441.

Kahrari, F., Rezaei, M., Yousefzadeh, F., Arellano-Valle, R.B. (2016). On the multivariate skew-normal-Cauchy distribution. Statistics and Probability Letters, 117, 80-88.

Kahrari, F., Arellano-Valle, R.B., Rezaei, M., Yousefzadeh, F. (2017). Scale mixtures of skew-normal-Cauchy distributions. Statistics and Probability Letters, 126, 1-6.

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Examples

```
rMSN(10, mu=c(0,0), Sigma=diag(2), lambda=c(1,-1)) ##bivariate MSN model
rMSNC(10, mu=0, Sigma=2, lambda=1) ##univariate MSNC model
rMSNC(10, mu=1:3, Sigma=2*diag(3), lambda=c(1,-1,0)) ##trivariate MSN model
```

`se.est` *Estimated standard errors for the estimated parameters in distributions belonging to the MSMN, MSMSN, MSSMN and MSMSNC classes.*

Description

se.est computes the estimated standard errors based on the hessian matrix for supported models in the multivariate scale mixtures of normal (MSMN), multivariate scale mixtures of skew-normal (MSMSN), multivariate skew scale mixtures of normal (MSSMN) and multivariate scale mixtures of skew-normal-Cauchy (MSMSNC) classes. See details for supported distributions.

Usage

```
se.est(P, y, X, dist = "MN", nu = 0.5, gamma = 0.5)
```

Arguments

`P`	the estimated parameters returned by a function of the form estimate.xxx, where xxx is a supported distribution.
`y`	The multivariate vector of responses. The univariate case also is supported.
`X`	The regressor matrix.
`dist`	a supported multivariate distribution. See details.
`nu`	nu parameter. Only for MSTT, MSSL2, MSCN2, MSTEC, MSSLEC and MSCEC distributions.
`gamma`	gamma parameter. Only for MSCN2 and MSCEC distributions.

Details

Supported models are:

In MSMN class: multivariate normal (MN), multivariate Student t (MT), multivariate slash (MSL), multivariate contaminated normal (MCN). See Lange and Sinsheimer (1993) for details.

In MSMSN class: multivariate skew-normal (MSN), multivariate skew-T (MSTT), multivariate skew-slash (MSSL2), multivariate skew-contaminated normal (MSCN2). See Zeller, Lachos and Vilca-Labra (2011) for details.

In MSSMN class: MSN, multivariate skew-t-normal (MSTN), multivariate skew-slash normal (MSSL), multivariate skew-contaminated normal (MSCN). See Louredo, Zeller and Ferreira (2021) for details.

In MSMSNC class: multivariate skew-normal-Cauchy (MSNC), multivariate skew-t-Expected-Cauchy (MSTEC), multivariate skew-slash-Expected-Cauchy (MSSLEC), multivariate skew-contaminated-Expected-Cauchy (MSCEC). See Kahrari et al. (2020) for details.

Note: the MSN distribution belongs to both, MSMSN and MSSMN classes.

Value

A vector with the estimated standard errors.

Note

For MSTEC and MSSLEC and distributions, nu is considered as fixed. For MSCEC distribution, nu and gamma are considered as fixed. This function is mainly for internal use.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

References

Kahrari, F., Arellano-Valle, R.B., Ferreira, C.S., Gallardo, D.I. (2020) Some Simulation/computation in multivariate linear models of scale mixtures of skew-normal-Cauchy distributions. Communications in Statistics - Simulation and Computation. In press. DOI: 10.1080/03610918.2020.1804582

Lange, K., Sinsheimer, J.S. (1993). Normal/independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics 2, 175-198.

Louredo, G.M.S., Zeller, C.B., Ferreira, C.S. (2021). Estimation and influence diagnostics for the multivariate linear regression models with skew scale mixtures of normal distributions. Sankhya B. In press. DOI: 10.1007/s13571-021-00257-y

Zeller, C.B., Lachos, V.H., Vilca-Labra, F.E. (2011). Local influence analysis for regression models with scale mixtures of skew-normal distributions. Journal of Applied Statistics 38, 343-368.

Examples

```
data(ais, package="sn") ##Australian Institute of Sport data set
attach(ais)
##It is considered a bivariate regression model
##with Hg and SSF as response variables and
##Hc, Fe, Bfat and LBM as covariates
y<-cbind(Hg,SSF)
n<-nrow(y); m<-ncol(y)
X.aux=model.matrix(~Hc+Fe+Bfat+LBM)
p<-ncol(X.aux)
X<-array(0,dim=c(2*p,m,n))
for(i in 1:n) {
    X[1:p,1,i]=X.aux[i,,drop=FALSE]
    X[p+1:p,2,i]=X.aux[i,,drop=FALSE]
}
##See the simulated regressor matrix X
##X
aux<-estimate.MN(y,X) ##Estimate the parameters for the MN regression model
aux$estimate[,2] ##Estimated standard errors (se) by the estimate.MN function
se.est(aux$estimate[,1], y, X, dist="MN") ##Estimated se for the MN model
```

---

tgamma *Truncated gamma distribution*

Description

Compute the probability density and quantile functions for the truncated gamma distribution with shape and scale parameters, restricted to the interval (a,b).

Usage

```
dtgamma(x, shape, scale = 1, a = 0, b = Inf)
qtgamma(p, shape, scale = 1, a = 0, b = Inf)
```

Arguments

x	vector of quantiles
p	vector of probabilities
shape	shape parameter
scale	scale parameter
a	lower limit of range
b	upper limit of range

Value

dtgamma gives the density function for the truncated gamma distribution. qtgamma gives the quantile function for the truncated gamma distribution.

Note

Auxiliary function to compute the E step for the Slash and Skew-slash models.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller

Examples

```
##probability density and quantile function of the truncated gamma
##model with shape and scale parameters equal to 1
##evaluated in 2 and 0.75, respectively
dtgamma(2, shape=1, a=1)
qtgamma(0.75, shape=1, a=1)
##standard gamma distribution with shape parameter 2 evaluated in 1
dtgamma(1, shape=2)
dgamma(1, shape=2)
```

---

vech — *Vectorize a symmetric matrix*

Description

vech takes the upper diagonal from a symmetric matrix and vectorizes it.

Usage

```
vech(x)
```

Arguments

x	a symmetric matrix.

Value

A vector with the components of the upper diagonal from the matrix, listed by row.

Note

For internal use.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

Examples

```
A<-matrix(c(1,2,2,5),nrow=2)
##vectorized A matrix
B<-vech(A)
B
##reconstitute matrix A using B
xpnd(B,2)
```

---

xpnd — *Reconstitute a symmetric matrix from a vector.*

Description

xpnd reconstitutes a symmetric matrix from a vector obtained with the vech function.

Usage

```
xpnd(x, nrow = NULL)
```

Arguments

`x`	vector with the components of the upper diagonal of the matrix
`nrow`	dimension of the matrix to be reconstitute.

Value

A symmetric matrix.

Note

For internal use.

Author(s)

Clecio Ferreira, Diego Gallardo and Camila Zeller.

Examples

```
A<-matrix(c(1,2,2,5),nrow=2)
##vectorized A matrix
B<-vech(A)
B
##reconstitute matrix A using B
xpnd(B,2)
```

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